75 minutes **The Staircase Problem: Consecutive Sums**

Materials:

- Vocabulary Chart: *Words that help me describe my mathematical thinking:*
- Centimeter grid paper for each participant
- Extended T-chart reproducible- 2 per participants (blank and Sums of 2 Consecutive addends/odd numbers)
- (6) 1-inch grid chart paper (Chart Paper)
- (30) 6-by-18 construction paper strips
- Numeral cards 1-30
- Newsprint
- Color Tiles bags per table (100 tile with 5 different colors)
- White labels for recording consecutive addends (4 per table)
- Powerpoint Presentation: NCTM_2023 Consecutive Sums
- Number Pattern Charts for Multiples of 2, 3 5, Powers of 2
- Literature Books: *Roosters off to See the World; Ten Black Dots, Five Little Monkeys (optional: Anno's Counting Book, and Anno's Multiplying Jar)*

Overview and Rationale:

Number sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world applications. In natural language many times we use numbers as adjectives but, in the mathematics landscape, numbers become nouns and they possess their own properties such as prime, composite, odd/even, multiples/factors, addends/sums, and more. In this problem-solving investigation participants will have the opportunity to engage in an in-depth study of patterns, number relationships, and properties that lay a foundation for future mathematics learning. This investigation will emphasize depth in mathematical thinking rather than superficial exposure to a series of isolated and fragmented topics. Relational understanding (knowing what to do and why) will be experienced by the participants through the rich interconnected web of mathematical concepts, ideas, tools, and language the Consecutive Sums/Staircase problem embodies. This web of interconnectedness will span the grade K-8 spectrum of mathematics learning.

It is important to emphasize the role mathematics plays in the real world and how the web of connected/ related information develops the disposition in a learner to find the sense in things and to consider the implications of choices and decisions that will impact their world.

Often students are moved very quickly from numbers 1-100 into the thousands and millions. As soon as students can count and identify numbers, they are immersed in algorithmic procedures. They learn rules and procedures and can get answers to problems that are already organized for them. However, we need to ask ourselves:

- What do students understand?
- What attitude are students developing toward mathematics?
- What do they believe about their ability to make sense of mathematics?
- Do they trust in their thinking?
- What evidence of student learning and understanding do we have to guide instruction?

During this investigation, participants are asked to critically examine the mathematical experience and the use of manipulatives to answer the above questions. The task requires interaction, discourse, and generalizations, a blank vocabulary chart titled "***Words that help me communicate my mathematical thinking:***" will be posted so that vocabulary being used to explain and describe discoveries can be written so learners can begin to use these terms in context. Possible words to develop, use, apply, and generated are:

generalization	relationship	finite Difference	formula
consecutive	even/odd	multiple	algorithm
addend	expanded T-chart	growth pattern	proof
sum	prime numbers	factors	

Important Mathematical and Pedagogical Ideas:

- The ability to **create, recognize,** and **extend patterns** is essential for **making generalizations.**
- Looking for patterns and enlisting **mathematical tools** to discover these patterns is a disposition that encourages students to persist in their problem solving.
- **Data analysis** requires a **visual display** of data to search for hidden patterns and relationships between numbers.

- **Representing a pattern both geometrically and numerically** helps students recognize a variety of relationships in pattern and make connections between arithmetic and geometry. (NCTM Standards p. 61)
- Manipulative materials help students make abstract ideas concrete and offer students the vehicle to reason with proof.
- Active learning creates discourse and language development as students find the need to communicate their findings or describe relationships emerging.

Directions:

1. Introduce ***Rooster's Off to See the World*** *and* ***Ten Black Dots.*** Talk about the question posed at the end of the book Ten Black Dots (*Count them are there really ten?)* Then show the growing staircase at the corner of each page as the characters enter the story in ***Rooster.*** Again, talk about what is being communicated about consecutive numbers. Much is assumed in both stories. Contrast this with *Five Little Monkeys* that models conservation of number yet uses the same melodic count of 1, 2, 3, 4, 5; 5, 4, 3, 2, 1. (Optional: Anno's Multiplying Jar/factorals; Anno's Counting Book/place value)

2. Introduce the Staircase Problem by reviewing a set of numbers:

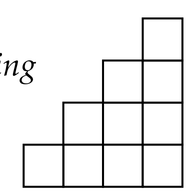
   I am going to show you a set of numbers. In your group discuss what these sets of numbers have in common.

3 4 5 6	**11 12 13 14**	**1 2 3 4**	**23 24 25 26**

(Possible comments…they are in counting order; they increase by one, they are in sequence with a difference of 1; each number is 1 more than they previous number; they are in increasing pattern; the two end numbers add to the sum of the middle numbers;)

These numbers describe a growth pattern. We have just described its growth. Each set of numbers is increasing by 1. If I build the set of numbers 1,2,3,4 with tiles, what will the design look like?

Build the design using one color of tiles:



What will the next step look like?
What is the difference between each step?

3. Have participants build the staircase. Participants are seeing pattern visually with color tiles and they can predict the number of squares in the next step. The visual representations allow learners to determine that each step increases by one so the next step will be one more than 4.

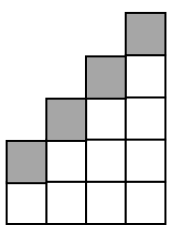
We can say that this staircase is increasing by a difference of 1. This growth pattern is growing in consecutive order.

All together we used 10 tiles to represent each number in consecutive order so we can say that 10 is the sum of four consecutive addends.

(*Record: 1 + 2 + 3 + 4 = 10)*

4. Ask participants to turn and tell a partner how the next 4-step staircase will look like and how many tiles it will take to build it as the sum of four consecutive addends.

5. Have participants share how they went about figuring out the total number of tiles. Use the original staircase and build the next staircase using a different color of tiles:



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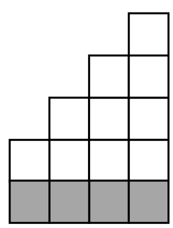


Record: *2 + 3 + 4 + 5 = 14*

Modeling Mathematical Tools

- Build a model
- Represent model with mathematical equation

If I made a listing of the equations that are consecutive addends for a 4-step staircase, my listing would look like this:

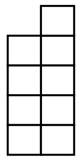
Record on chart paper: *1 + 2 + 3 + 4 = 10*
2 + 3 + 4 + 5 = 14

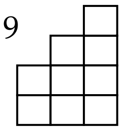
- Make an organized listing.
- Identify the difference between **sums** in the increasing number pattern.

Label the mathematical tools that have been used to describe and discover sums of consecutive addends.

6. Pose a part of the problem or a similar but smaller problem.
Let's investigate 9 as the sum of consecutive addends. Can 9 be built into a staircase that grows in consecutive order? Get 9 tiles and build a staircase.

Note: There are two ways that 9 can be built as the sum of consecutive addends:

4 + 5 = 9 

2 + 3 + 4 = 9 

7. Present the problem to be solved:

    Using color tiles find all the different ways the numbers from 1-30 can be built into staircases as sums of consecutive addends. Record your staircases on grid paper. Label each staircase with an equation that describes the number as a sum of consecutive addends. (This is an important point in the investigation)

    Find patterns that can help you predict possible arrangements…*Remember, the power of pattern is to predict beyond the physical evidence.*

    Decide how your group will divide the work to complete the task and how you are going to represent your discoveries on one large sheet of paper.

    Write summary statements describing the patterns you found in your investigation and give examples to support these statements.

8. Pass out directions for each pair of participants to read and get clarity of task. Circulate and observe the process. As you observe and circulate among groups notice how groups decide who does what. It may help to jot down differences you see groups approach the task for the concluding discussion. At this time you may observe patterns in organizing tiles to find sums of consecutive addends. Note this down, too. As you walk around, listen for vocabulary being used by participants. Jot them down on the chart:

Words that help me describe my <u>mathematical thinking:</u>

9. After 10 minutes, bring group to whole group discussion. Ask them if they have found any patterns. (Most of the time, participants find out that all odd numbers are sums of two consecutive addends.) This is the time to model your expectations for the group recording.

***What summary statement or generalization can we make about sums of 2 consecutive addends?* (Model shared writing)**

Your directions tell you to write summary statements describing the patterns you found in your investigation and give examples to support these statements.

EX: Record summary statement with data to support it.

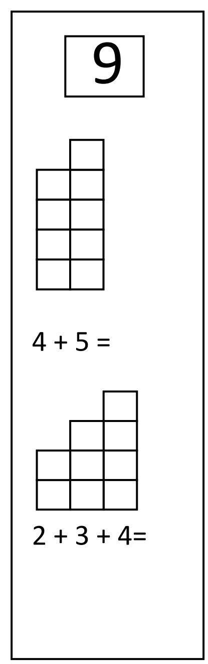
(Optional- use acetate to cover the numbers on the 1-100 pocket chart to show pattern emerging and allowing participants to predict the next sum. Have 1-100 miniature charts for participants to color in)

All odd numbers are sums of 2 consecutive addends.

1 + 2 **= *3***
2 + 3 = ***5***
3 **+** *4* **= *7***
4 **+** *5* **= *9***
5 + 6 = ***11. . .****and so on (until everyone is convinced that the statement is true)*

<u>Making a Class Graph</u>

10. Make a class graph. (Have number 9 ready to show on large inch grid paper and then cut and create a strip with all possible staircases. Each number is represented on the class graph. (Numbers 1-30 are printed or written on 3″x 3″ squares of paper). The class graph is displayed so that later in the processing all participants may verify findings with the data on the graph.



As you work with your group to write summary statements I will be passing some number cards and your group will represent each number given to the group as staircases with the mathematical equation that describes the number as a sum of consecutive addends so that we can have a class graph with this data.

11. Offer the challenge of finding out why some numbers can be built into staircases and some are impossible.

12. **Processing of the Learning.**

Ask participants to reflect on how the work was done in their group. How was the work divided? Inquire into the different approaches they took in the investigation. Some may say that they first started individually trying to figure out the task, then joined in the group discussion. Others might share that they began right away working cooperatively. It is important that participants become aware of the conditions that allowed them access to this learning experience. These same conditions are necessary for students.

- *What were your experiences as a learner?* **Individually record your response to this question. Once everyone has had time to record their responses, time will be given to share with the group.**

- **Mathematical Content:** ***What discoveries did you make while investigating numbers that can be built into staircases as sums of consecutive addends?***

Have group share their findings. As one group shares one discovery or pattern then ask if any other group discovered that same pattern and ask them to read how they described it. (NOTE: the language we use to describe our understandings is dependent on our previous experiences. Utilizing one's personal language fosters confidence in one's knowledge. Also, it models the different ways that one concept can be described and represented.)

Do this until you have exhausted all the possible ways of describing the specific pattern. Do this with other discoveries. This will also be the time that participants will share the difficulty they had in expressing or describing their patterns. Carl Luty's quote is good to share. Post summary statements next to graph. Have participants put their work in their folder for later use.

Luty's quote: **". . .we don't write to display understanding, but to acquire understanding. Writing teaches. That simple fact explains why students need as much writing in the content areas as possible. The process of composing their thoughts moves students away from the muddle of isolated facts toward the order of integrated knowledge. That's usually called <u>understanding.</u>**

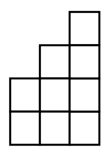
The Staircase Construction

Sums of Consecutive Addends

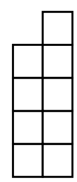
You need: color tiles, grid paper, large newsprint paper, markers

<u>Group Task</u>

- using color tiles find all the different ways numbers 1-30 can be built as staircases and can be described as sums of consecutive addends;
- record staircases on grid paper and represent each staircase with a mathematical equation to show the number as the sum of consecutive addends. **Example:**



2 + 3 + 4 = 9



4 + 5 = 9

- find patterns that can help you predict possible arrangements…

  <u>Remember:</u>

  The power of pattern is to predict beyond the physical evidence. When will you feel the power of pattern?

- decide how your group will divide the work to complete the task and how you are going to represent your discoveries on one large sheet of paper;
- <u>write a summary statement</u> describing the patterns you found in your investigation and <u>give examples</u> to support these statements.

Note: **Some numbers can be represented in only one staircase;
some numbers can be represented in more than one way;
and finally, some numbers are <u>impossible.</u>**