SCAN Me

slide deck

Torres' Rights of the Learner

A Framework for EMPOWERING ALL Students in Mathematics

Crystal Kalinec-Craig
Olga G. Torres

NCTM Annual Meeting & Exposition
Oct. 25-28, Washington, DC

*Creating Spaces for Change Through Community: It Starts With **You***

Introductions

Building community.
Empowering students.
Disrupting stereotypes.

What does it mean to empower students?

What does it mean to empower students?

empower. . . make someone more stronger and more confident, in controlling their life and claiming their rights.

empowered. . . having the knowledge, confidence, means, or ability to do things or make decisions for oneself.

What are the learning conditions you need to do your best work?

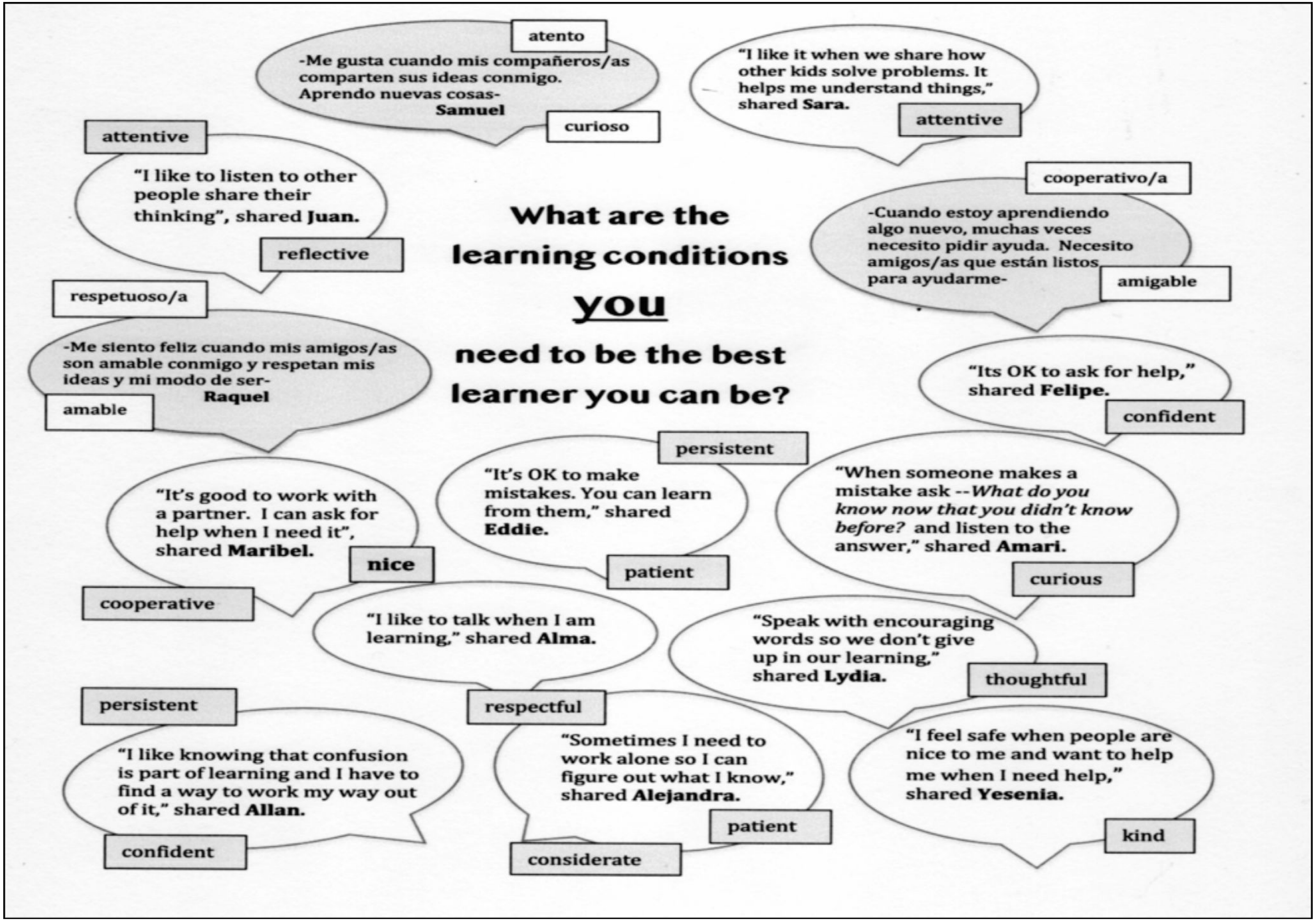
what are the learning conditions you need to be the best learner you can be?

Use the link to record what learning conditions you need to do your best work as a math learner.

Menti.com
Code: 29127458

Show results and talk to your partner about what you see and notice

- https://www.mentimeter.com/app/presentation/al6hc42so97in54z11y8c3v35v8sbsfv


What are the learning conditions you need to be the best learner you can be?
atento
-Me gusta cuando mis compañeros/as comparten sus ideas conmigo. Aprendo nuevas cosas- Samuel
curioso
"I like it when we share how other kids solve problems. It helps me understand things," shared Sara.
attentive
attentive
"I like to listen to other people share their thinking", shared Juan.
reflective
cooperativo/a
-Cuando estoy aprendiendo algo nuevo, muchas veces necesito pidir ayuda. Necesito amigos/as que están listos para ayudarme-
amigable
respetuoso/a
-Me siento feliz cuando mis amigos/as son amable conmigo y respetan mis ideas y mi modo de ser- Raquel
amable
"Its OK to ask for help," shared Felipe.
confident
persistent
"It's OK to make mistakes. You can learn from them," shared Eddie.
patient
"When someone makes a mistake ask --What do you know now that you didn't know before? and listen to the answer," shared Amari.
curious
"It's good to work with a partner. I can ask for help when I need it", shared Maribel.
nice
cooperative
"I like to talk when I am learning," shared Alma.
"Speak with encouraging words so we don't give up in our learning," shared Lydia.
thoughtful
persistent
"I like knowing that confusion is part of learning and I have to find a way to work my way out of it," shared Allan.
confident
respectful
"Sometimes I need to work alone so I can figure out what I know," shared Alejandra.
patient
considerate
"I feel safe when people are nice to me and want to help me when I need help," shared Yesenia.
kind

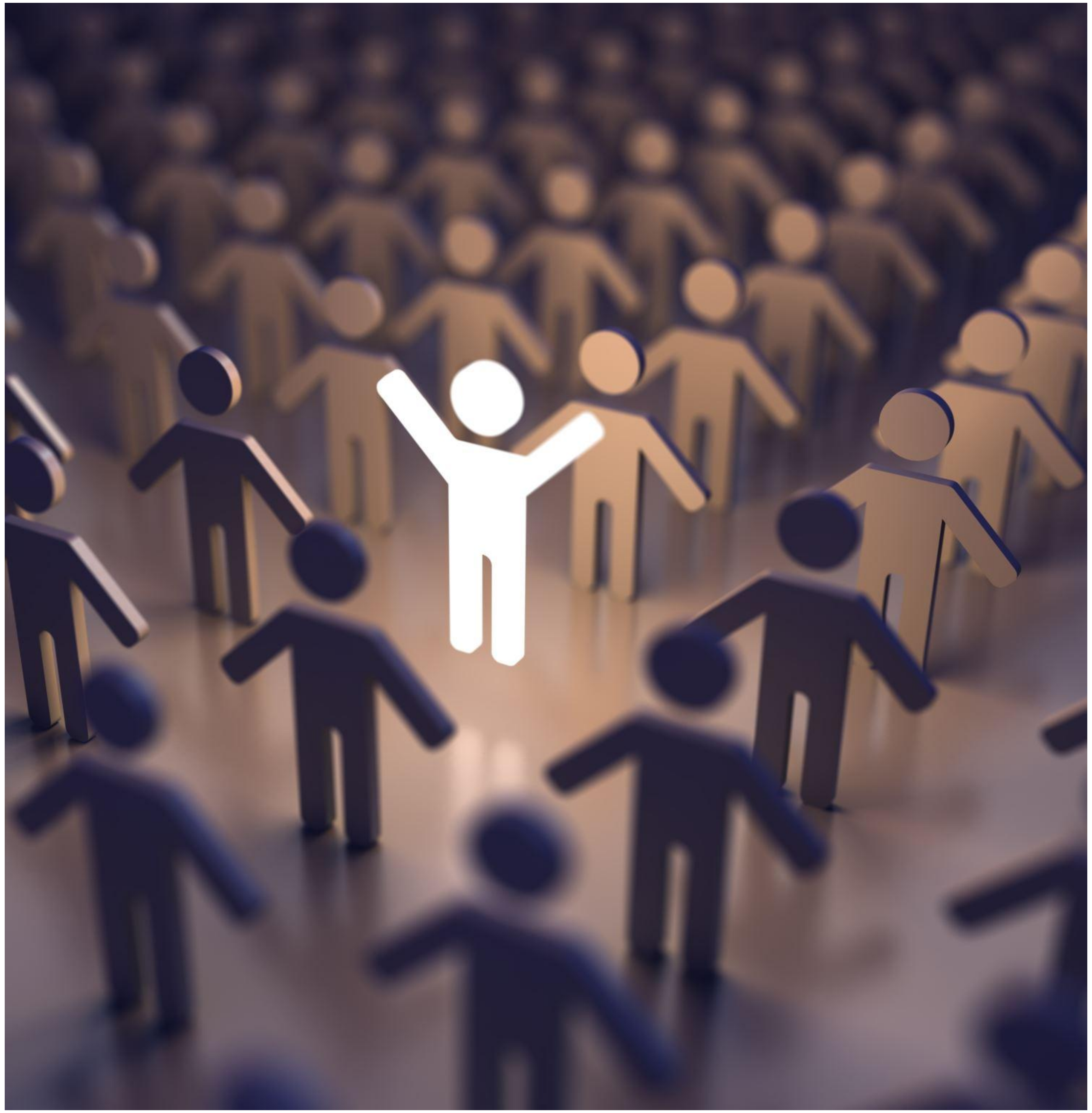
The Rights of the Learner

All human beings are born free and equal in dignity and rights.

They are endowed with reason and conscience and should act toward one another in a spirit of brotherhood.

Declaration of Human Rights (1948)

Article 1

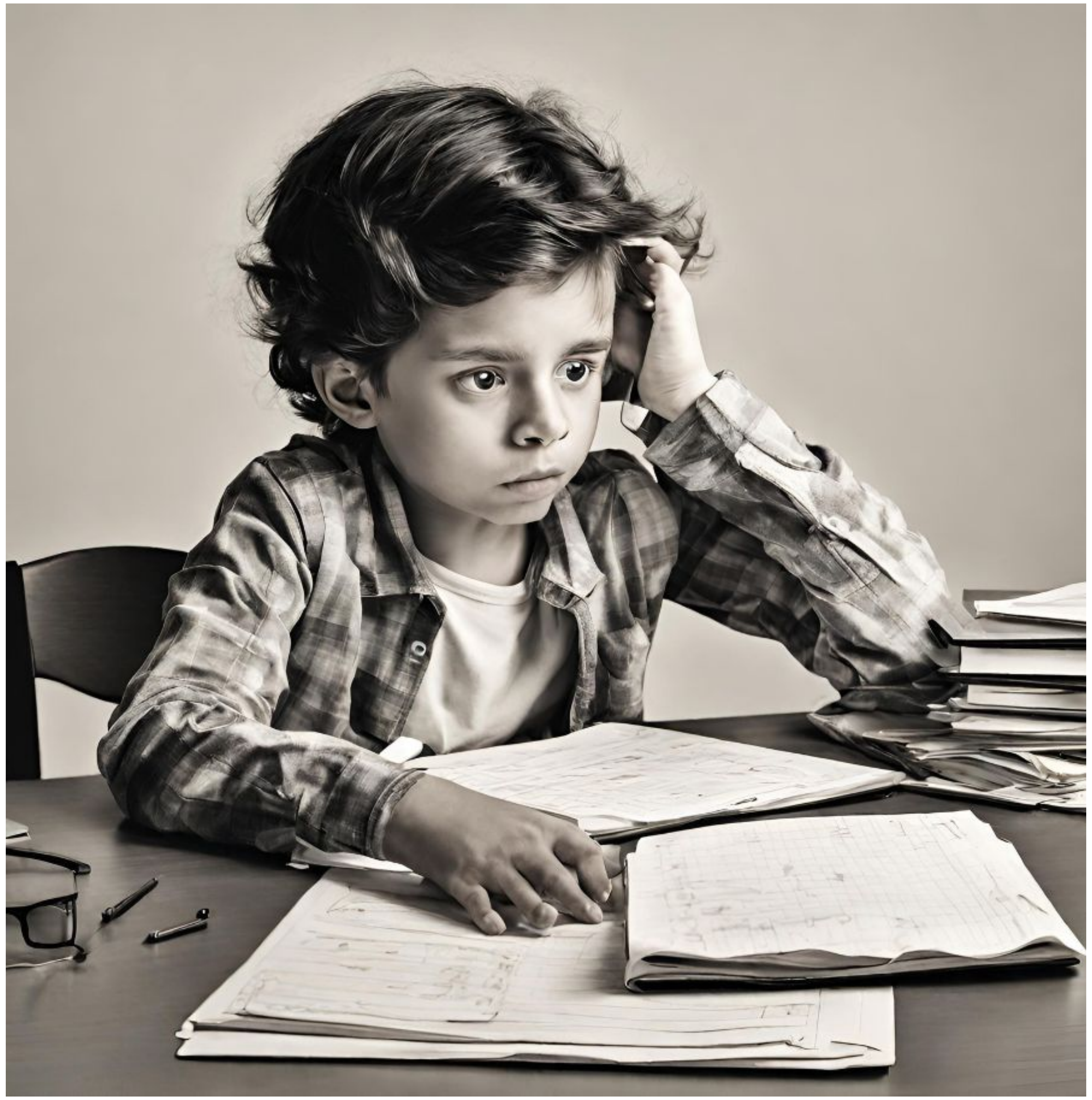


The right to freely express an opinion in all matters affecting him/her and to have that opinion taken into account.

The UN Convention on the Right of the Child (1989) Article 12

You have the right

to be confused



You have the right

to claim a mistake and revise your thinking

You have the right

to speak, listen, and be heard

You have the right

to write, do and represent only what makes sense to you

You have the right

to feel safe and respected

Use the link to record MORE RIGHTS that we can add to this list.

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There is always a language experience approach when teaching and learning of mathematics.

In a *democratic* learning community with equity and social justice permeating throughout every citizen is given a voice in the governance of the community and feels a sense of ownership for the mathematics learning that goes on in the classroom.

***There is always a language experience approach when* teaching and learning of mathematics.**

In a *democratic* learning community with equity and social justice permeating throughout <u>every citizen</u> is given a voice in the governance of the community and feels a sense of ownership for the mathematics learning that goes on in the classroom.

Our job as teachers is TO DISCOVER YOU.

A Framework for Empowering ALL Students in Mathematics:
An Equity and Social Justice Perspective

A ***perspective*** is a way of thinking (growth mindset) that encompasses values and beliefs that:

- guide what a teacher will pay attention to,
- how students' needs will be accommodated,
- how instruction will be differentiated to meet and enhance the needs of students, and
- to anticipate the knowledge and skills students will need to connect new ideas and concept to prior knowledge.

In a ***democratic*** learning community with equity and social justice permeating throughout <u>every citizen</u> is given voice in the governance of the community and feels a sense of ownership for the mathematics learning that goes on in the classroom;

A ***language experience approach*** is in effect in the teaching and learning of mathematics.

What does research say.

"…**the roots of the term**
education
imply
drawing out children's <u>potential</u>
and making them more than they were."

-Professor Mary Ashworth
University of British Columbia
Speaking at the Ontario TESL Conference and quoted by
Jim Cummins **<u>Empowering Minority Students</u>**

Mathematically speaking,
this definition encourages one to consider the mathematical stance of ***making more***
which implies that there has to exist an *<u>equality factor</u>* before ***more*** can be determined.

This ***<u>equality factor</u>*** implies that the foundational core of the instructional curriculum begins with ***students' knowledge, skills, and experiences.***

. . . . **Empowerment**
derives from the process
of negotiating identities in the classroom.

Identities are not static or fixed
but rather are constantly being shaped
through
experiences and interactions.

Quote taken from **Negotiating Identities: Education for Empowerment in a Diverse Society by Jim Cummins (p.1 and 15-16)**

Task

The Staircase Problem

What numbers from 1-30 can be represented as sums of consecutive addends?

Investigate all the ways the set of numbers assigned to your group can be represented as sums of consecutive addends. What discoveries will you make to help you answer these questions:

- What numbers can be sums of **two** consecutive addends?
- What numbers can be sums of **three** consecutive addends…**four** consecutive addends…and so on?
- Record your equations for all numbers that can be represented as sums of consecutive addends.

The Staircase Problem

Investigating Sums of Consecutive Addends

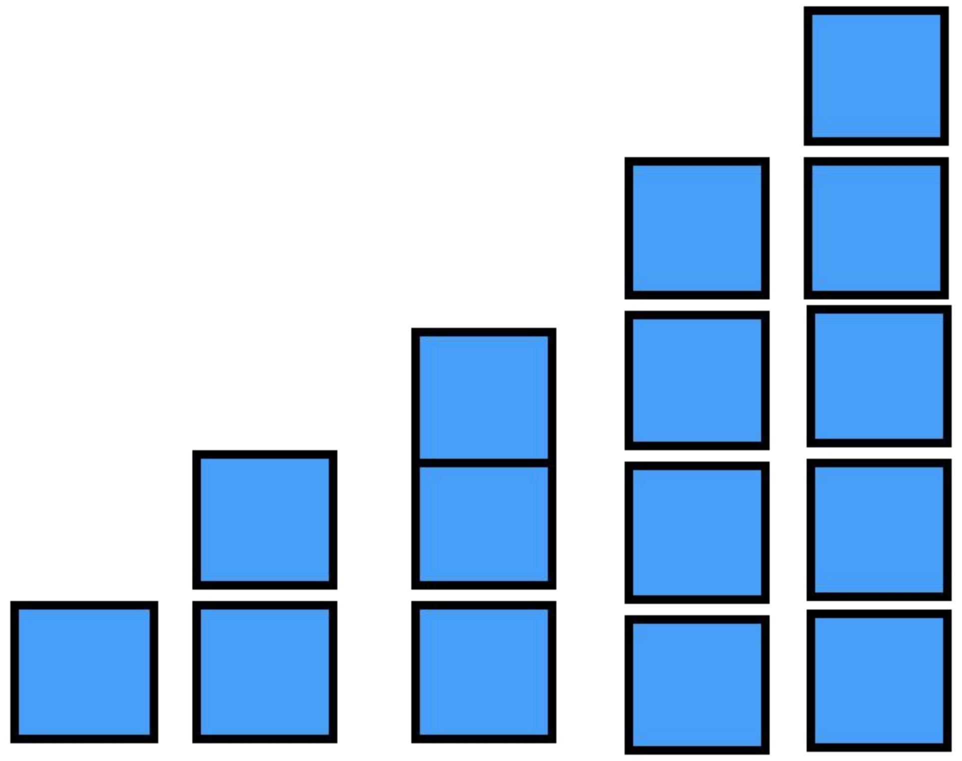
Five Little Monkeys Jumping on the Bed by Eileen Christelow

The doctor said,
"No more monkeys jumping on the bed!"

1 + 4 = 5
2 + 3 = 5
3 + 2 = 5
4 + 1 = 5
5 + 0 = 5

Additive expressions of equivalence for 5.
Conservation of number.

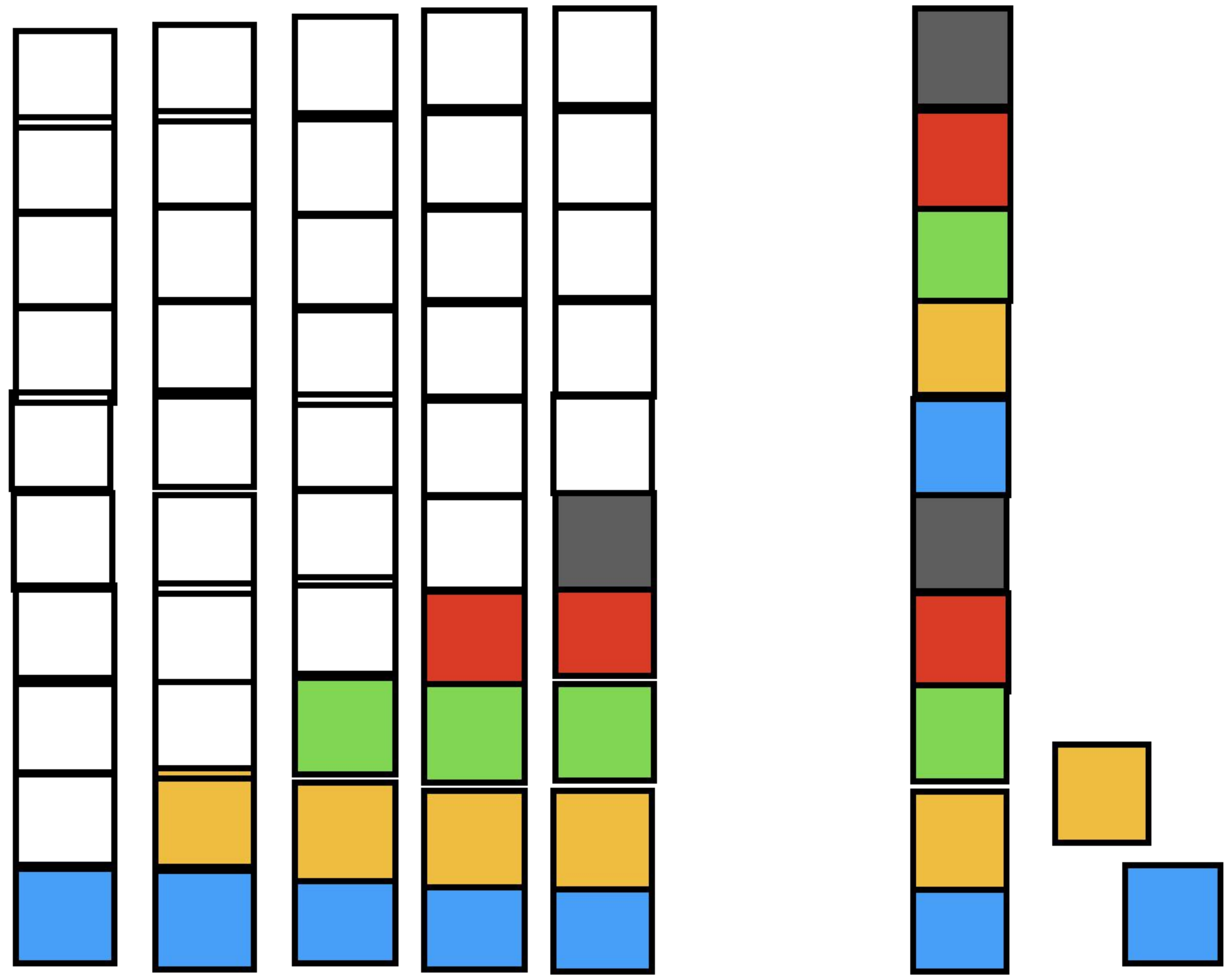
Rooster's Off to See the World By Eric Carle



1 + 2 + 3 + 4 + 5

The author introduces the reader to the growth pattern of consecutive numbers.

Anno's Counting Book
By Mitsumasa Anno



1 2 3 4 5 . . . 12

Digit value in relationship to 10 . The book introduces the decimal system.

Mathematics Task:

"What humans do
with the language of mathematics
is to describe patterns.

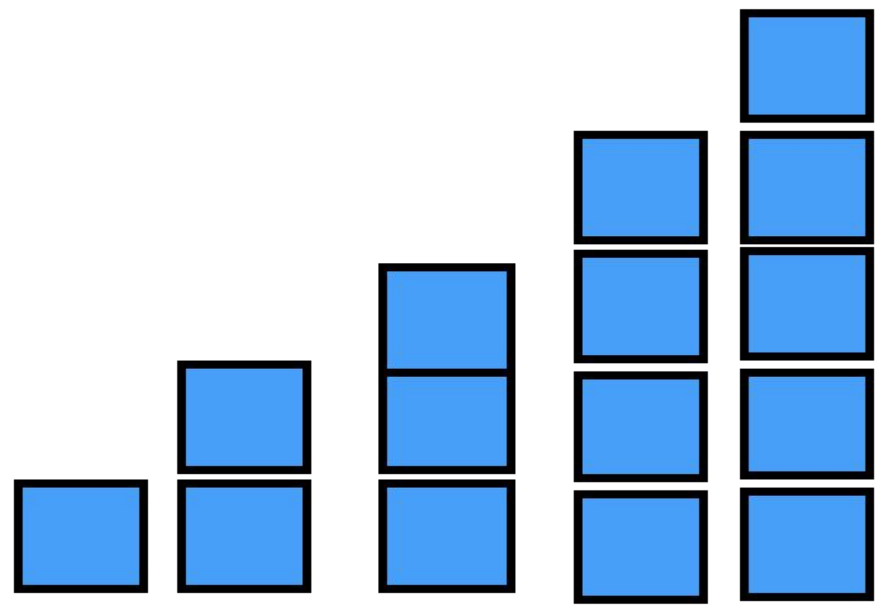
Mathematics is
an exploratory science that seeks to understand
every kind of pattern. . ."

To grow mathematically,
children must be exposed to a rich variety of patterns
. . . in their own lives through which
they can see variety, regularity, and
interconnections.

- *Lynn Arthur Steen*
- *On the Shoulders of Giants: New Approaches to Literacy (1990)*

1	2	3	4	5

If I build each number in a set of 5 consecutive numbers with tiles/cubes, what will the design look like?



1 + 2 + 3 + 4 + 5 = 15

What will the next step look like?

What is the difference between each step?

15 is the sum of 5 consecutive addends.

How many ways can you represent 15 as the sum of consecutive addends? Will you discover a pattern to support your findings?

The Staircase Problem

What numbers from 1-30 can be represented as sums of consecutive addends?

Look for patterns that can help you predict possible arrangements.
Talk with group members about how the square tiles help you visualize possible staircases for specific numbers and answer these questions:

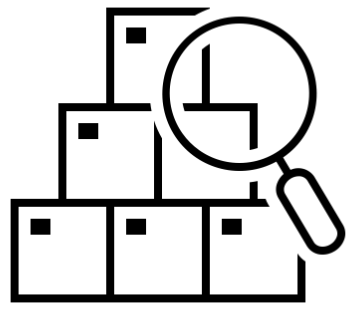
- Why can some numbers be done only one way?
- Why can some numbers be done in more than one way?
- Why are some numbers impossible?

Remember, the power of pattern is to predict beyond the physical evidence.

The Staircase Problem

What numbers from 1-30 can be represented as sums of consecutive addends?

Look for patterns that can help you predict possible arrangements and write summary statements about the patterns you find.



Talk with group members about how the square tiles help you visualize possible staircases for specific numbers and answer these questions:

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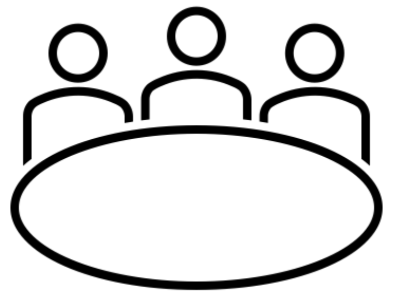

Remember, the power of pattern is to predict beyond the physical evidence.

The Staircase Problem

Investigating Sums of Consecutive Addends

What numbers from 1-30 can be represented as sums of consecutive addends?

1. Look for patterns that can help you predict possible arrangements.
2. Talk with group members about how the square tiles help you visualize possible staircases for specific numbers and answer these questions:
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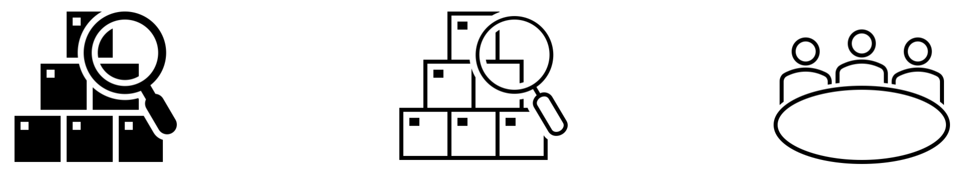


Time to work

- Olga’s slide of the children’s literature connection and different ways to consider consecutive addends.

- GO BACK to working in groups to get more inspired.

- Shared writing and offer evidence to prove claim

Return to whole group

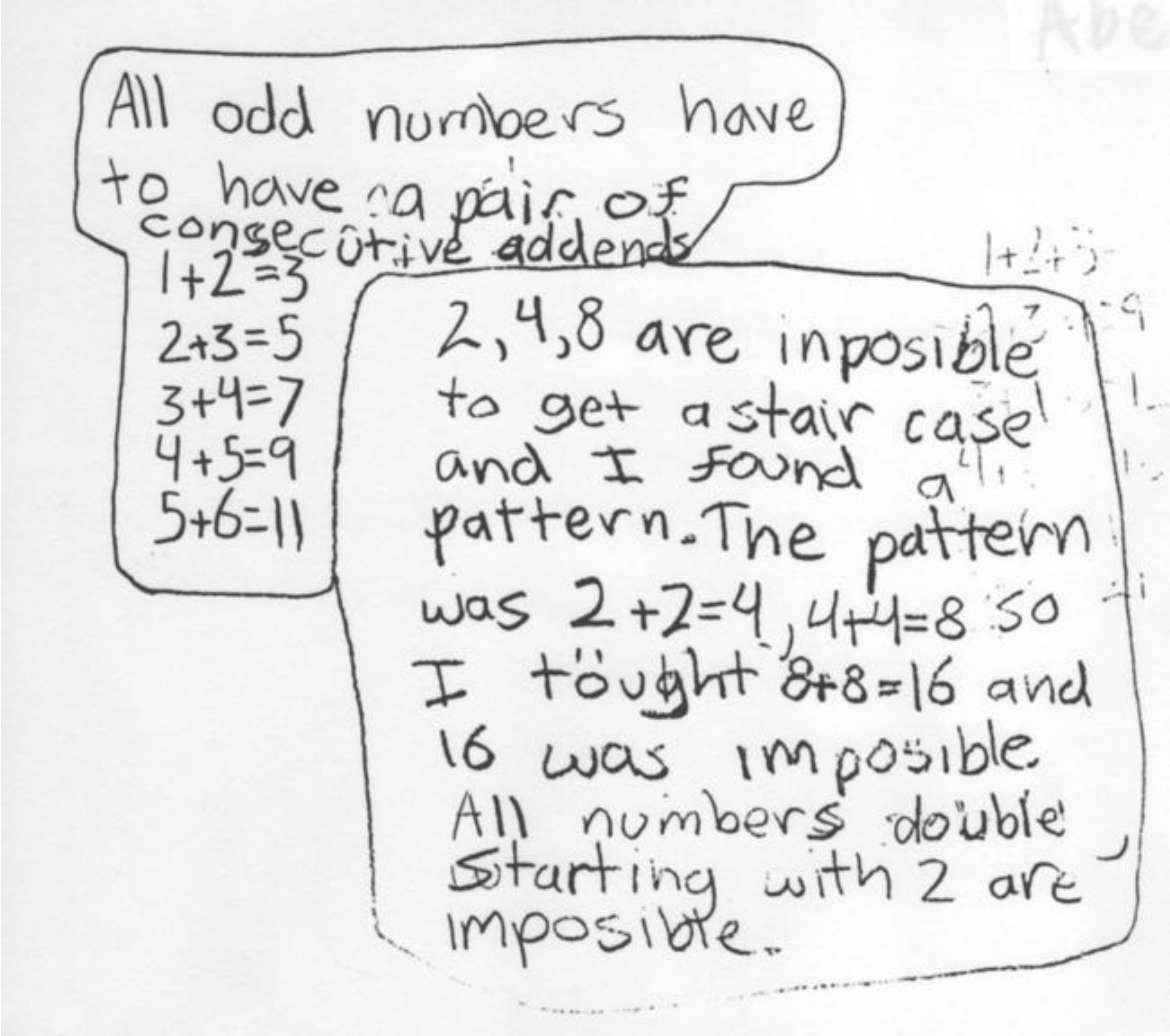
Reflect on your experiences as a learner.

What did you discover about:

yourself
about your neighbors
and about mathematics through this task?

Examining student work

What discoveries did you make about impossible numbers?



Number	Ways to Count	How many ways
2	1, 2	2
4	1, 2, 4	3
8	1, 2, 4, 8	4
16	1, 2, 4, 8, 16	5
6	1, 2, **3**, 6	4
10	1, 2, **5**,10	4
24	1, 2, **3**, 4, 6, 8, 12, 24	8

Prime Factors

2
4 = 2 x 2
8 = 2 x 2 x 2
16 = 2 x 2 x 2 x 2

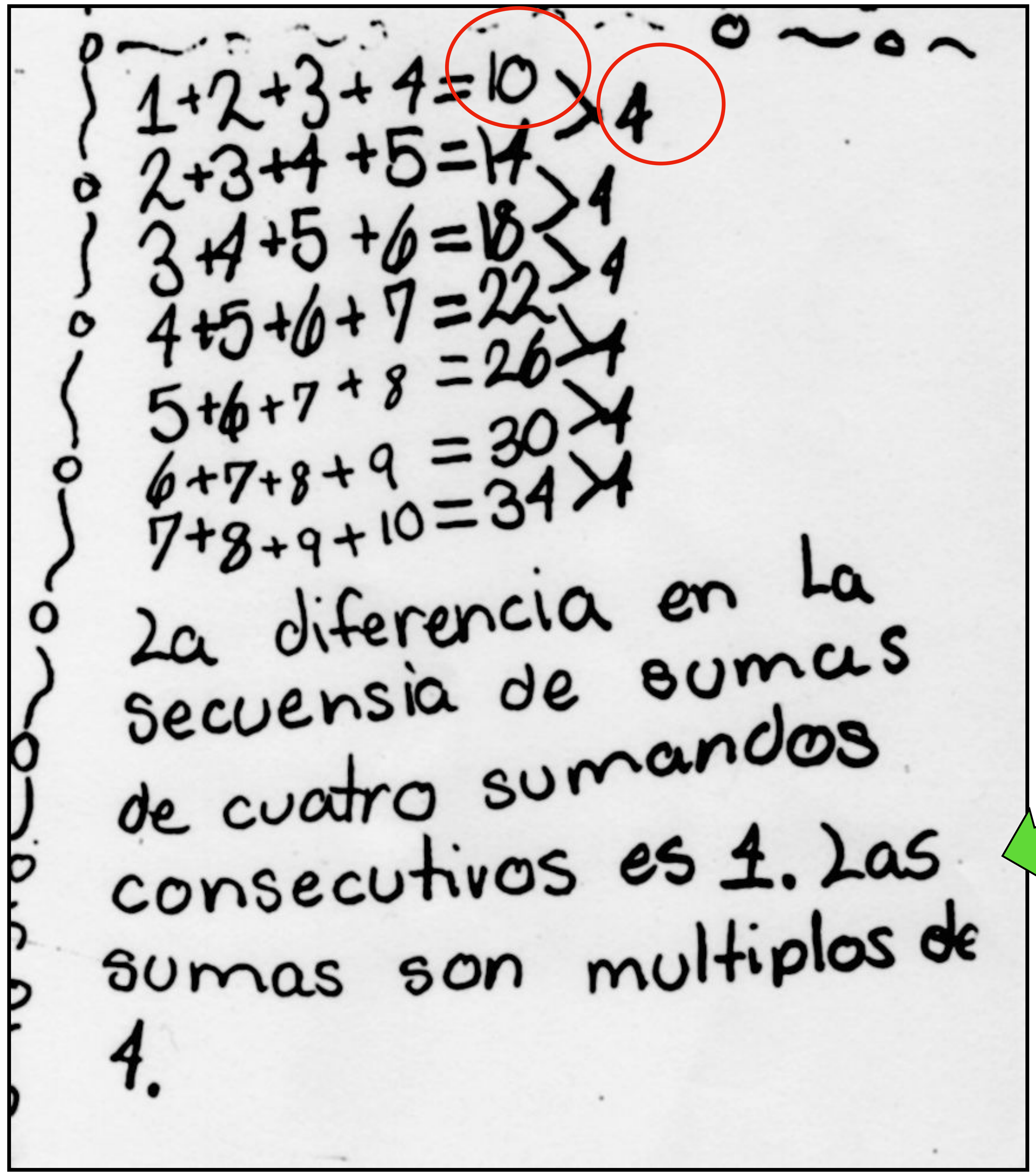
Powers of 2

Prime Factors

6 = 2 x 3
10 = 2 x 5
12 = 2 x 2 x 3

Students' Right to Have Wonderful Mathematical Ideas:

What action would you take to make this mistake a celebration of learning?


1+2+3+4=10 > 4
2+3+4+5=14 > 4
3+4+5+6=18 > 4
4+5+6+7=22 > 4
5+6+7+8=26 > 4
6+7+8+9=30 > 4
7+8+9+10=34 > 4

La diferencia en la secuensia de sumas de cuatro sumandos consecutivos es 4. Las sumas son multiplos de 4.

"The difference in the sequence of sums of 4 consecutive addends is 4. ***The sums are multiples of 4."***

Celebration of Learning

3	> 3	1 x 3 = 3
6	> 3	2 x 3 = 6
9	> 3	3 x 3 = 9
12		4 x 3 = 12
15		
18		
21		
24		
27		
30		

What multiple of 4 is close to 10 but not more than 10?

(4 x 2) + 2 = 10

What multiple of 4 is close to 14 but not more than 14?

(4 x 3) + 2 =14

What multiple of 4 is close to 18 but not more than 18?

(4 x 4) + 2 = 18

Let's make a listing of what we know:

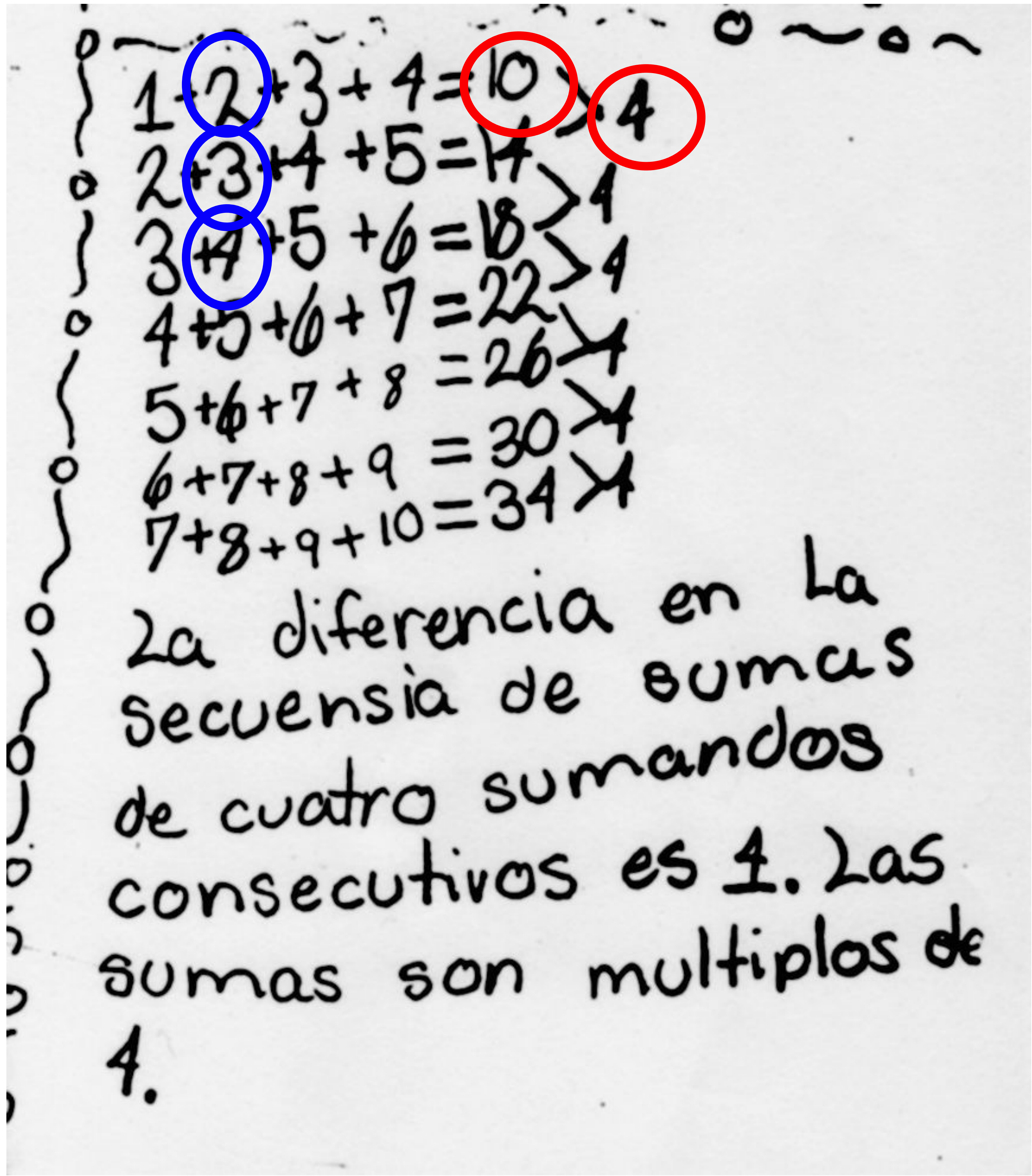
(4 x 2) + 2 = 10
(4 x 3) + 2 = 14
(4 x 4) + 2 = 18

Do you see a pattern?

What is changing?
What is staying the same?

4n + 2 = S

What action would you take to make this mistake a celebration of learning?



"The difference in the sequence of sums of 4 consecutive addends is 4. The sums are multiples of 4."

What multiple of 4 is close to 10 but not more than 10?

(4 x 2) + 2 = 10

What multiple of 4 is close to 14 but not more than 14?

(4 x 3) + 2 =14

What multiple of 4 is close to 18 but not more than 18?

(4 x 4) + 2 = 18

Let's make a listing of what we know:

(4 x 2) + 2 =10
(4 x 3) + 2 = 14
(4 x 4) + 2 = 18

Do you see a pattern?

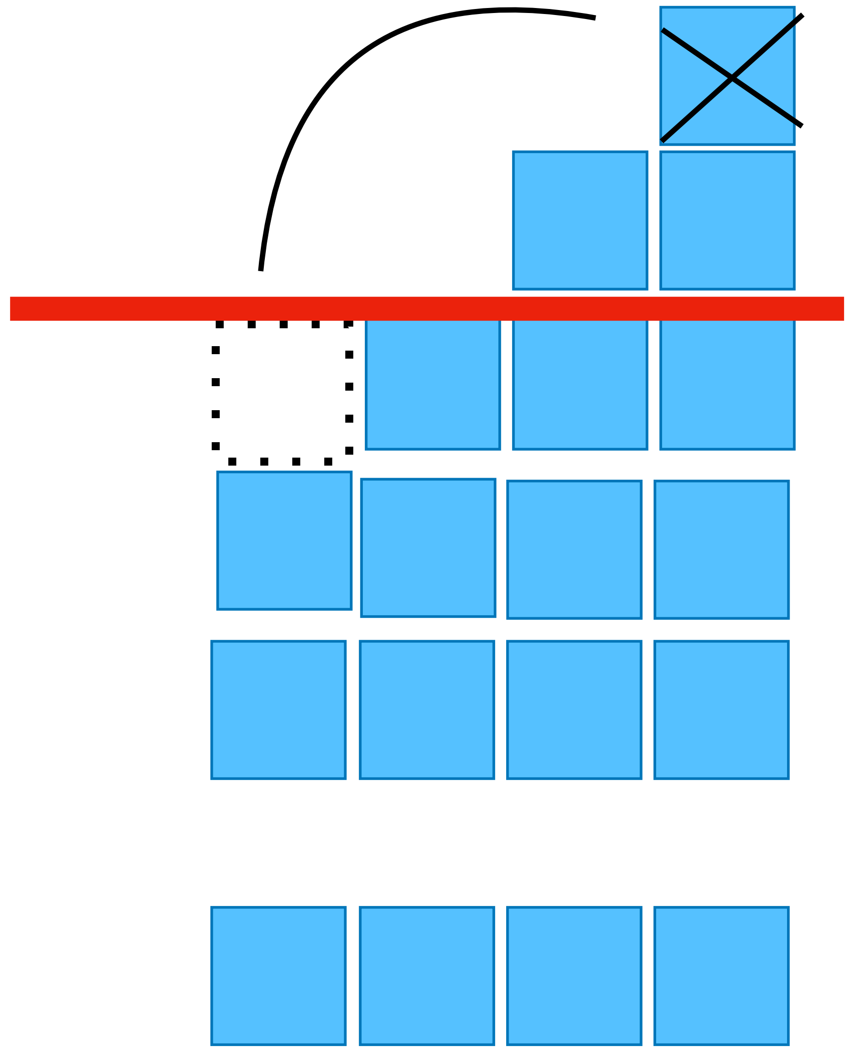
What is changing?
What is staying the same?

4n + 2 = S

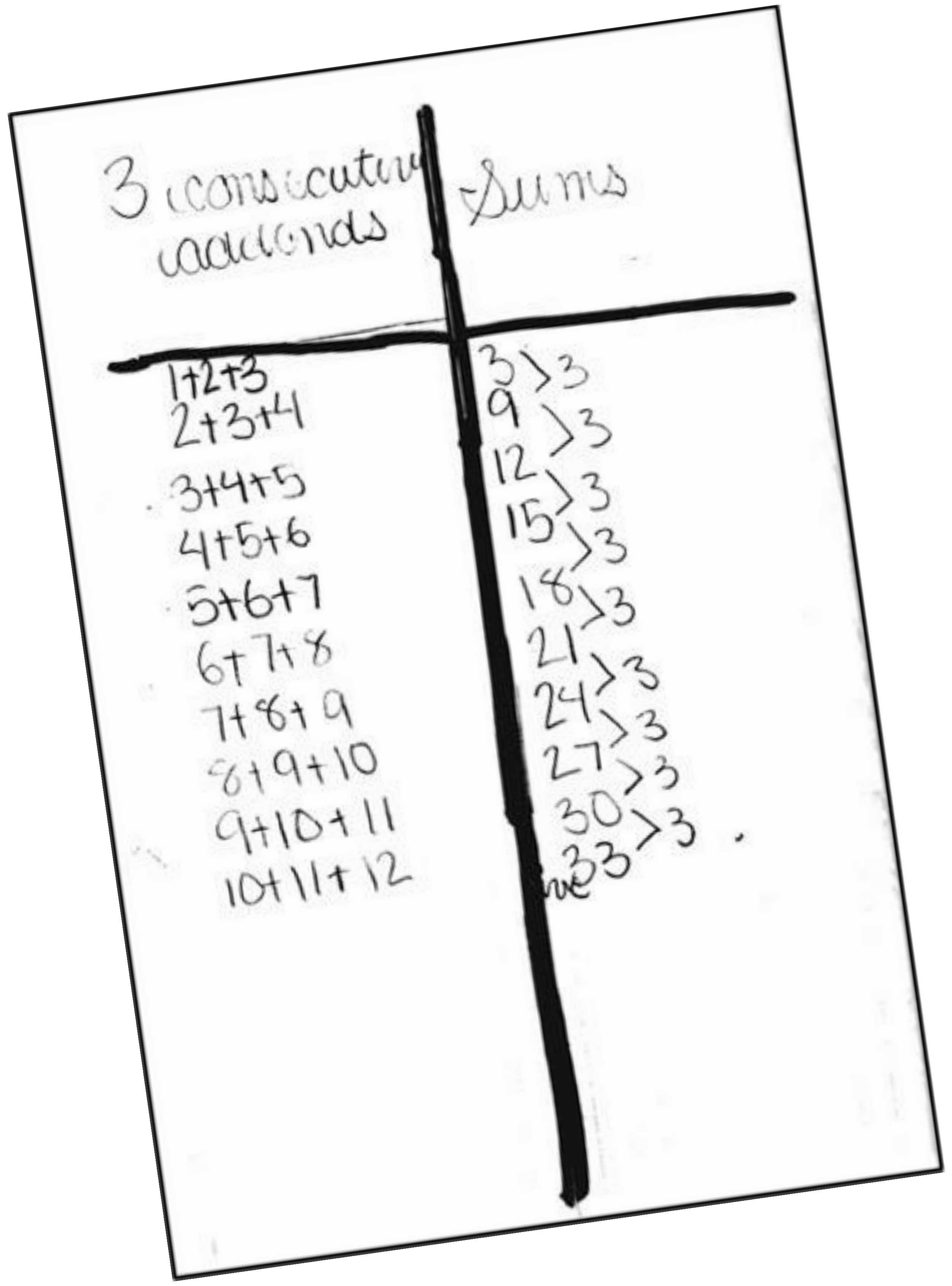
What is **n** ?

n = second addend of 4 consecutive addends

Why?



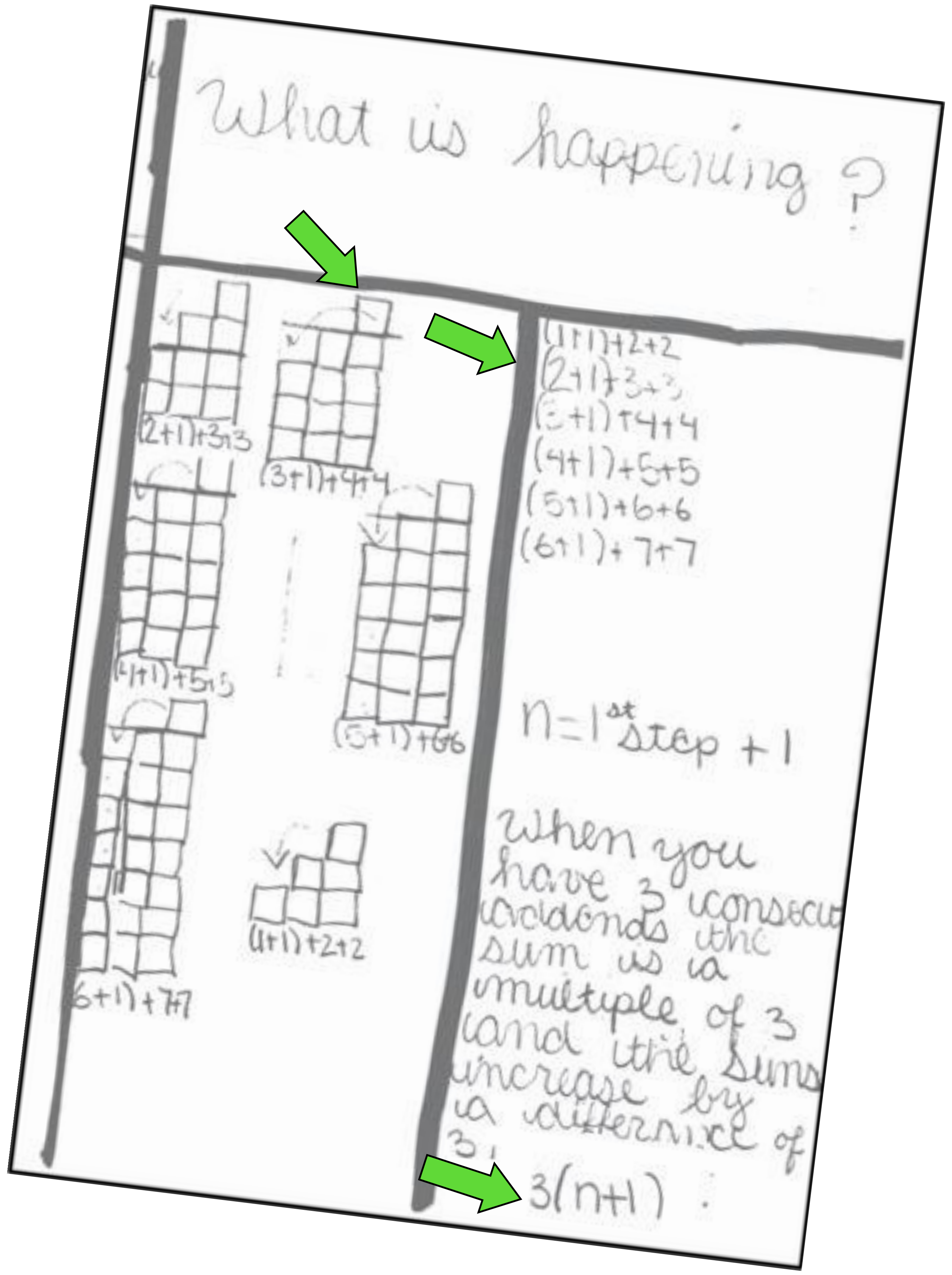
The Right to Have Wonderful Mathematical Ideas

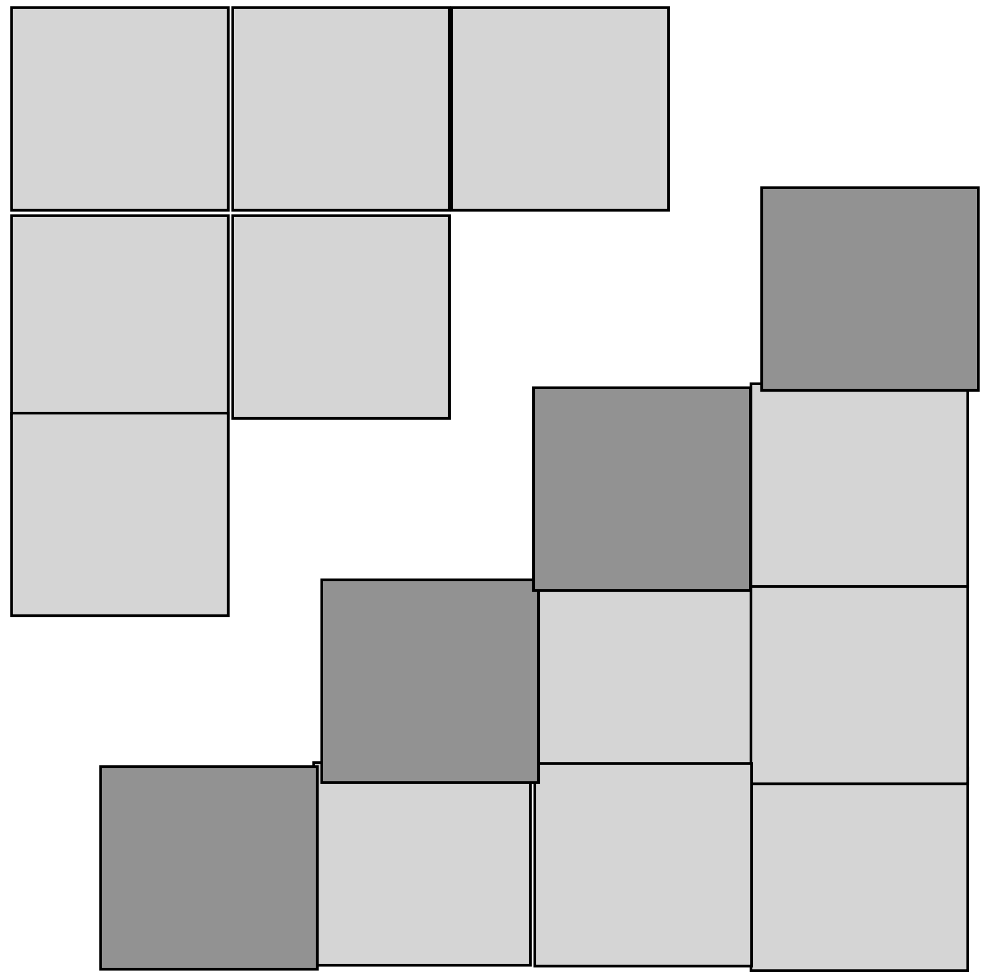


"… an **algorithm** reflects a fundamental **generalization of a pattern.**

We want students to build a tool kit of useful procedures, understanding and underlying patterns so well that they recognize algorithms that help them solve problems."

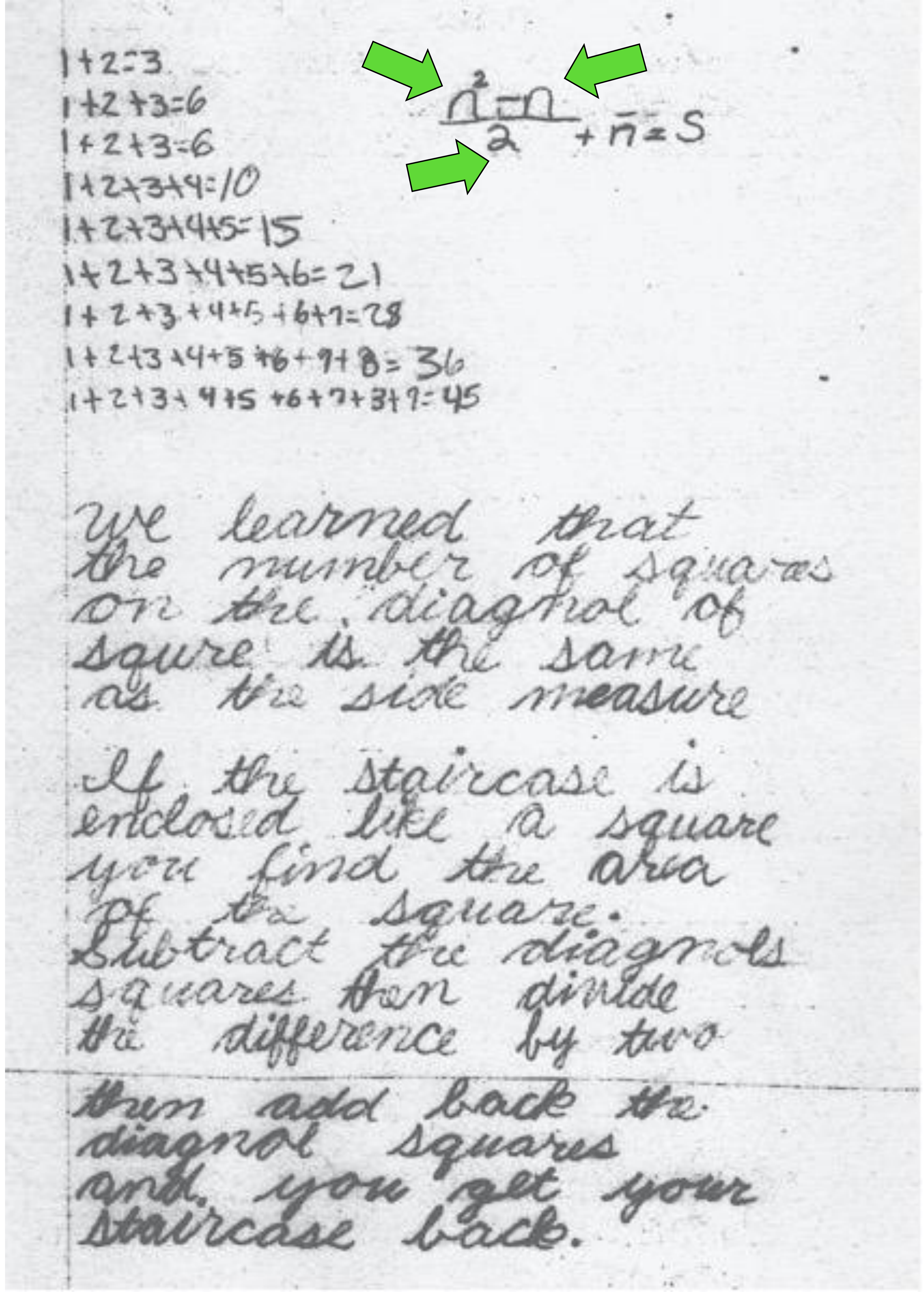
-Cathy L. Seeley
Faster Isn't Smarter





"The more we help students to have their wonderful ideas and to feel good about themselves for having them, the more likely it is they will someday happen upon wonderful ideas that no one else has happened upon before..."

Eleanor Duckworth

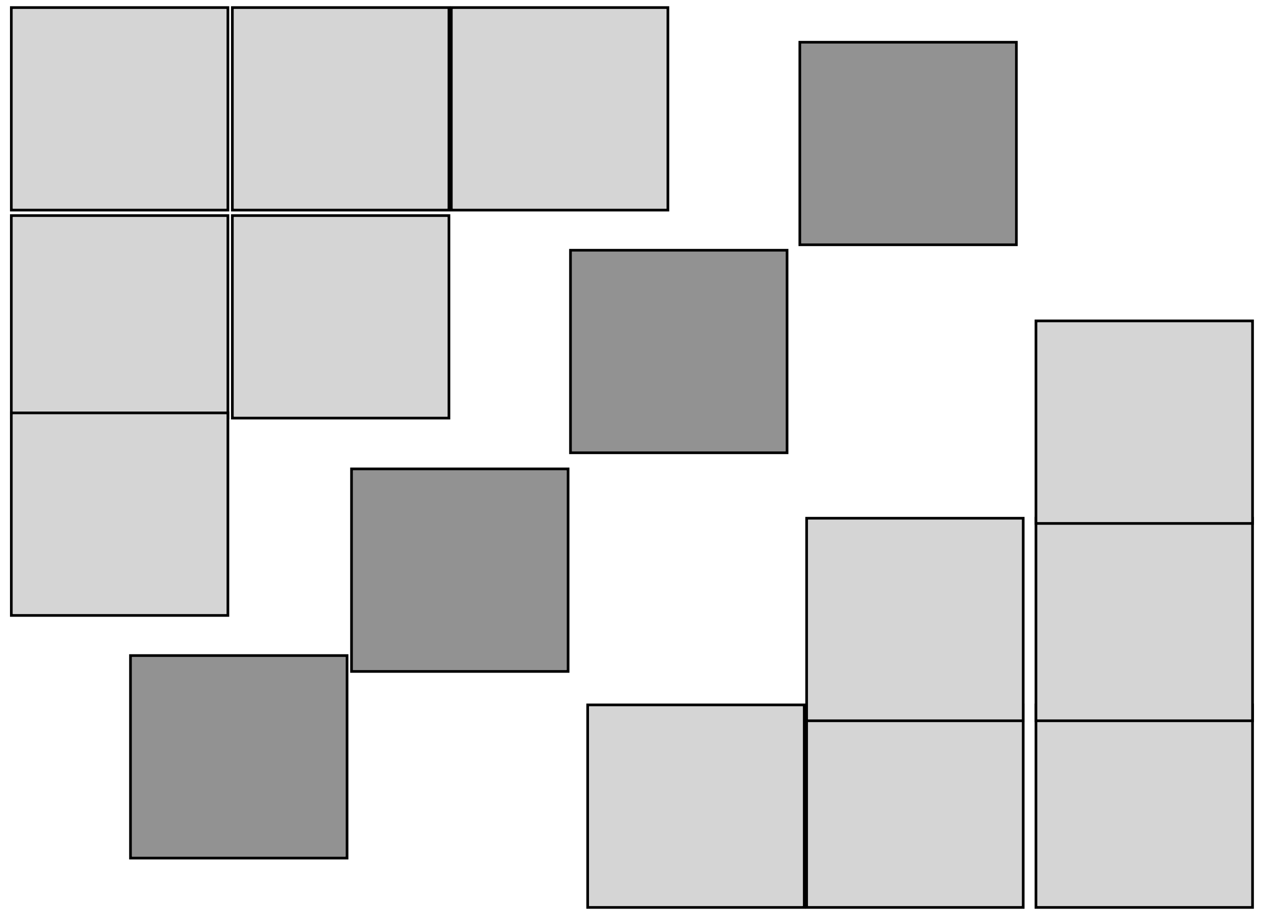


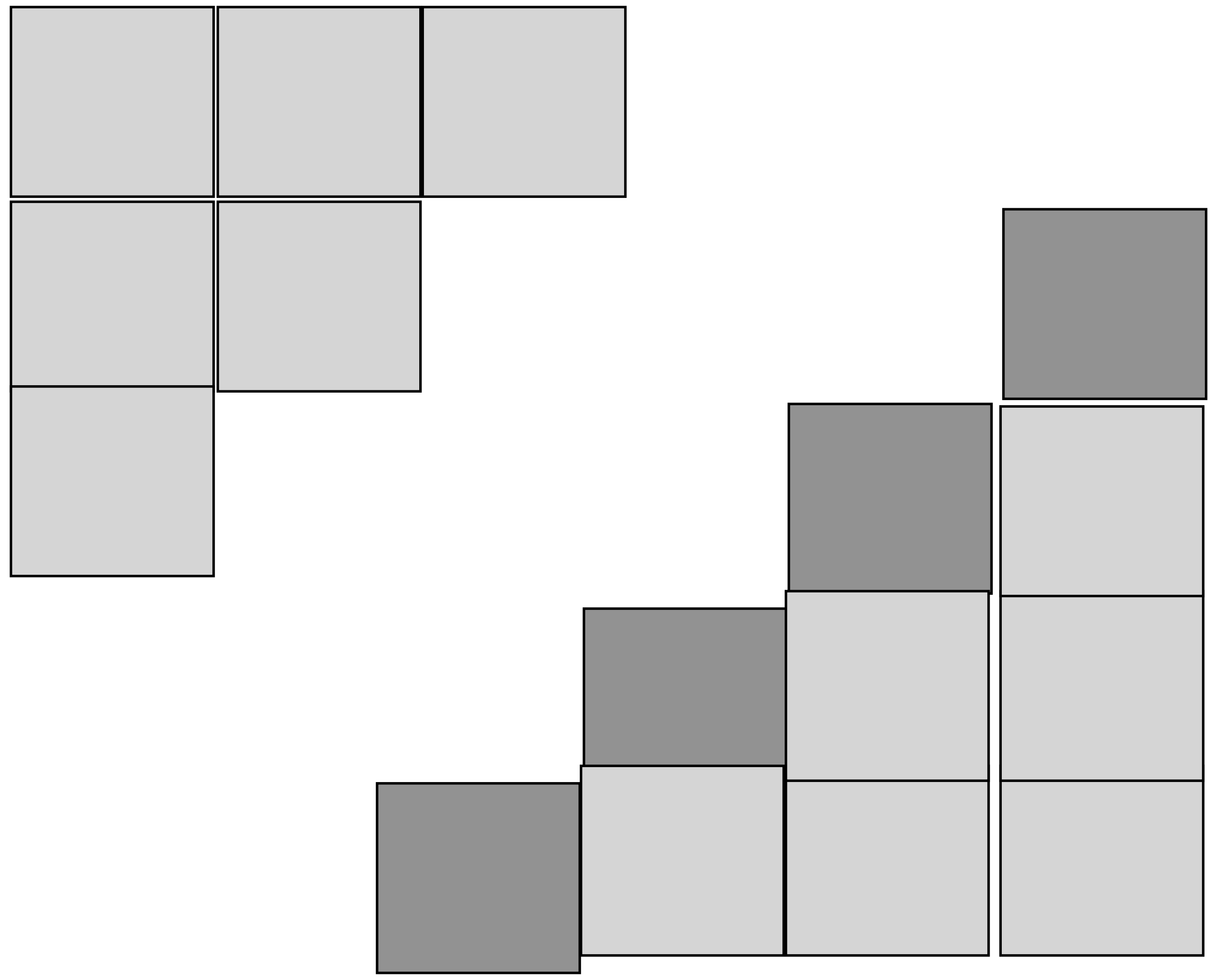
1+2=3
1+2+3=6
1+2+3=6
1+2+3+4=10
1+2+3+4+5=15
1+2+3+4+5+6=21
1+2+3+4+5+6+7=28
1+2+3+4+5+6+7+8=36
1+2+3+4+5+6+7+8+9=45

$$\frac{n^2-n}{2}+n=S$$

We learned that the number of squares on the diagnal of squre is the same as the side measure

If the staircase is enclosed like a square you find the area of the square. Subtract the diagnols squares then divide the difference by two then add back the diagnol squares and you get your staircase back.





What will you do now?

What are your beliefs doing?

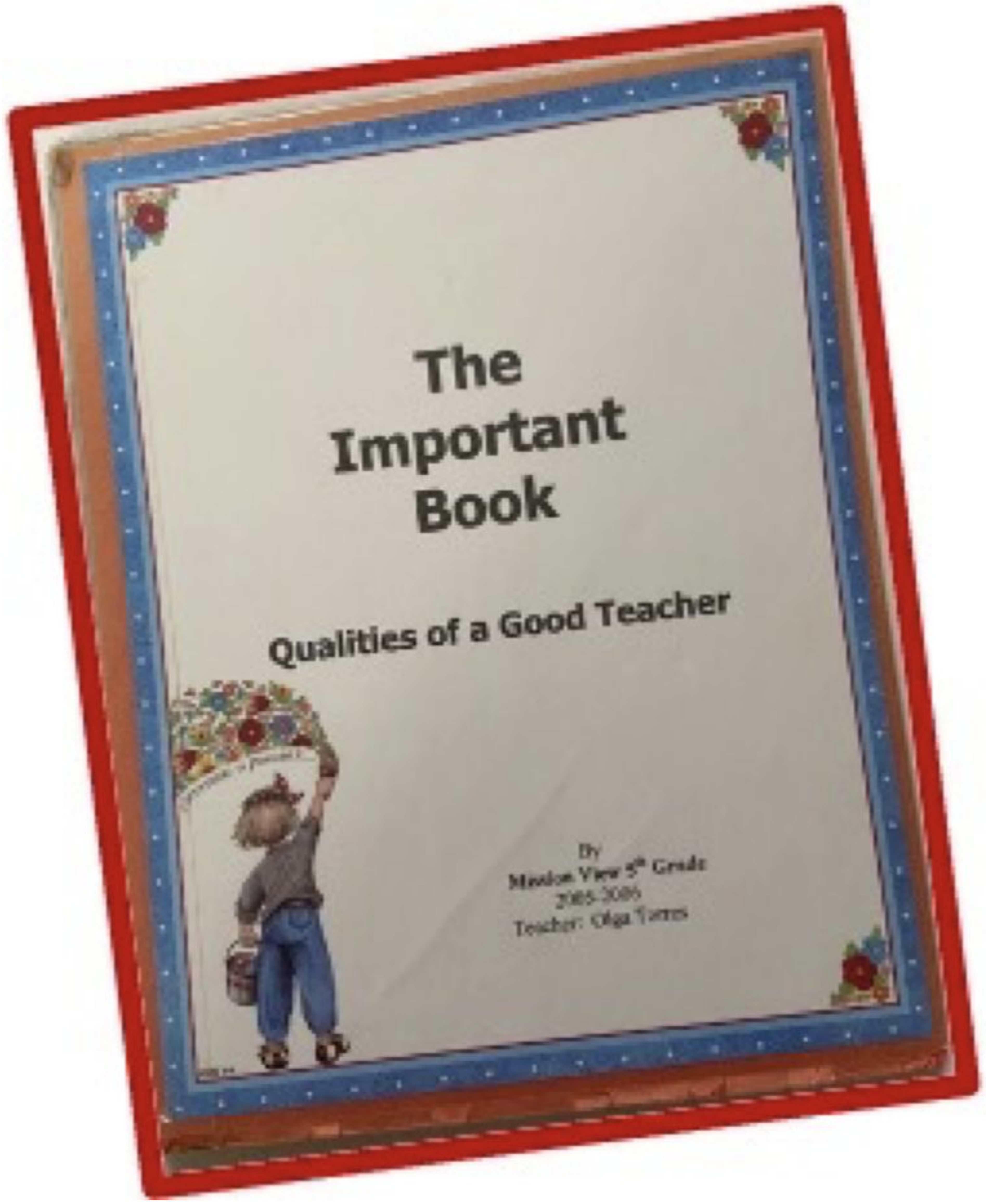
your beliefs — can trap you — or help you grow

If . . . then. . .The Logic Behind Teaching with Heart and Mind

If ***you take the time to discover your students,***

then ***you will have time to discover and teach them in ways they can understand you.***

5th grade student (2006)



THE IMPORTANT BOOK
Qualities of a Good Teacher

The important quality of a good teacher
is to be curious.

A curious teacher is inquisitive
about discovering
students' ideas.

Being curious also means
that a teacher will ask questions
and students will talk in groups,
share their ideas
and
the teacher listens.

But, the important quality of a good teacher
is to be curious.

5th Grade Student

Kalinec-Craig and Robles (2020)

Classroom Rules Reimagined as the Rights of the Learner

Fifth graders learned to graph and interpret nonlinear data by exercising their rights: to be confused; to claim a mistake; to speak, listen, and be heard; and to write, do, and represent what makes sense.

Crystal Kalinec-Craig and Rose Ann Robles

Citations and Resources

Resources cited in Presentation

Duckworth, E. (2006) *The Having of Wonderful Ideas.* Teacher College Press: Columbia University, New York.

Cummins, J. (2001). *Negotiating Identities: Education for Empowerment in a Diverse Society 2nd Edition,* California Association for Bilingual Education, Los Angeles, CA 90017

Seeley, C. (2009). *Faster Isn't Smarter.* Math Solutions: Sausalito CA

Links to Torres' RotL Papers

- Kalinec-Craig (2017)
- Kazemi (2018)
- Boaler and Anderson (2018)
- Hintz et al (2018)
- Prasad and Kalinec-Craig (2021)
- Tyson et all (2021)
- Jansen et al (2021)
- Kalinec-Craig and Robles (2020)

Social Media

@OlgaGTorres1

@CrystalKCraig

TikTok: @EmbraceMajorRevisions

Podcast and Webinar

Torres on 180 Days Education
Torres Casio Webinar

Contact for In-Person or Virtual Speaking Engagements:

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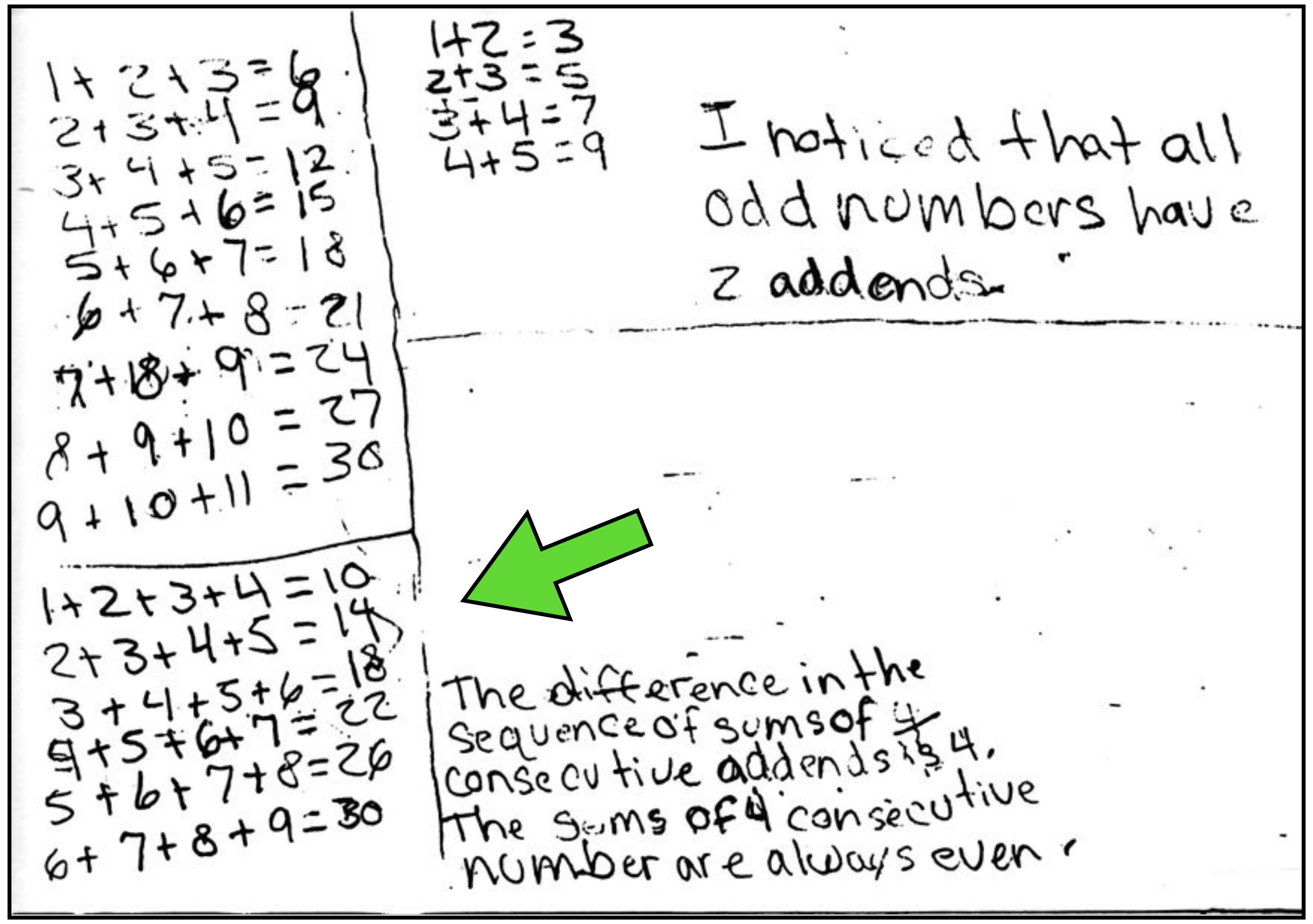
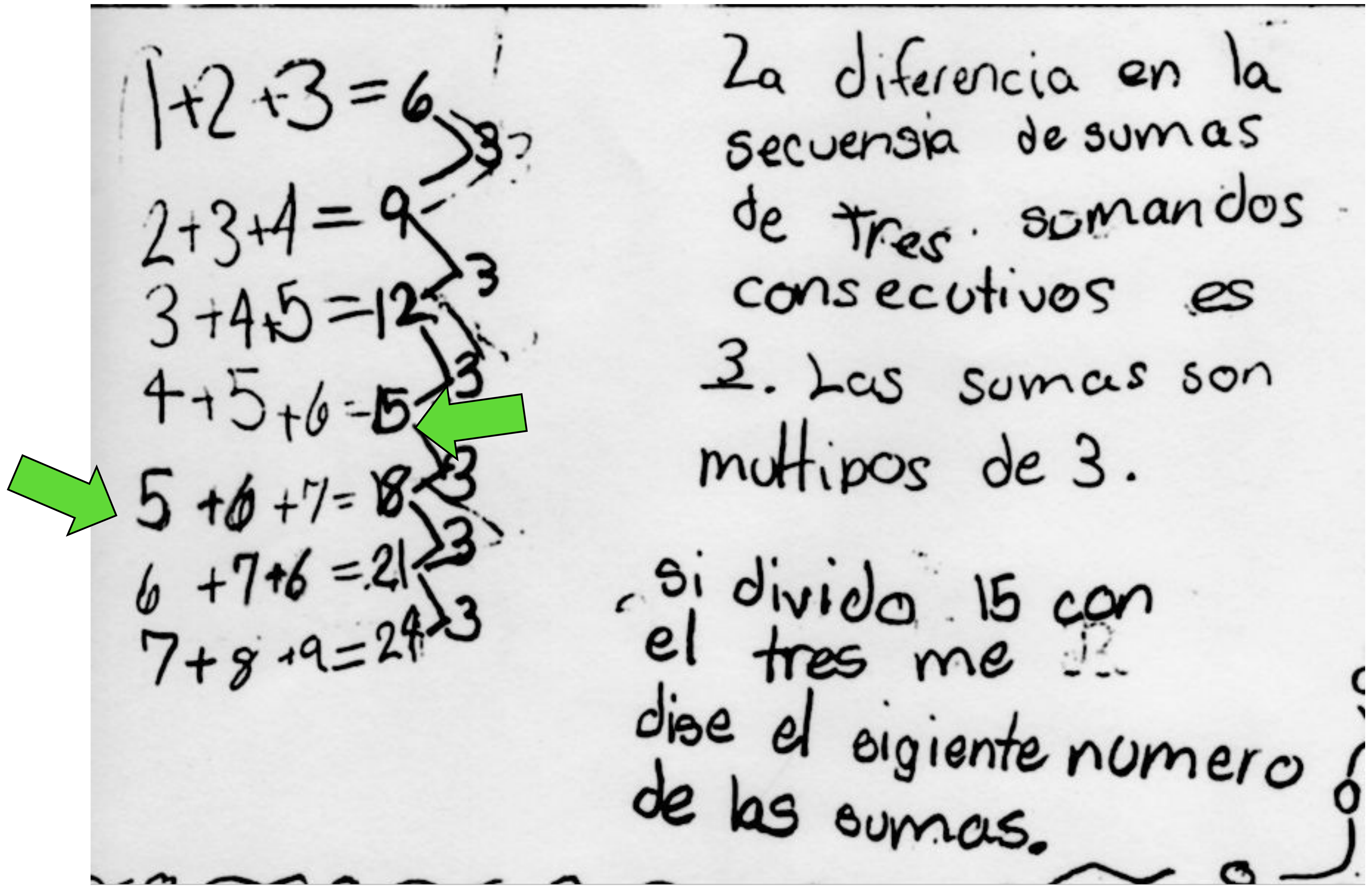
Extra student work

Students' Right to Have Wonderful Mathematical Ideas:
An Equity and Social Justice Perspective

Trust is an essential component of a culture of equity
with social justice permeating throughout—for the student and the teacher.

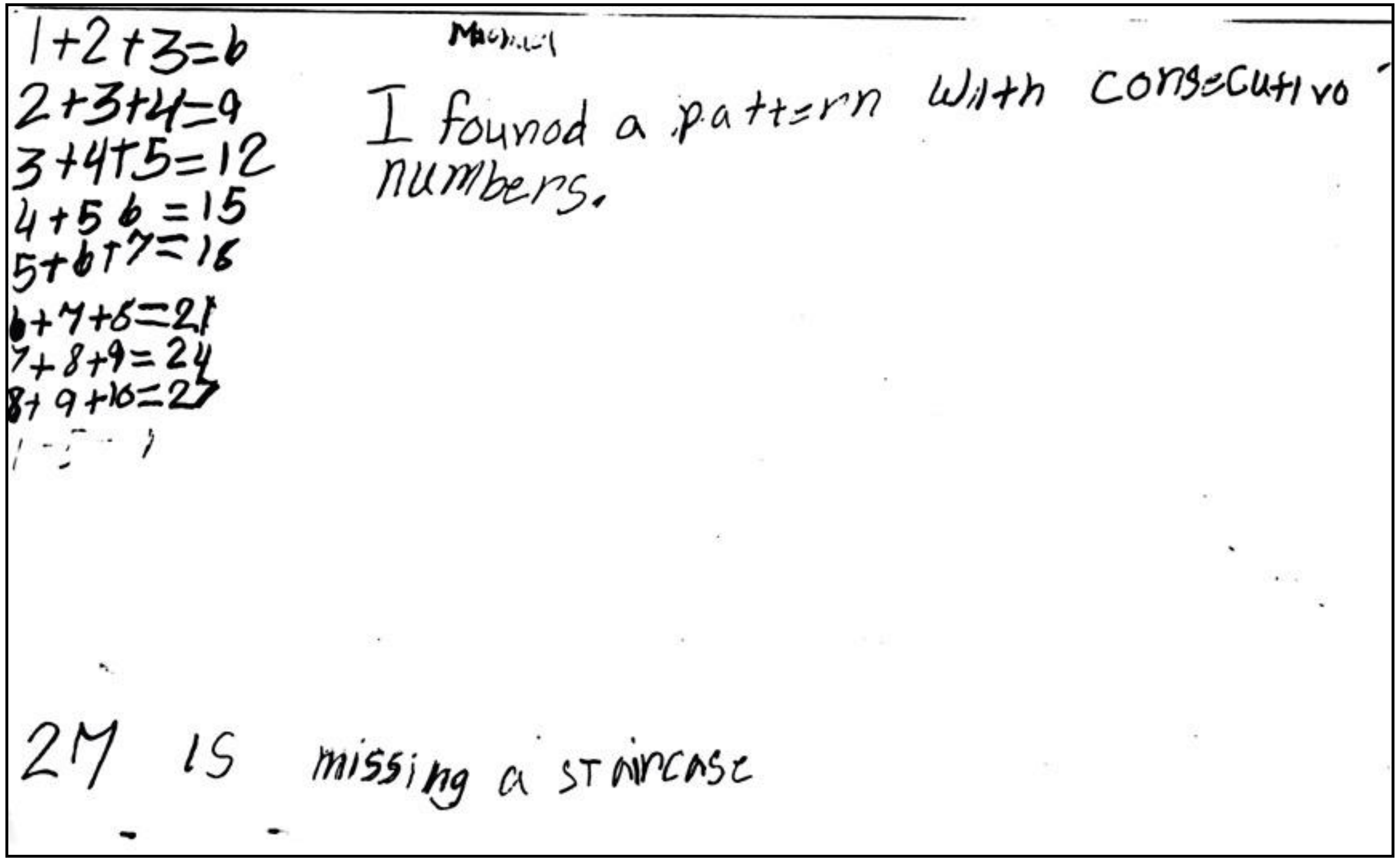


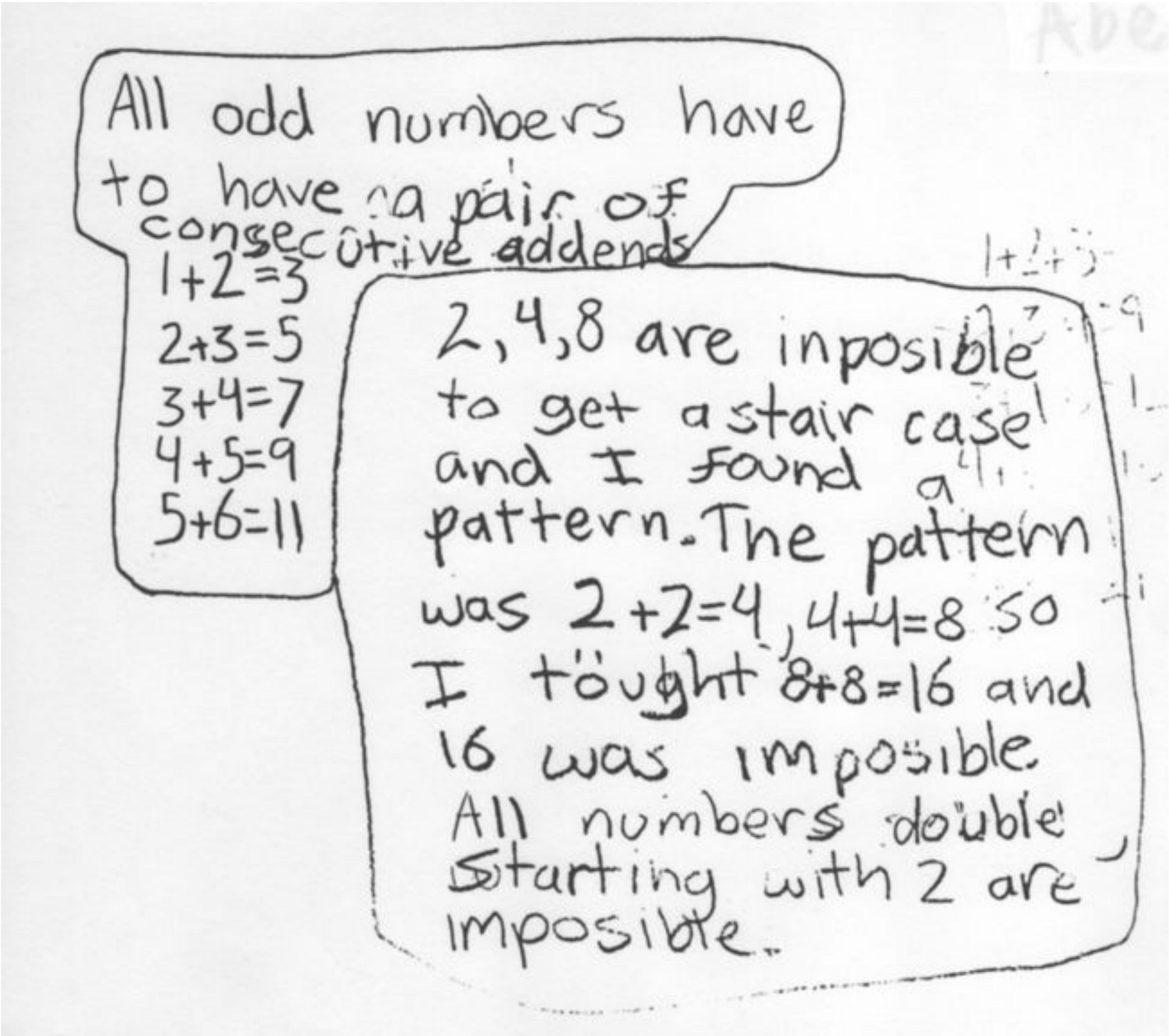


"The difference in the sequence of three consecutive addends is 3. If you divide 15 by 3 you will get the next number of sums."

Students' Right to Have Wonderful Mathematical Ideas:

An Equity and Social Justice Perspective



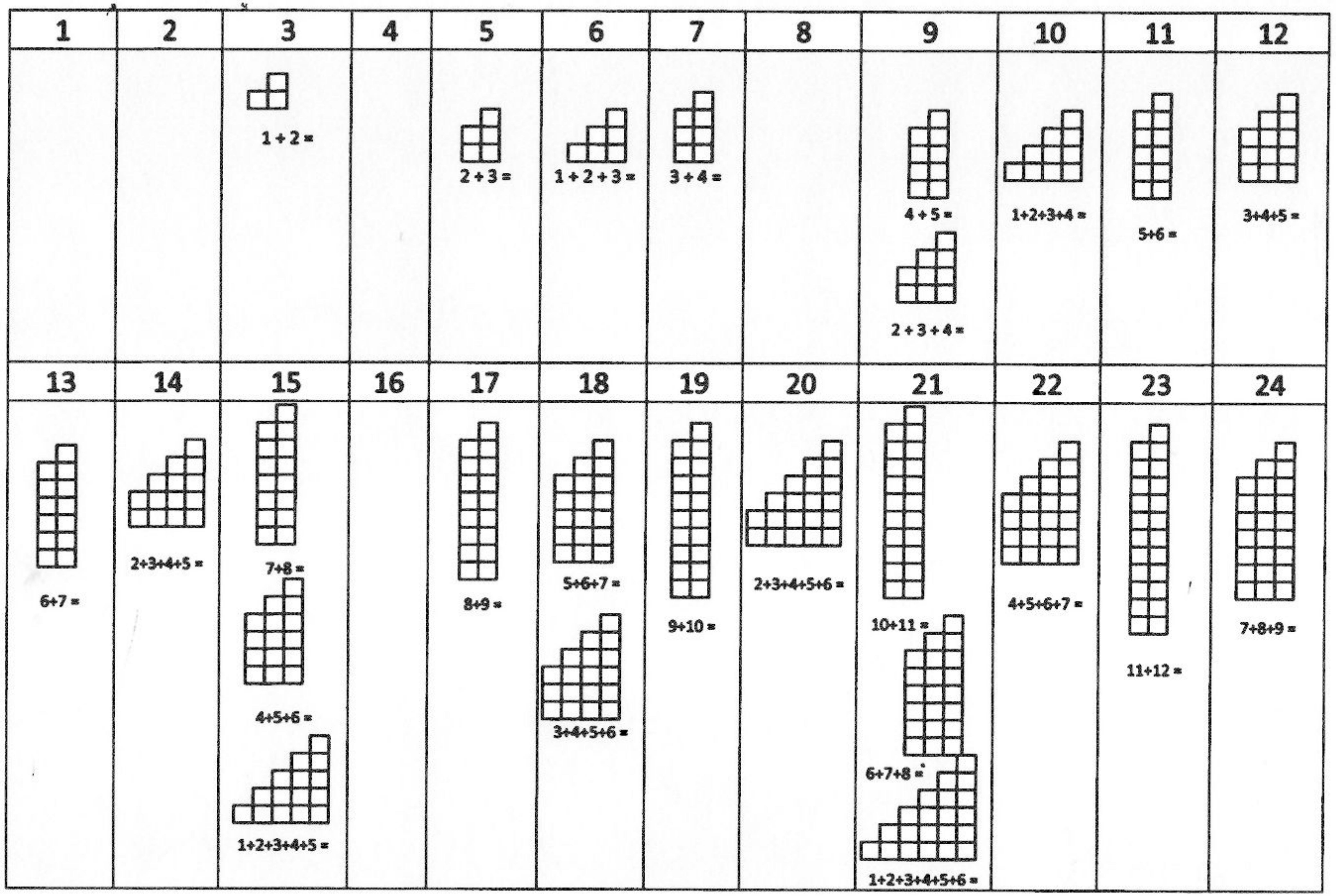


Prime Factors

2 prime factor
4 = 2 x 2
8 = 2 x 2 x 2
16 = 2 x 2 x 2 x 2

Prime Factors

6 = 2 x 3
10 = 2 x 5
12 = 2 x 2 x 3



Number	Ways to Count	How many ways
2	1, 2	2
4	1, 2, 4	3
8	1, 2, 4, 8	4
16	1, 2, 4, 8, 16	5
6	1, 2, **3**, 6	4
10	1, 2, **5**,10	4
24	1, 2, **3**, 4, 6, 8, 12, 24	8

Prime Factors

2 2^1

4 = 2 x 2 2^2

8 = 2 x 2 x 2. 2^3

16 = 2 x 2 x 2 x 2 2^4

Powers of 2

Prime Factors

6 = 2 x 3

10 = 2 x 5

12 = 2 x 2 x 3

The Staircase Problem: Sums of Consecutive Addends

Extended T-Chart: *Making the Invisible Visible*

_2_ consecutive addends	sum
1 + 2 =	3
2 + 3 =	5
3 + 4 =	7
4 + 5 =	9
5 + 6 =	11

Representing a pattern both geometrically and numerically helps students recognize a variety of relationships in the pattern and make connections between arithmetic and geometry.

NCTM Standards 1989-p. 61

1 + 2

2 + 3

3 + 4

4 + 5

5 + 6

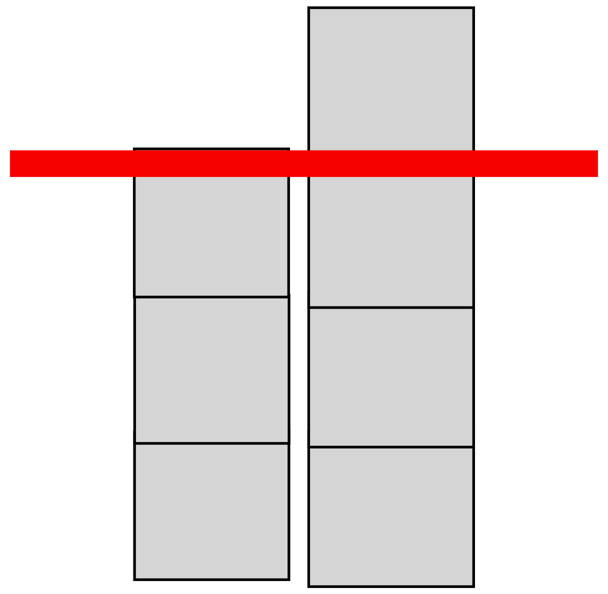
6 + 7

7 + 8

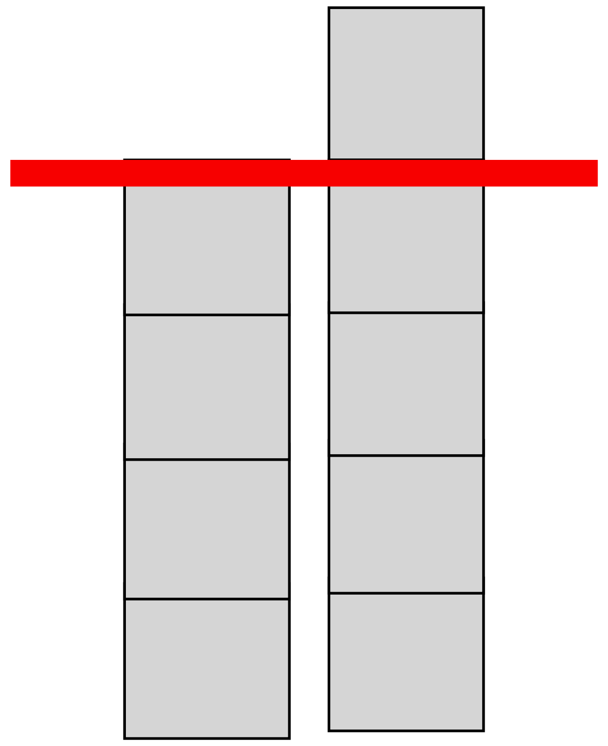
8 + 9

9 + 10

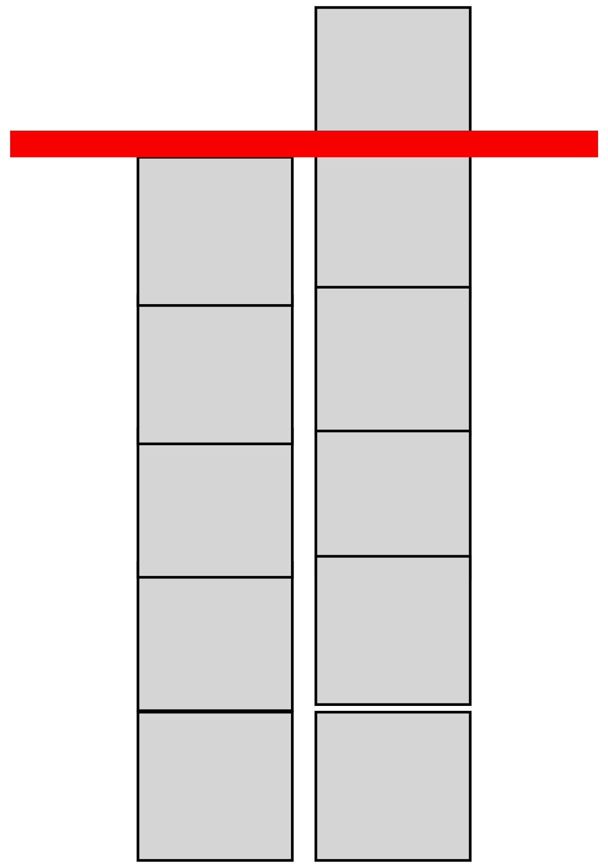
10 + 11



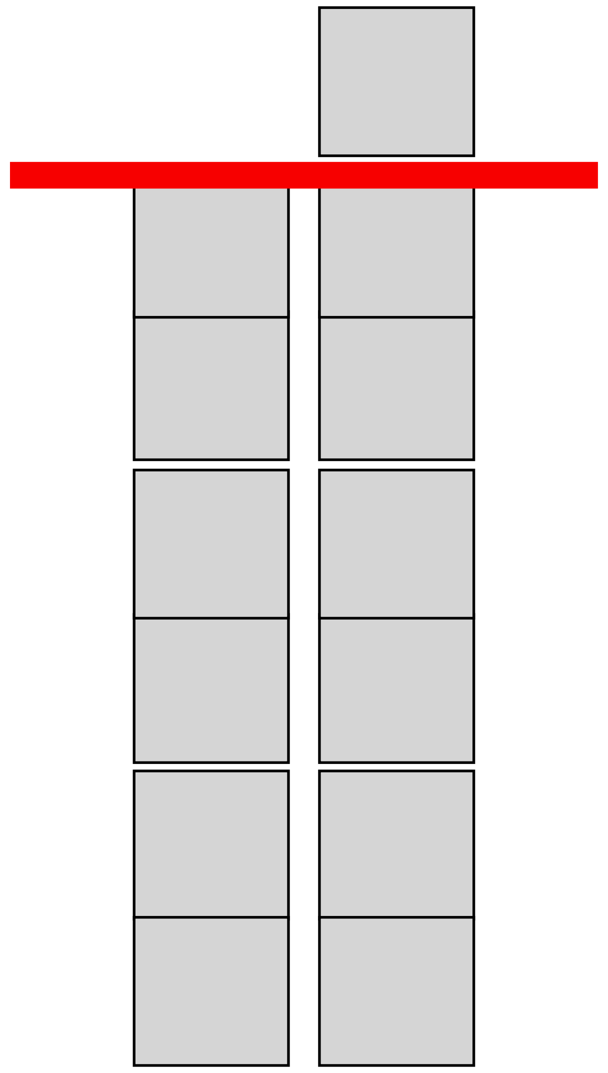
3 + 3 + 1 = 7



4 + 4 + 1 = 9



5 + 5 + 1 = 11



6 + 6 + 1 = 13

(2 x 3) + 1

(2 x 4) + 1

(2 x 5) + 1

(2 x 6) + 1

3

5

7

9

11

13

15

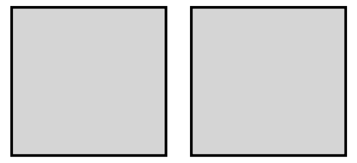
17

19

21

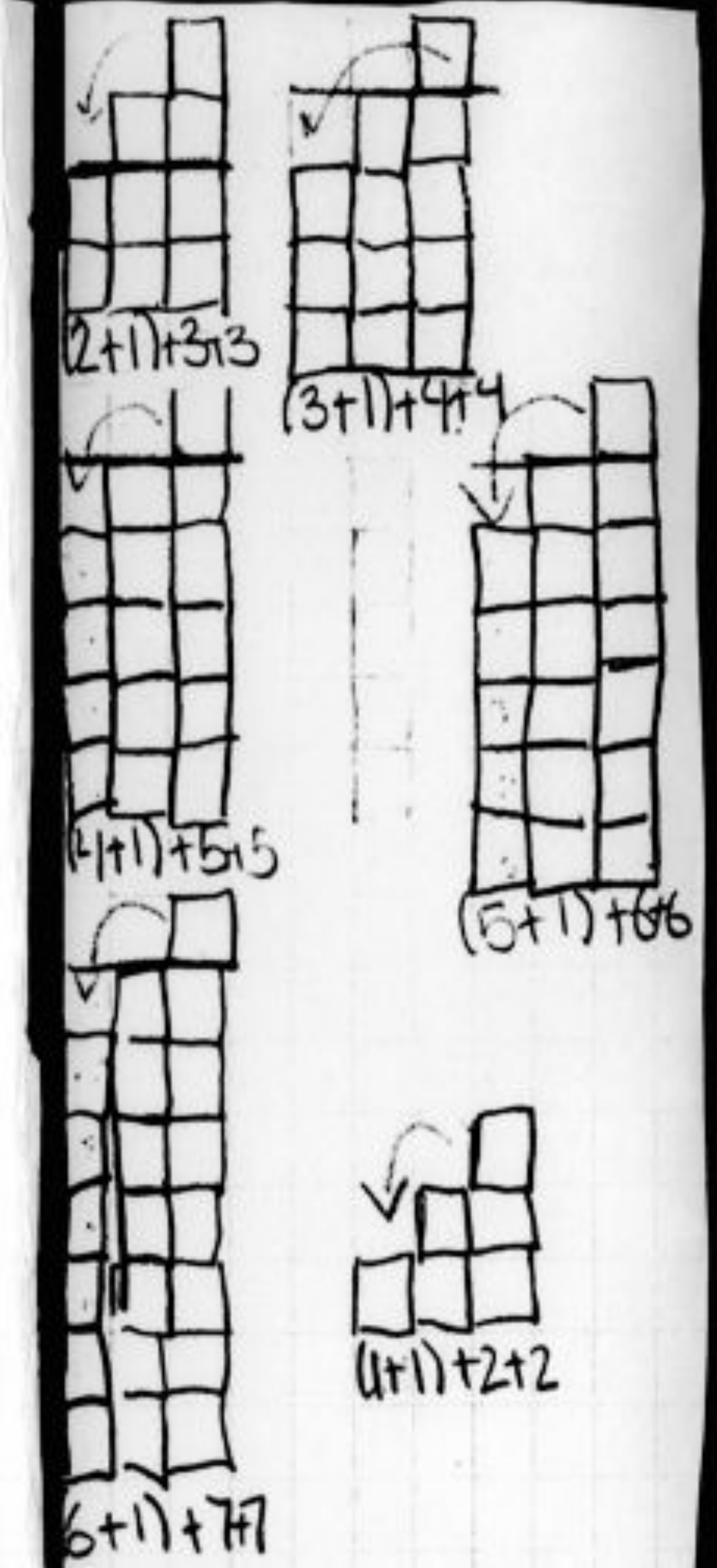
n = first addend

$$2n + 1 = S$$



3 consecutive addends	What is happening?	Sums
1+2+3	(1+1)+2+2	3
2+3+4	(2+1)+3+3	9
3+4+5	(3+1)+4+4	12
4+5+6	(4+1)+5+5	15
5+6+7	(5+1)+6+6	18
6+7+8	(6+1)+7+7	21
7+8+9		24
8+9+10		27
9+10+11		30
10+11+12		33

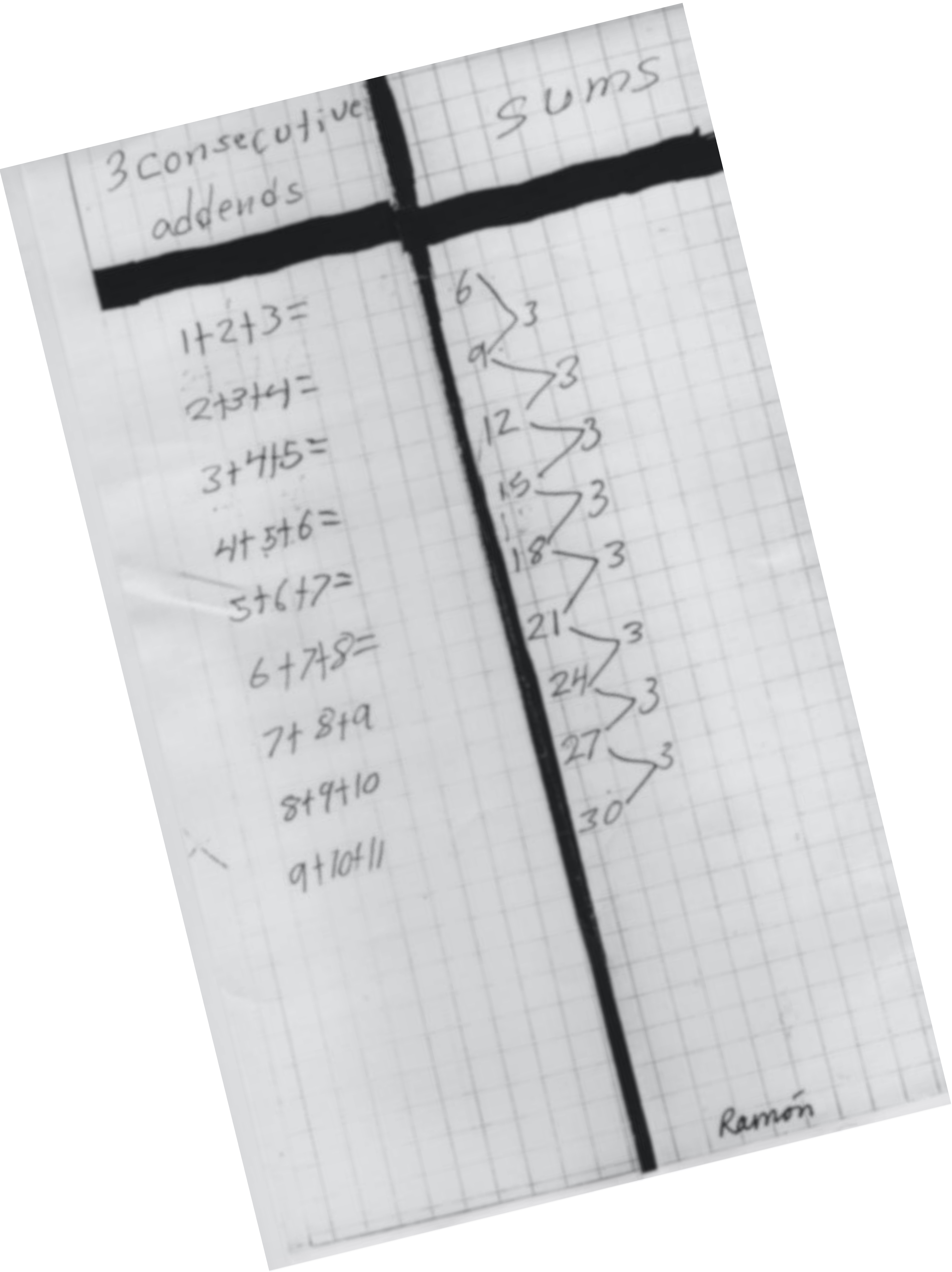
Sums increase: >3 >3 >3 >3 >3 >3 >3 >3 >3 >3

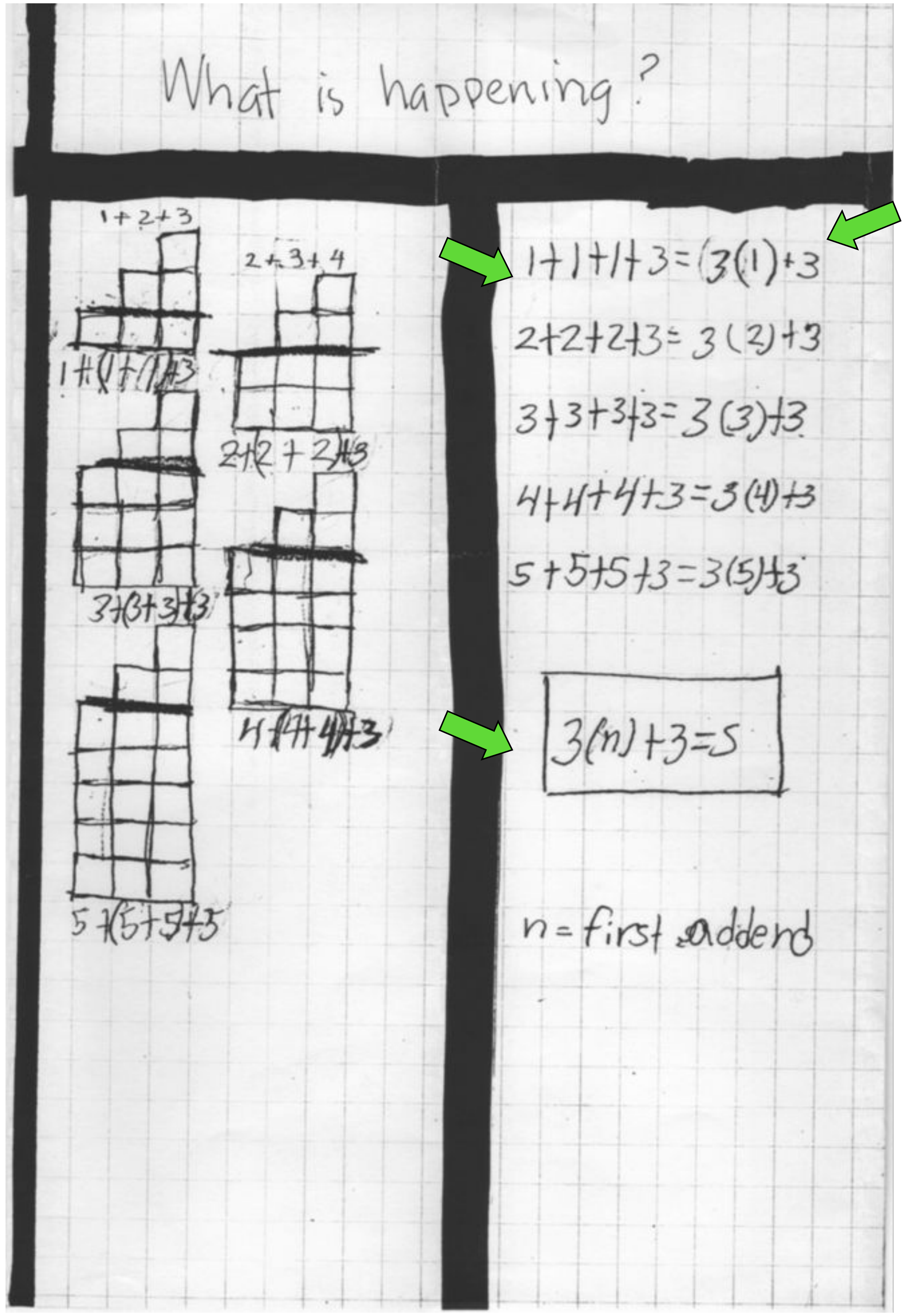


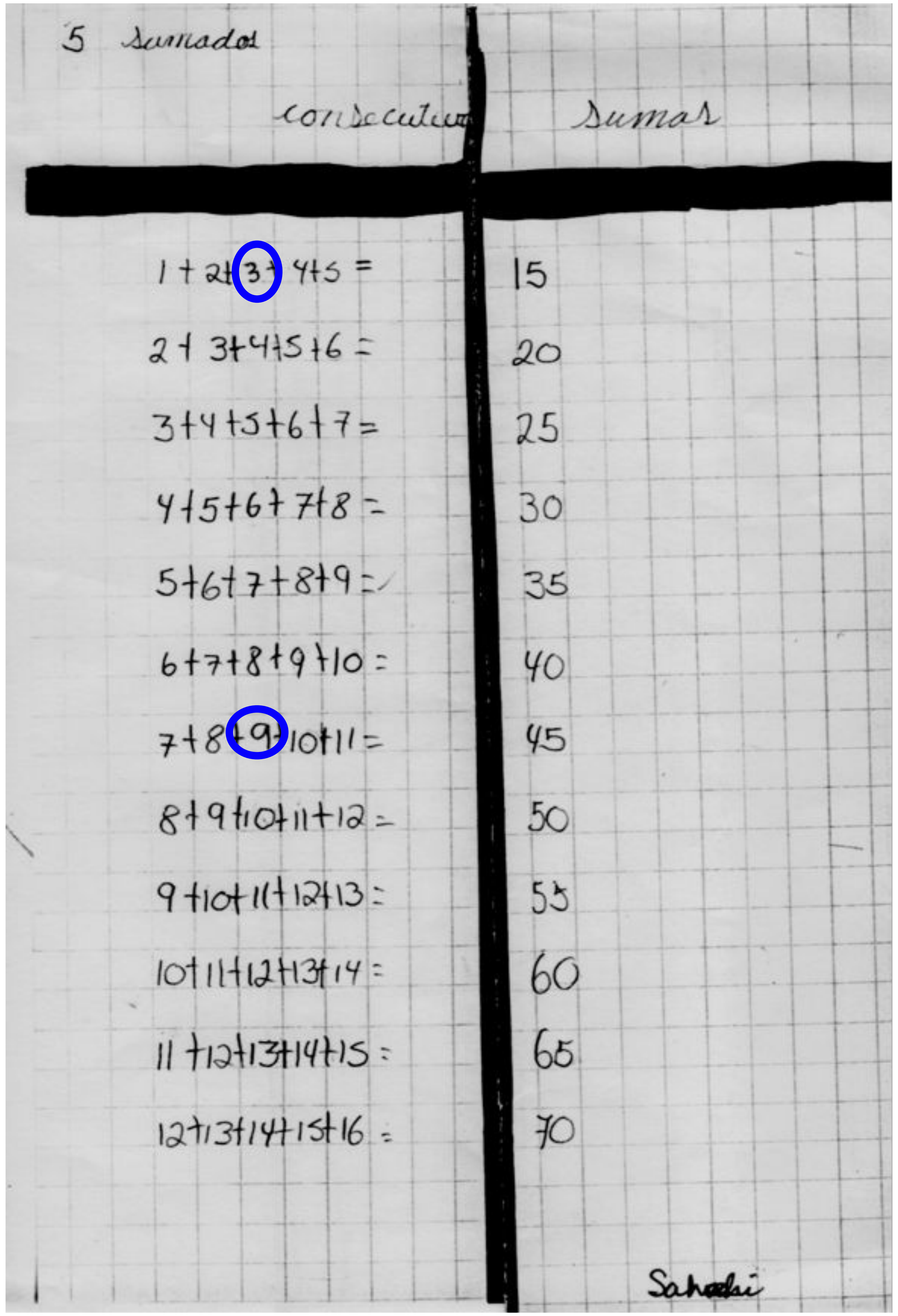
n=1st step +1

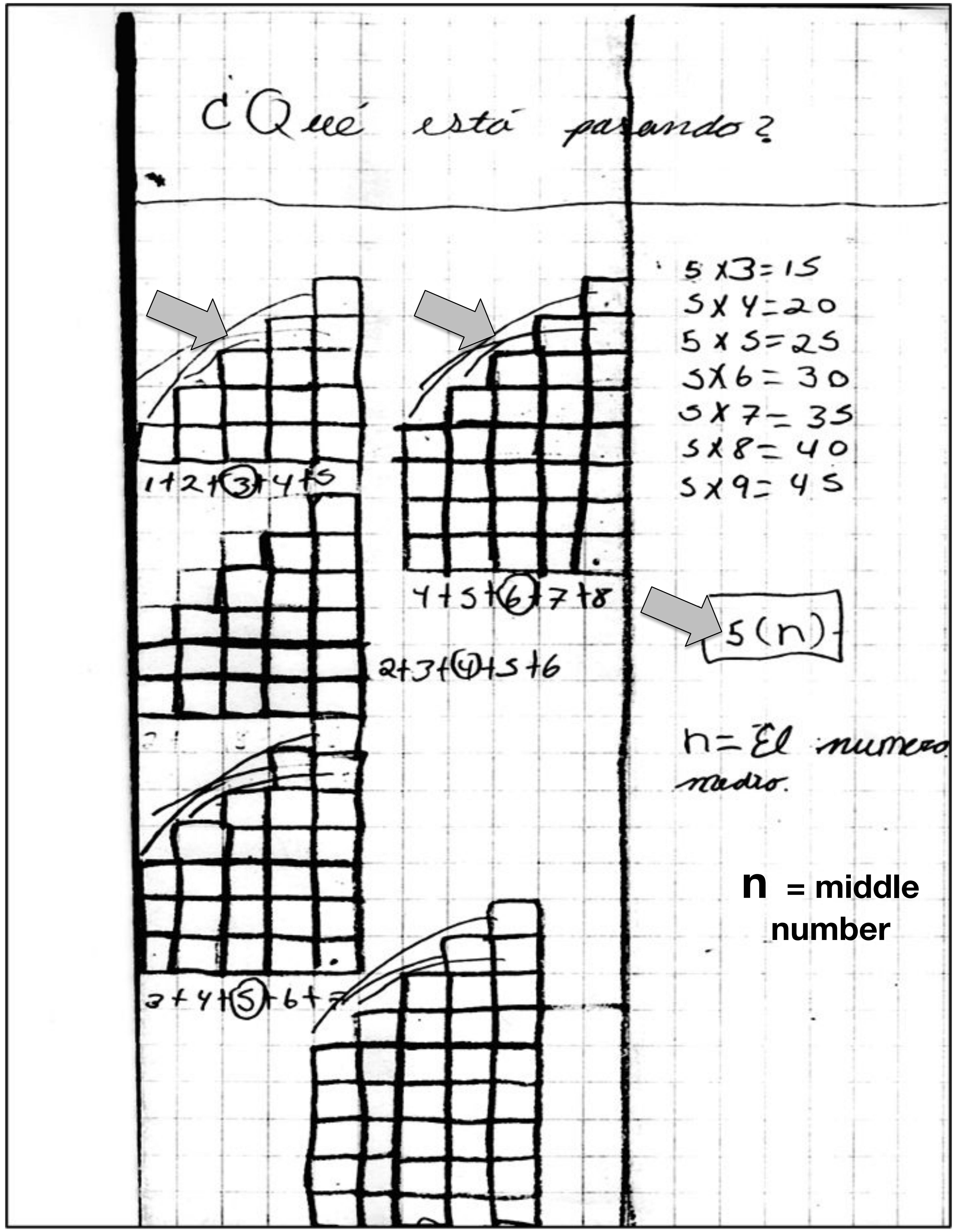
When you have 3 consecutive addends the sum is a multiple of 3 and the sums increase by a difference of 3.

3(n+1)


3 consecutive addends
sums
1+2+3=
2+3+4=
3+4+5=
4+5+6=
5+6+7=
6+7+8=
7+8+9
8+9+10
9+10+11
6
9
12
15
18
21
24
27
30
3
Ramón


What is happening?
1+2+3
2+3+4
1+1+1+3=3(1)+3
2+2+2+3=3(2)+3
3+3+3+3=3(3)+3
4+4+4+3=3(4)+3
5+5+5+3=3(5)+3
3(n)+3=S
n=first addend


5 sumados
Sumas
1+2+3+4+5 = 15
2+3+4+5+6 = 20
3+4+5+6+7 = 25
4+5+6+7+8 = 30
5+6+7+8+9 = 35
6+7+8+9+10 = 40
7+8+9+10+11 = 45
8+9+10+11+12 = 50
9+10+11+12+13 = 55
10+11+12+13+14 = 60
11+12+13+14+15 = 65
12+13+14+15+16 = 70


¿Qué está pasando?
5x3=15
5x4=20
5x5=25
5x6=30
5x7=35
5x8=40
5x9=45
1+2+3+4+5
4+5+6+7+8
2+3+4+5+6
3+4+5+6+7
5(n)
n = El numero medio.
n = middle number