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Modeling with Geometry: What is it and how is it used in mathematical modeling?

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Let’s warm up…Think, Pair, Share

1. How many peanut M&Ms are in the jar?

- How do you know?
- How confident are you in your answer?
- What questions do you have?

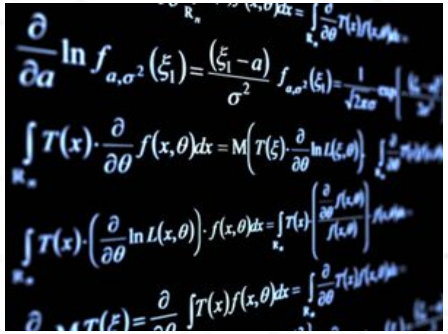
2. What is geometric modeling?

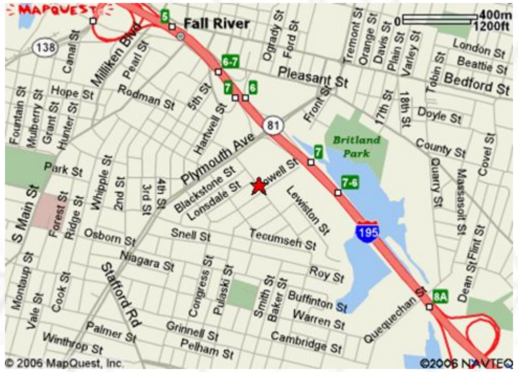
Discuss your ideas with a partner.

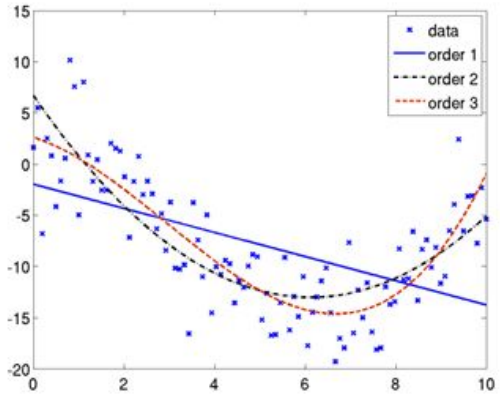
Let’s Back Up…

What is a model?

A *model* is a representation of a target that capture features or aspects of the target but is not identical to the target.







What is a mathematical model?

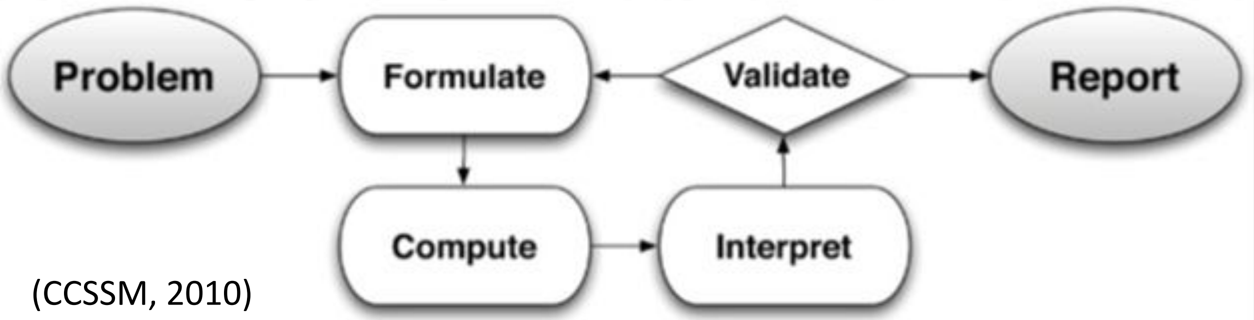
A *mathematical model* is a set of ideas that describe a process, pattern, or phenomenon in the real world expressed using mathematical objects.

We build mathematical models to explain, predict, or control some process, pattern, or phenomenon in the real world.

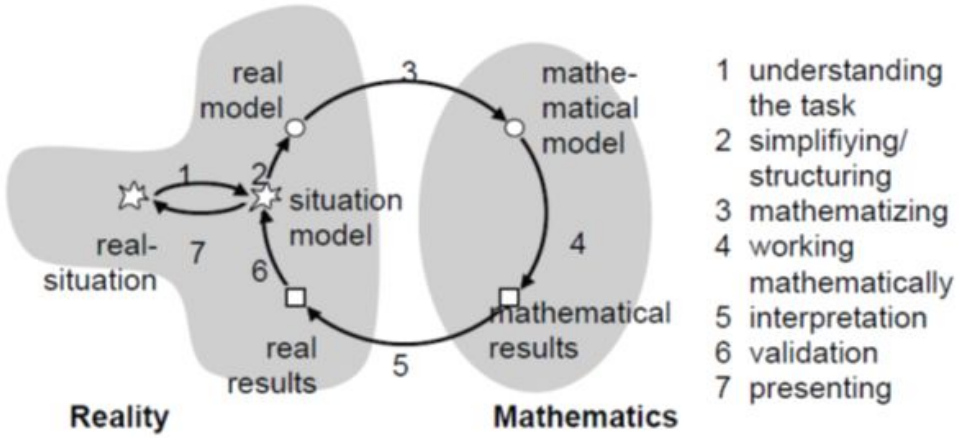
What is mathematical modeling?

- Mathematical modeling is the process of constructing, analyzing, and revising mathematical models.

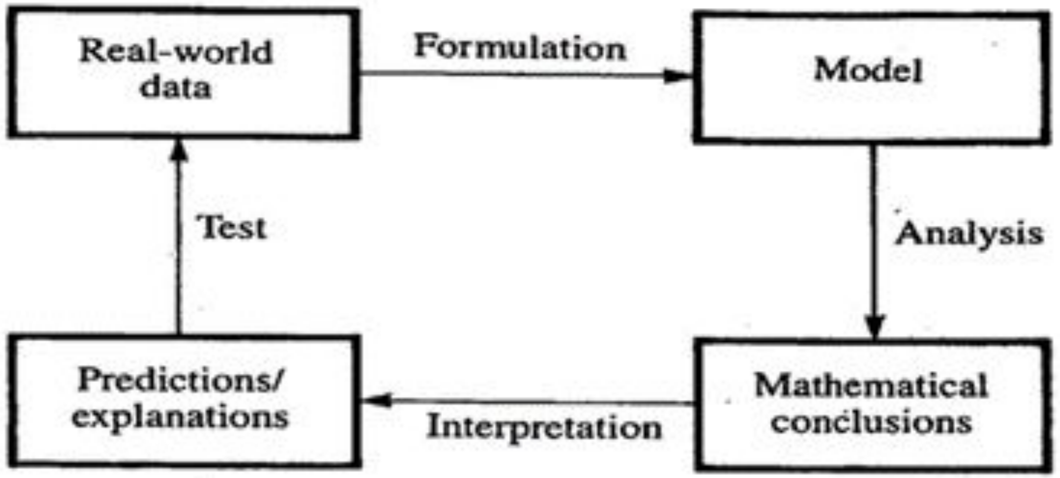
Mathematical Modeling Depicted as a Process/Cycle



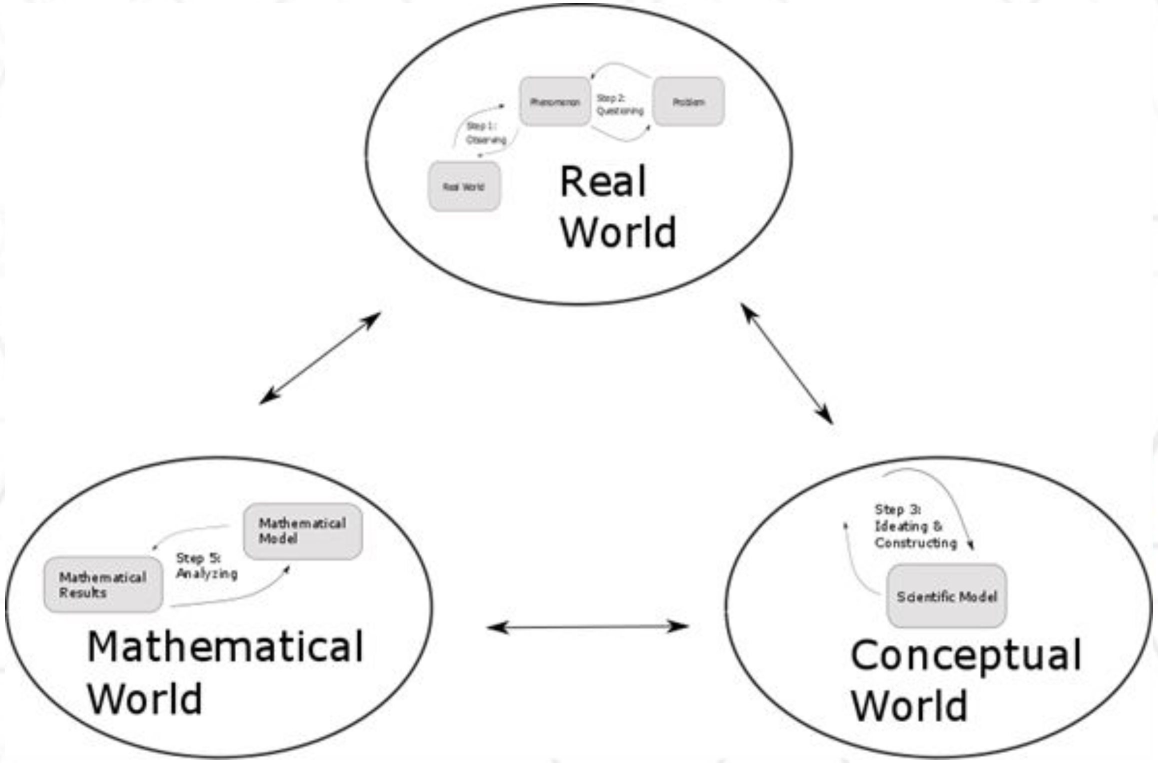
(CCSSM, 2010)



(Blum, 2011)



(Dossey et al., 2002))



From Cirillo & Pelesko (2022)

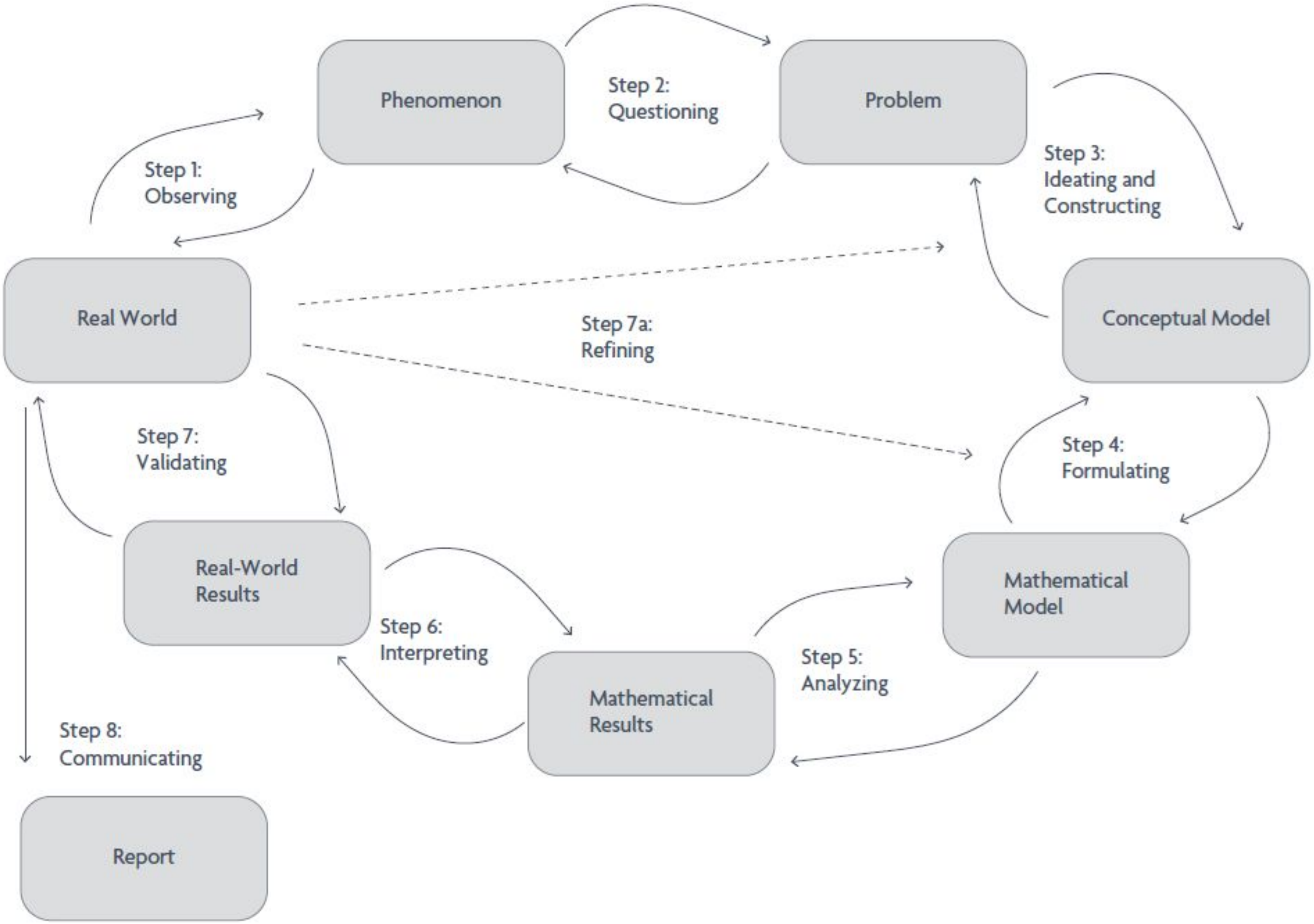


Figure 3–13
Our depiction of the mathematical modeling cycle.

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What is geometric modeling?

From CCSSSM:

Apply geometric concepts in modeling situations

CCSS.MATH.CONTENT.HSG.MG.A.1
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

CCSS.MATH.CONTENT.HSG.MG.A.2
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

CCSS.MATH.CONTENT.HSG.MG.A.3
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

What is geometric modeling?

From Cirillo & Pelesko (2022):

- Geometric modeling alone is not the same as mathematical modeling.
- Rather, it is a valuable tool that helps us represent real-world phenomena by making simplifying assumptions about real-world objects.
- **Geometric modeling is the act/process of making simplifying assumptions by choosing a geometric object to represent a real-world object to help the modeler formulate a mathematical model as part of the mathematical modeling process.**

Geometric Modeling Situation #1

You’re All Wet!

Problem: *How should one dry oneself when they get out of the swimming pool?*

(Davies, 1978)

Problem: *How should one dry oneself when they get out of the swimming pool?*

- When drying ourselves after getting out of the swimming pool, people tend to do so in a definite order.

- How do you dry yourself?

Drying Order	Number of People
Torso, arms, legs	11
Torso, legs, arms	7
Arms, torso, legs	38
Arms, legs, torso	5
Legs, arms, torso	5
Legs, torso, arms	4

Table 12–1
Preferred drying order data from our informal survey.

Problem: *How should one dry oneself when they get out of the swimming pool?*

- When drying ourselves after getting out of the swimming pool, people tend to do so in a definite order:

  head, arms, then torso, then legs.

Two Hypotheses

Hypothesis #1

Drying order is the result of people trying to minimize their total heat loss, meaning, the order is chosen so as to minimize the total amount of thermal energy that the body loses through being wet.

Hypothesis #2

It's not the total energy loss that is minimized, but rather the total discomfort, where discomfort is some measure of the temperature drop the person experiences.

A situation for geometric modeling…

- From day to day, our total surface area is roughly constant.
- What matters is drying ourselves quickly.
- It turns out that change in temperature is not simply dependent on the surface area of the body, but instead is dependent on the ratio of the surface area to the volume A/V.
- The two hypotheses differ in this regard.
- The task of the modeler is to explore whether this difference in dependence leads to a difference in the ability of the two hypotheses to explain the observed drying order.

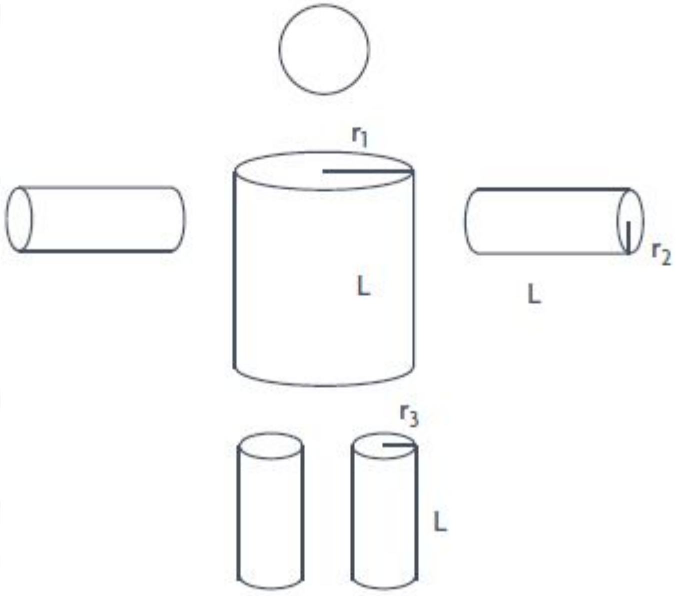

Figure 12–1
Our approximate human form.

A situation for geometric modeling…

Parts	Surface Area	Volume	Drying Time
Two arms	$4\pi r_2 L$	$2\pi r_2^2 L$	$4\pi r_2 L/R$
Two legs	$4\pi r_3 L$	$2\pi r_3^2 L$	$4\pi r_3 L/R$
Torso	$2\pi r_1 L$	$\pi r_1^2 L$	$2\pi r_1 L/R$

Table 12–2
Surface area, volume, and drying time for various body parts.

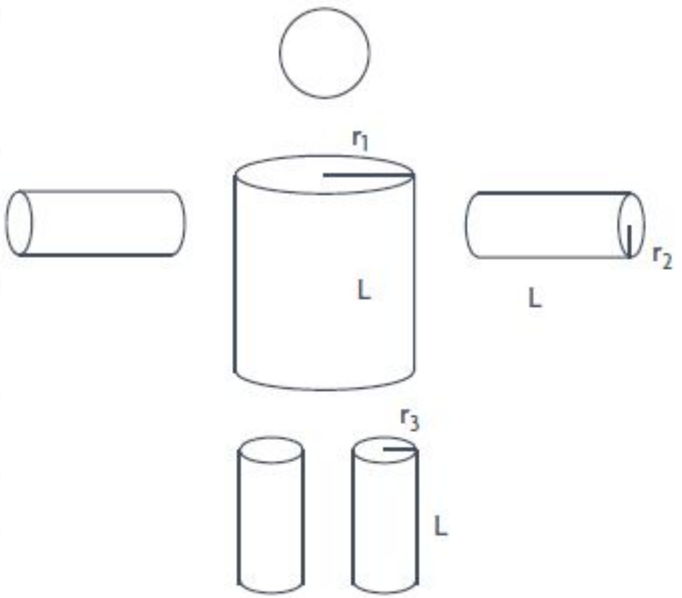

Figure 12–1
Our approximate human form.

Geometric Modeling Situation #2

A Hot Topic

A Hot Topic

- Modelle is an aficionado of hot sauce.
- When Modella sent him to the grocery store, he noticed the prices of different sized bottles of the same flavor of hot sauce.

Container Size	Price	Price per Ounce
⅛ ounce	$0.55	$4.40
2 ounces	$1.90	$0.95
64 ounces	$21.50	$0.34
128 ounces	$40.95	$0.32

Table 4–2
Modelle's revised table of data on hot sauce.

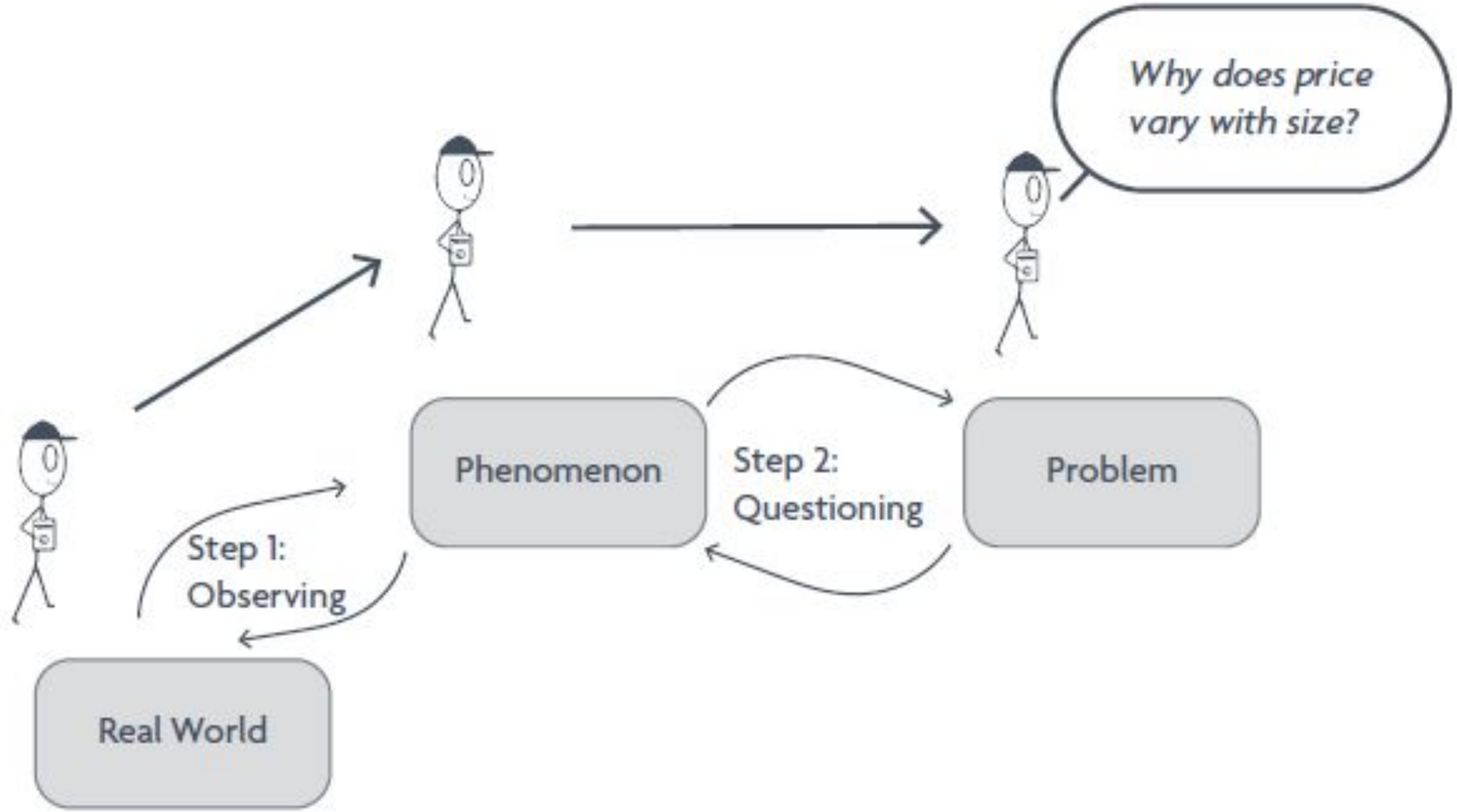

Figure 4–3
Modelle's first few steps through the modeling process.

Modelle Makes a Simplifying Assumption to Explore this Phenomenon

How might Modelle use geometric modeling in service of mathematical modeling?

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Modeling Competency: *Choosing Functions and Other Mathematical Objects*

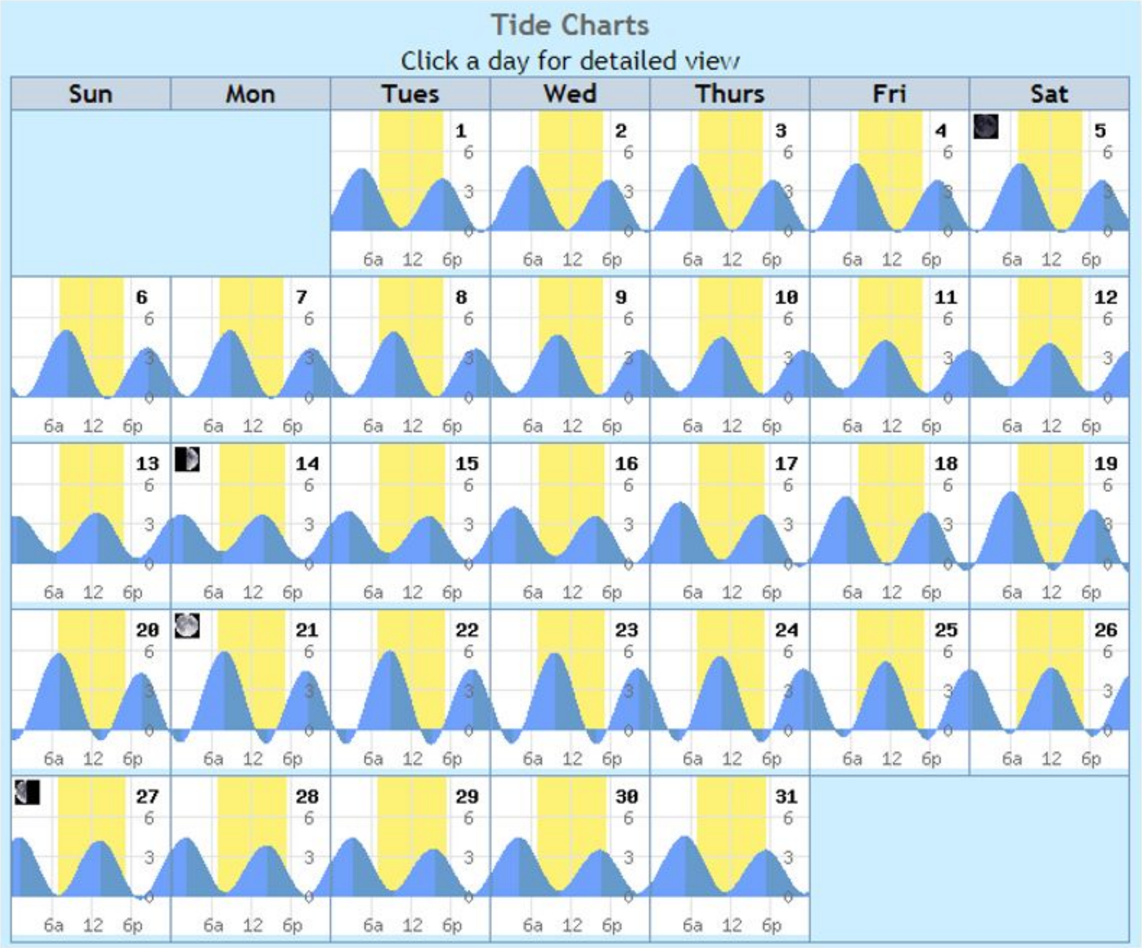

Figure 9-16: A graphical representation of the tidal data represented in Table 9-8 from tides.net.

Geometric Modeling Situation #3

Candy Estimations

Let’s Revisit the M&M Jar

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How might we think about this task as a mathematical modeling task?

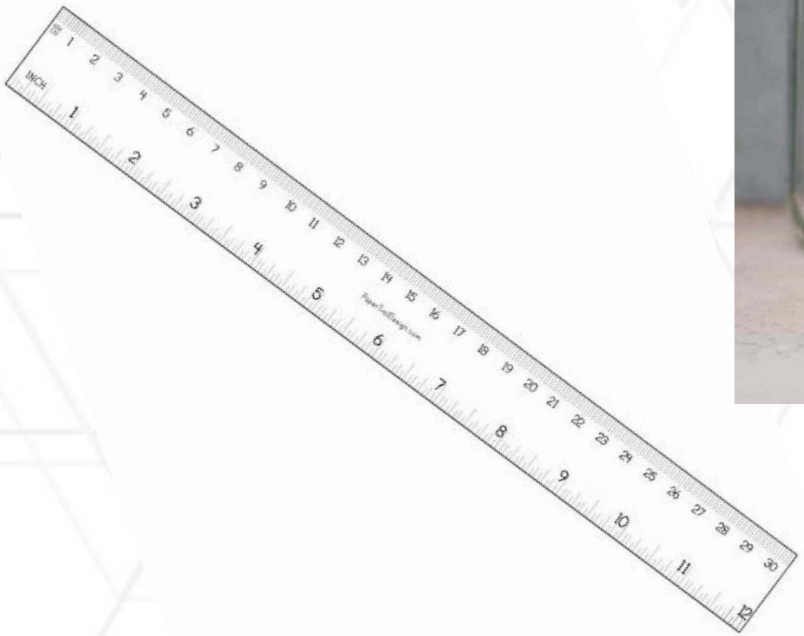
How might we use geometric modeling in service of mathematical modeling?

Suppose we wanted to build a mathematical model that would help us predict how many M&Ms are in *any* sized jar.

How would you do it?

How might we conduct experiments to solve this problem?

What questions would you want answered?



Conducting Experiments can be Part of the Modeling Process

Conducting Experiments can be Part of the Modeling Process

m&m	diameter	radius	Volume (cm^3)
1	1.5	0.75	1.77
2	1.75	0.88	2.81
3	1.7	0.85	2.57
4	1.3	0.65	1.15
5	1.5	0.75	1.77
6	2	1.00	4.19
7	1.2	0.60	0.90
8	1.7	0.85	2.57
9	1.8	0.90	3.05
		mean =	2.31

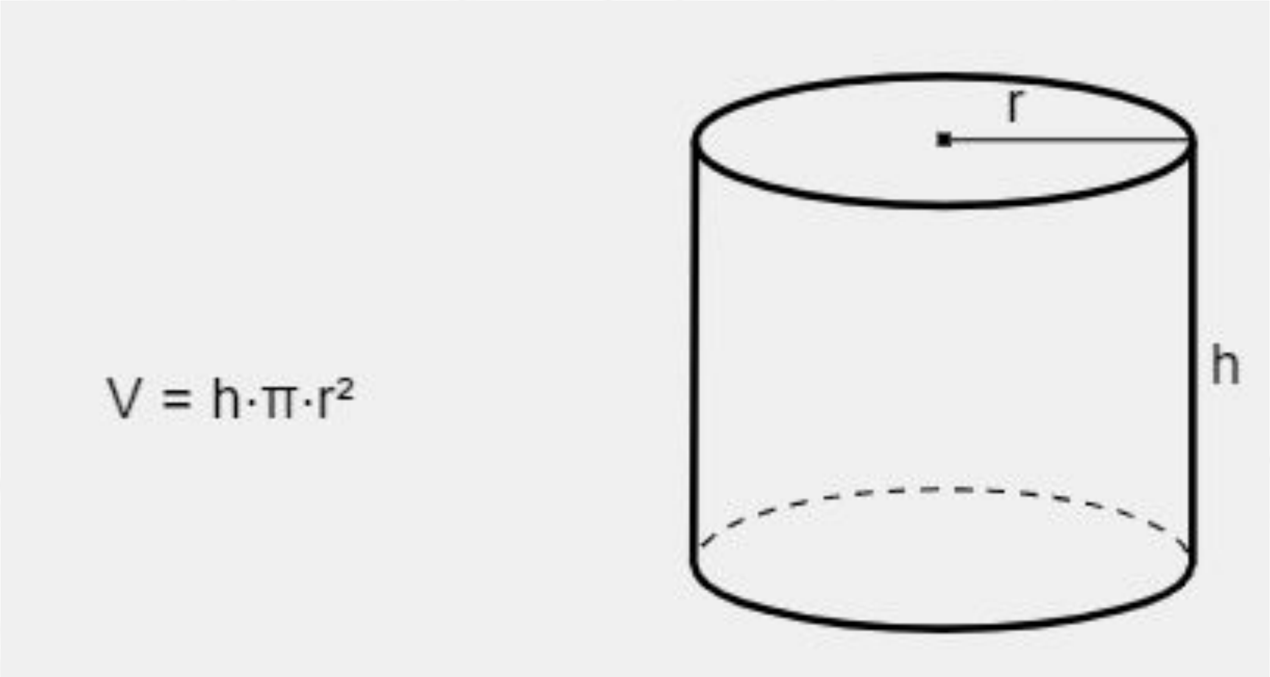
Conducting Experiments to Build My Model

Jars	radius	height	Volume (cm^3)
Small	3.25	6	199.098
Medium	3	11.25	318.086
Large	3.75	13.5	596.412

Building a mathematical model

of m&ms in any jar =

$$\frac{\pi {r_1}^2 h_1}{2.31} = m$$



Model Predictions versus Actual m&ms

Mathematical Model for the Number of m&ms that could fit into any sized jar:

Volume (cm^3)	# of m&ms (predicted)	# of m&ms (actual)
199.098	86	40
318.086	138	70
596.412	258	160

What am I missing?

Accounting for the Void!

What percentage of a jar do you think accounts for the void?

Model Predictions versus Actual m&ms

Mathematical Model for the Number of m&ms that could fit into any sized jar:

Volume (cm^3)	# of m&ms (predicted)	# of m&ms (actual)
199.098	86	40
318.086	138	70
596.412	258	160

What am I missing?

46%

51%

62%

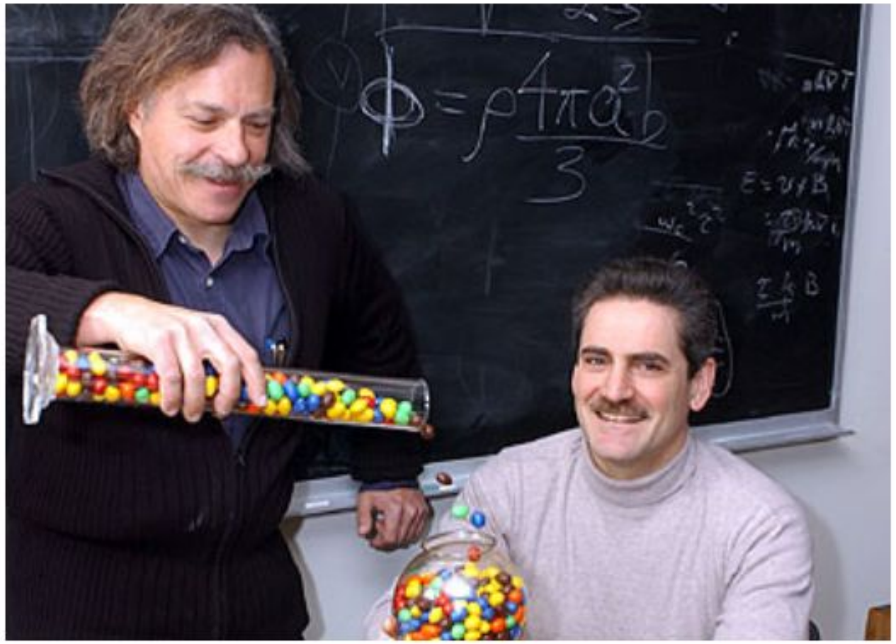
Refining My Mathematical Model

Particle Packing - A Long-Term Scientific Interest

- The study of particle packing dates to the 16th century when physicist and mathematician Johannes Kepler investigated ordered arrangements of spheres.
- It was not until 1998 that a mathematician proved that the densest possible arrangement of spheres fills 74.04% of the total space, as Kepler had predicted.
- The packing of randomly assembled particles is less well understood.

Taking the Void Into Account

For equal spheres in three dimensions, the densest packing uses approximately 74% of the volume. A random packing of equal spheres generally has a density around 63.5%.[1]



Paul Chaikin (left), professor of physics, and Salvatore Torquato, professor of chemistry, used M&M's candies to reveal fundamental principles governing the random packing of particles at Princeton University. Photo: Denise Applewhite (2004)

For most people, a regular lunch of M&M's and coffee would lead to no good. For Princeton physicist Paul Chaikin and collaborators, it spurred fundamental insights into an age-old problem in mathematics and physics.

Chaikin and Princeton chemist Salvatore Torquato used the candies to investigate the physical and mathematical principles that come into play when particles are poured randomly into a vessel. While seemingly simple, the question of how particles pack together has been a persistent scientific problem for hundreds of years and has implications for fields such as the design of high-density ceramic materials for use in aerospace or other applications.

The researchers discovered that oblate spheroids, the shape of M&M's chocolate candies, pack

Improving the Density of Jammed Disordered Packings Using Ellipsoids

Aleksandar Donev,[1,4] Ibrahim Cisse,[2,5] David Sachs,[2] Evan A. Variano,[2,6] Frank H. Stillinger,[3] Robert Connelly,[7] Salvatore Torquato,[1,3,4*] P. M. Chaikin[2,4]

m&ms poured randomly occupy 68% of the volume

The structure of liquids, crystals, and glasses packings, as are the transitions between these

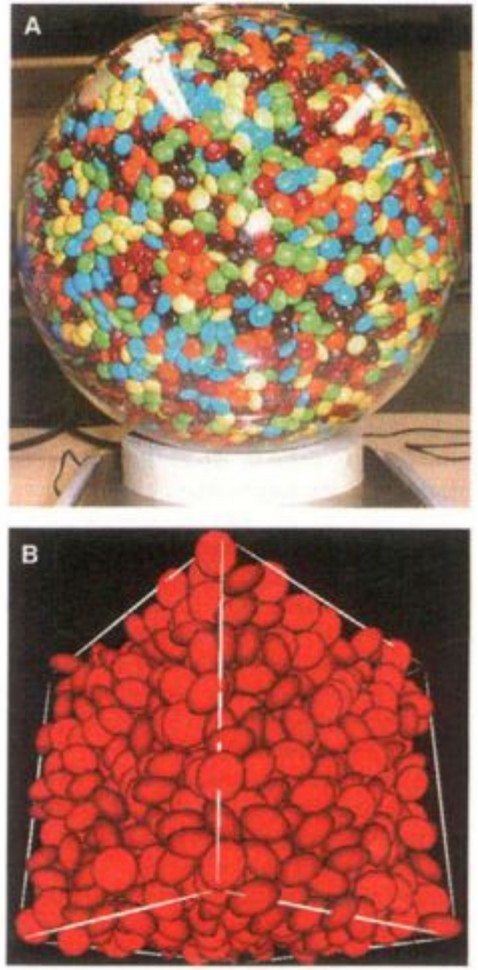


Fig. 1. (A) An experimental packing of the regular candies. (B) Computer-generated packing of 1000 oblate ellipsoids with $\alpha = 1.9^{-1}$.

MUNDO DILBERT

HOW MANY M&M'S ARE IN THE JAR?

08/08/2004 KIRSCH 77 COMMENTS

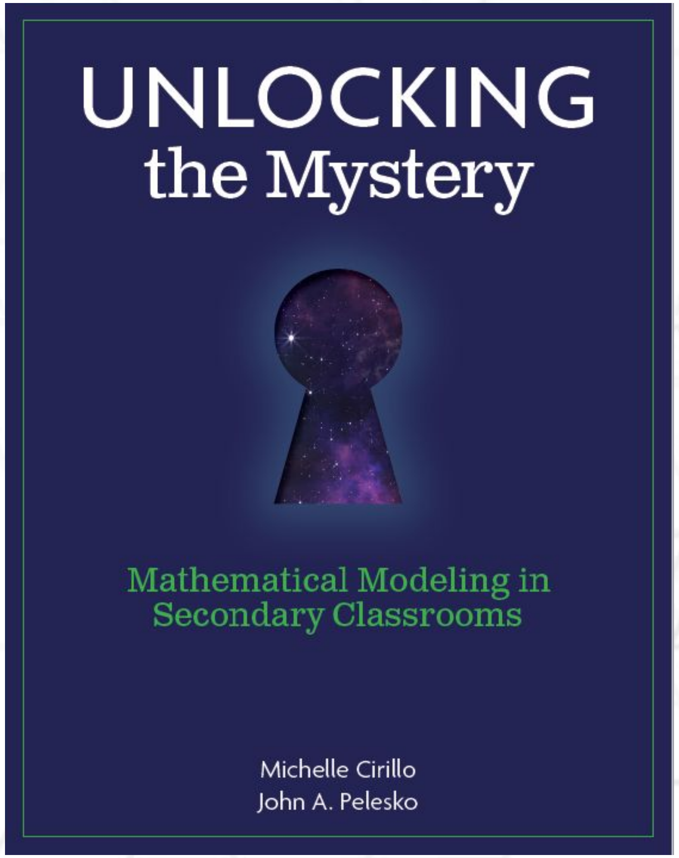
Last Thursday, there was a company-wide party. That morning, they put a big jar full of M&M's (regular, not peanuts) at the lobby of the Truchard Design Center. You were supposed to estimate how many M&M's were in there. Whoever makes the best guess, wins a $50 Gift Certificate at Best Buy (and everlasting glory).Continue reading for details on the complicated process involved in this chocolatey adventure.**Step 1: What is the size of an M&M?**

This is the jar full of m&m's

Thank you for attending!

Unlocking the Mystery:

Mathematical Modeling in Secondary Classrooms



(Cirillo & Pelesko, 2022)

- Part I: *The Power of Mathematical Modeling*
- Part II: *Unlocking the Mystery of Mathematical Modeling*
- Part III: *Building a Foundation for Mathematical Modeling in the Classroom*
- Part IV: *Developing a Classroom of Mathematical Modelers*

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