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Big Idea 3: Modelers Exercise Judgment

In chapter 7, we talked about managing the open, complex nature of mathematical modeling. Now we turn our focus to the ways modelers exercise judgment, and especially to the ways teachers can give their students opportunities to make consequential decisions throughout the modeling process. Before we dive back into classroom modeling, take a moment to think about the last time you used mathematics in your own decision-making. Maybe a close look at your budget helped you evaluate your progress on your spending or savings goals. Perhaps you estimated how many people your favorite recipe could feed as you considered what to bring to a potluck. In these contexts, and a host of others, the ability to understand and interpret mathematics supports the ability to make judgments about situations, and judgments help us make informed decisions. Now, take a moment to think about decisions your students make. How does quantitative reasoning play into their judgments? When do students learn to use mathematics in their decision-making?

Part of exercising judgment is making decisions. In classroom modeling, both the teacher and the students make decisions. Some decision-making happens because it is a feature of modeling (i.e., the modeler makes decisions as part of the modeling process) and some decision-making happens as a feature of *teaching* modeling (i.e., the teacher makes decisions as part of effectively and efficiently facilitating modeling in classrooms). Although these are distinct types of decisions, together they support Big Idea 3: Modelers Exercise Judgment. In this chapter, we invite you to think about how you position students as the agents for making modeling decisions. Empowering students as decision makers is fundamental to equitable teaching practice (Fernandes, Crespo, and Civil 2017) and is at the heart of what makes modeling distinct from other curricular mathematics. By the end of this chapter you will be able to do the following:

- Describe the relationship between teacher and student decisions in mathematical modeling.
- Identify decisions your students can make as they engage in mathematical modeling.
- Empower students to make decisions in each phase of the modeling process.

Spotlight on Access, Equity, and Empowerment

Making decisions and discussing them as a class helps to build collective mathematical agency. It highlights that "different students can contribute different elements to collective agency" (Aguirre, Mayfield-Ingram, and Martin 2013, p. 17) as their models progress.

DECISIONS MODELERS MAKE

Modelers make decisions in every part of the modeling process. They decide what mathematical question to ask, what to bring into the model and what to ignore, what mathematics to use, and when their model is good enough. Each of these decisions requires modelers to make sound judgments. Modelers exercise and exhibit judgment as they answer questions such as "What is important about this situation, and for whom?", "What values are present in the way the problem is formulated and solved?", "What information will I maintain and what information will I discard as I work on my mathematical solution?", and "How do I know when my solution is good enough?"

These four questions are part of all modeling, including classroom modeling. Secondary school students, supported by their teachers, can participate in asking and answering these questions. In this chapter, we will follow Lule and her eighth-grade students as they make decisions in a modeling task that is motivated by their interest in school sports.

> Lule is developing a task during which students make predictions through modeling, while tapping into the content of linear functions that is central in eighth-grade mathematics. Several students in her class run on the eighth-grade cross-country team, and she has often overheard students in class discussing their races. Lule is a coach of the high school cross-country team, and every year the coaches must enact a strategy for selecting the seven runners who will represent the school at the state meet. Lule considers how this strategic decision-making relies on making assumptions about the speed of runners, on choosing fairly, and on communicating the decision-making process transparently. She decides that the selection strategy for choosing runners for the school cross-country team will make an engaging context for her students.
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> As she thinks about the modeling task, Lule knows the problem will engage students in learning about linear functions. The races throughout the season are of different lengths, and the state meet is a 2.5-mile race, so comparing runner speed is more complex than just ordering runners' times from lowest to highest. Lule expects that her students will predict times that runners would achieve for a 2.5-mile race, and she knows they can use historical data to check their predictions. She is also aware that the problem can be tackled with a statistical model, and she will have the opportunity to revisit the same context with different content later in the year.

Teachers consider both the content and the context when formulating modeling lessons. Because her purpose was to address linear functions through modeling, Lule developed a task that she knew was well suited to it.

Her knowledge of the context matters, too. If all races were the same length, then comparing runners' speeds would be the same as comparing their times. Because she knows that the races are of different lengths, she anticipates that students will use the length of the race and the time to run to compute a runner's speed, using *distance* = *rate* × *time*. The *distance* = *rate* × *time* equation is a simple linear model, with a *y*-intercept of 0.

What Is Important, and What Do We Value?

The process of composing teams for high school sports is routine, and yet each team is the result of a coach's decisions. In chapter 3, we suggested that modeling tasks can address issues of justice by considering the equitable or inequitable outcomes of decisions. Composing a team might not seem like it is about justice, but consider the kinds of judgments modelers might make as they decide on a strategy for composing a team of runners. What does it mean to be the fastest runner? Is a runner's speed determined by their fastest time, or their average time? Is it consistency that matters? If an average is used, should times that are abnormally slow due to illness or injury be discarded? Each of these judgments can change the outcome of the decision, and to ensure fairness, the process of making that decision should be transparently communicated. The state team selection process, which at the outset seems to value *speed*, upon further inspection relies on the underlying values of *fairness* and *transparency*. Being aware of how choices are connected to what is deemed important helps teachers link mathematics to sound judgment and informed decision-making.

> **Spotlight on Access, Equity, and Empowerment**
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> When teachers consider values embedded in mathematical choices, they help students see that their perspectives matter.

When she launches the task with students, Lule is careful to describe the authentic situation while leaving the act of posing the mathematical problem as the responsibility of the students. Lule wants to ensure that students concentrate on the important aspects of the situation as they pursue solutions, so she makes use of the client strategy. By positioning herself as the client of the model, students will be able to consider her point of view directly.

Lule presents the problem to the class:

> "I am a coach of the high school cross-country team. The team has 12 runners who are among the fastest, but only 7 can represent the school at the state meet. How do I decide which 7 runners to assign to the state team?"

She asks the students to brainstorm in their small groups about some possible solutions. As she circulates around the room, students record their ideas. Among their ideas, they record these:

- Choose the 7 runners who have had the fastest times over the whole season.
- Choose the 7 runners who have had the 7 fastest average times.
- Choose the 7 runners who had the 7 fastest times in the most recent race.

Lule chooses to focus the whole class's attention on these three ideas because they demonstrate different interpretations of what makes a runner "fastest" over the course of a season. As the students discuss potential selection strategies, she asks questions and makes comments to focus their attention. She asks, "Is consistency more important than a one-time exceptional performance?" Some students think it is. She remarks, "Runners can peak during a season, so does it matter if someone's fastest race was at the beginning of the season instead of the end?" Students discuss this idea for a little while and agree that there is no clear-cut answer.

Lule knows that deciding how to form the state team is not straightforward, and her goal with this model is to focus the students on a deterministic approach in which an individual runner's speed is arrived at with some assumptions, and then the team is selected on the basis of whose times are determined to be the fastest. From a mathematical point of view, any one of the methods proposed for determining a speed to assign to a runner would work (fastest single time, average time, or most recent time), but in the context of the solution, each of those choices would result in a different team of seven runners. Lule wants students to examine how the different assumptions about what makes someone the fastest will result in a different decision about the team. Students could make other reasonable assumptions, too, such as averaging times over the last half of the season, or dropping a runner's slowest time and averaging the remaining times; but students did not generate these through their initial brainstorming. Lule is content to leave those options unexplored, because the three options that students have proposed provide plenty of rich mathematical approaches.

Lule wants to be sure that students acknowledge the values that are embedded in each of their assumptions, so next she tells students that because she is a coach who would use the model, she is the client and wants them to recommend a strategy for team selection that she can share with her fellow coaches. "Are there questions you'd like to ask about what is important to me when making this kind of coaching decision?" In the approaches they have brainstormed, students have placed a value on a fast team. But she knows there are additional values that are a part of selecting runners for the state meet, and she would like to see those highlighted. Students each write down as many questions as they can think of in three minutes, and then take turns asking their questions. Students ask a variety of questions, and she can tell they are having fun interviewing her.

Some of their questions are about what happens in various races:

"Does it matter if someone gets sick and doesn't compete?"

"If a runner had a bad race because they fell in the mud one time, do you hold that against their average time?"

"Is the same person always the fastest on the team?"

"How much does the order of finishing change from race to race?"

"What course during the season is most similar to the course that the runners will be running for the state meet?"

Some of their questions are about fairness:

> "What if a fast runner doesn't have good attendance at practice—do you still want them to represent the school at the state meet?"
>
> "Is it fair to put a runner on the team who hasn't been as fast on average as someone who isn't put on the team?"
>
> "Will runners get mad if they aren't selected for the state team?"

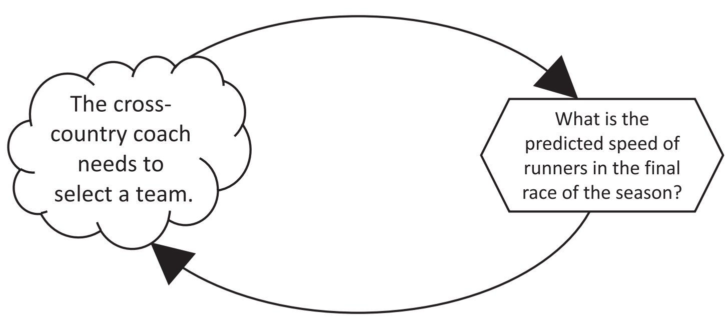
Lule is not surprised that students ask about fairness, or that students wonder if the top seven runners are always the top seven. In fact, she knows that the top four are almost always the top four, and the next five runners are consistently in the top nine, but not always in the same order. It is the last three spots that are hardest to assign, and this is what she is able to articulate by taking on and representing the role of the coach. Speed, fairness, and transparency are all values highlighted in the whole-class discussion.

What Information Do I Need? What Mathematical Tools Will I Use?

Lule gives students enough information to get started but does not provide all the facts that might be pertinent to the situation. Lule's choices empower students as decision makers and as modelers who will shape the direction of the task and its solution. In fact, the decision that is at the heart of the issue becomes the posed problem. Students have translated her initial prompt, "How should the coach select 7 runners for the state team?" to the mathematical question, "What is the predicted speed for each of the 12 runners on the team, and who are the 7 fastest runners?" (figure 8.1). This marks the *pose a mathematical problem* phase of the mathematical modeling process.

FIG. 8.1.

In the *pose a mathematical problem* phase of the modeling process, Lule's class translated her initial prompt into a mathematical question.



Students work on a model that will predict a racing time for each of the 12 runners in consideration and then order the 12 runners based on that time. Lule decides to give students data from the team's regular season four years ago, with runners given pseudonyms. Because it is real data, she can understand some of the quirks of it—a runner was sick, a runner fell, a runner peaked in the middle of the season—and that is the advantage of using her empirically collected data to assign the parameter value of speed that is at the focus of the model.

Students work in groups with different assumptions to build their models, and then they compare their models. Lule organizes the students in their usual groups of 3 students (she has a class of 30 students), and groups choose which assumptions they will use in their models. Some groups assign each runner's speed as their average time, and other groups assign each runner's speed as their fastest time. One group decides to assign speed as the average time over the second half of the season. Regardless of what assumptions they make about how to assign a speed, each group computes a prediction of race times for the 12 runners for a 2.5-mile race and orders the runners to identify the 7 fastest.

In facilitating classroom modeling, teachers can guide students as they encounter mathematical content that is the focus of the lesson. Lule ensures that each group computes a unit speed and a total time, even as they explore different model assumptions in order to arrive at that unit speed. She provides opportunities for groups to present their developing models to the whole class.

How Did Our Decisions Lead to Different Solutions?

Lule asks each group to articulate what their assumptions are in determining a runner's speed. For example, the groups computing an average speed have to decide what to do with a missed race. Groups take turns to explain how they are ensuring that the process of identifying a runner's speed is fair and that all of their assumptions are made apparent.

Next, each group explains what is included and what might be "lost" in their solution. One group says a model that disregards a race time earned when a runner falls down might lose the ability to capture a runner's nimbleness. Lule points out that this is a consequence of ignoring elements of authentic experience in formulating a mathematical model and that it is part of the modeling process. Another group wonders if their model ignores fatigue. After some discussion, the class decides that using real two- to three-mile race times to calculate average minutes per mile actually takes fatigue into account. They note that they could not use their averages to predict race times that were much longer, like a marathon, or much shorter, like a single-mile run. A third group has considered terrain and used races that are most like the state course in computing their runners' average. The last group explains that at first they wanted to exclude data from races run in bad weather, but then they decided that since no one knows what the weather will be like on the day of the state meet, they should compute their averages for races in all kinds of weather.

The final step for Lule's class is for each group to use the historical data to predict a 2.5-mile race time for each of the 12 runners and compare their prediction to the runner's actual state meet time. Each group's set of predictions is different from the others because of the different assumptions that went into their models. The students do not have a complete set of data to compare to, because they only know the final times of the 7 who raced in the state meet, not the 12 who might have raced. But because each group has computed a time for the 2.5-mile race, they can see how close each way of predicting race time came to being accurate for the runners whose data they have.

To conclude the modeling process, each group writes a report detailing their model, including a paragraph comparing the predicted times to the actual times. Each report addresses the basic model, *time = distance/rate*, along with an explanation of how the rate was assigned for each runner. By explaining their decisions, the students agree that their models are both fair and transparent. Lule shares these reports with the coaches of the high school team.

TEACHER CHOICES AND MODELER DECISIONS

Teachers have a great deal of discretion when they develop and implement modeling tasks. They make choices about how to set up and launch tasks, what information to provide and withhold, how to organize and monitor students, and how to bring classroom activities to a close—to name just a few! In modeling, teacher choices that give students opportunities to make decisions and exercise judgment are particularly salient (table 8.1). After all, learning how to make good decisions using mathematics requires practice with making decisions in the context of learning mathematics.

Table 8.1. Teacher Choices That Support Student Decisions

When Teacher Facilitators . . .	Student Modelers . . .
. . . involve students in using mathematics to plan and carry out school-related activities, then	. . . use mathematics to make decisions *and* experience the results of those decisions.
. . . make the values that inform judgments explicit, then	. . . practice putting values into action.
. . . selectively withhold information that might be helpful in formulating a model, then	. . . decide what information is important in their model and what information can be discarded.
. . . give options with respect to manipulatives and techniques, then	. . . learn to select mathematical tools because they are useful.
. . . give students strategies to discuss and evaluate models, then	. . . see how varied decisions lead to different solutions.

Teacher Choices

Teachers decide how to keep the class moving forward in modeling while still engaging students in the phases of the modeling process. They decide how to describe the authentic situation to students in a way that captures the important features of the situation but without posing the mathematical problem for students.

For example, Lule was careful to describe the situation of team selection, without indicating selecting fastest runners or fastest average times, which are mathematical choices. Teachers decide which mathematical approaches to pursue as a class. Lule had considered ahead of time ways that students might tackle the problem and knew that any collection of ideas that included computing a unit speed would address the mathematical content she was targeting. Teachers decide how to provide students with the data they need to pursue a solution. They decide how to assign groups to work on problems, and how long students should work in groups. Lule had developed a group work structure as a routine in her classroom, so this was a natural part of how her class proceeded in modeling and nonmodeling lessons. Teachers also decide which mathematical ideas to revisit later in the year to address different mathematical content. Lule knows that the variability in the speed students assign to each runner is an excellent problem to tackle once the class has studied statistics.

Modeler Decisions

Modelers make decisions about how to represent the authentic situation mathematically, and they decide which elements of the situation the mathematical solution must address. Lule's students value speed and investigated different ways to assign a speed to a runner. They also value fairness and transparency, and these values are reflected in their explanations of how they went about assigning a speed to a runner. Modelers articulate assumptions made in their models. Lule's students made judgments about how to handle a missed race in assigning a runner a speed.

SUMMARY

One of the biggest challenges and biggest opportunities in mathematical modeling is the way it extends the responsibility for consequential decision-making to students. Positioning students as mathematical decision makers requires teachers to navigate the tension between being careful, intentional planners and responsive facilitators. When they are modeling, students use their mathematical skills, identify the values embedded in a problem, and exercise judgment as they examine solutions to problems. The modeler emerges from the modeling process knowing that different perspectives of a problem highlight different points of view that different people hold. A complex problem does not have a right or wrong answer; instead, it has a problem statement and a proposed solution that capture something important about what another human being values. Two different models of the same situation can be useful, and these different models capture the perspectives of different human beings. A student who understands this tenet has learned empathetic critical thinking skills.

REFLECT AND DISCUSS

Continue to work with the mathematical problem you identified in chapter 7.

1. When will your students have opportunities to exercise judgment as they represent and explore the mathematical problem?
2. Write down a few questions you could ask to help students decide what is important in the problem.
3. It can be tempting to jump in and make decisions for students. How will you avoid this pitfall? What strategies will you use when you feel as though students need support making decisions? Identify one or two choices from table 8.1.
4. How might student choices lead to different solutions? How will you help students notice and discuss those differences?
5. Lule wanted to be sure that her students acknowledged the values embedded in their choices and invoked her role as the client. What strategies will you use to ensure that your students are considering the values of their decision-making?