WHAT MADE YOU SAY THAT?

Promoting Mathematical Literacy with Annotation and Recitation

NCTM Annual Meeting
Friday, October 27, 2023

Today's song: "What Made You Say That?" by Shania Twain

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and
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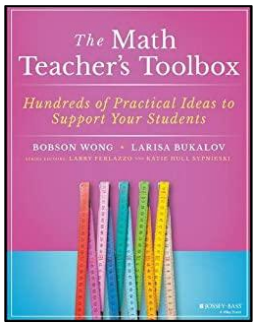
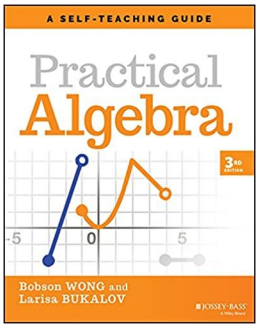
[redacted]

ABOUT US

- Math teachers at Bayside HS (NYC)
- Authors, *The Math Teacher's Toolbox: Hundreds of Practical Ideas to Support Your Students* (Jossey-Bass, 2020) and *Practical Algebra: A Self-Teaching Guide* (3rd edition, Jossey-Bass, 2022)
- Recipients of Math for America Master Teacher Fellowship and the MfA Muller Award for Professional Influence in Education





MATH CLASS OR LANGUAGE CLASS?

As you watch this video, think silently about the following questions:

- What is happening in the class?
- Based on what people are doing, how can you tell whether this is a math class or a language class?

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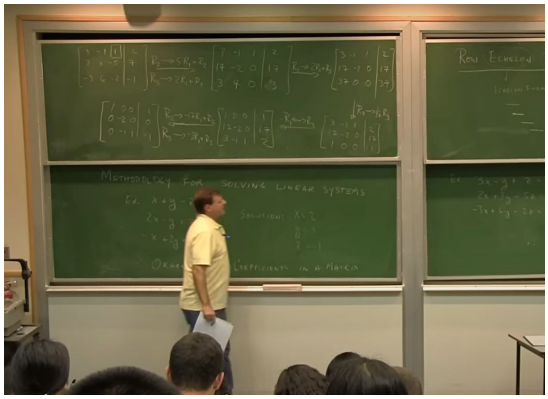
WHAT HAPPENS IN A LANGUAGE CLASSROOM?

- Speaking, listening, acting and interacting
- Experiencing culture, not just language
- Actively engaged in communication
- "A child that is…not allowed to use language will learn no language."

WHAT HAPPENS IN A MATH CLASSROOM?



- Listening to the teacher
- Copying from the board
- Direct instruction → guided practice → independent practice
- Worksheets, worksheets, and more worksheets…

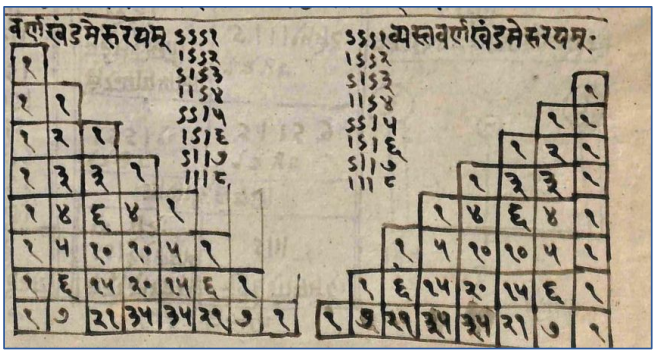
IS MATH A LANGUAGE?

A language is a systematic way of communicating ideas using symbols, sounds, and words that follow a set of accepted rules.

(Oxford Dictionary, Merriam-Webster Dictionary)

Math:

- Is systematic
- Is used to communicate ideas
- Uses symbols, sounds, and words
- Follows a set of accepted rules



MATH VS. OTHER LANGUAGES

Any language

SPEAKING

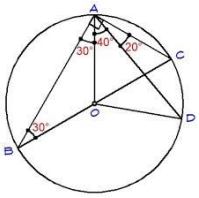
LISTENING

READING

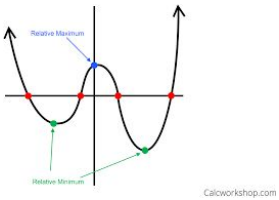
WRITING

Mathematics

INTERPRETING CHARTS



INTERPRETING DIAGRAMS



INTERPRETING GRAPHS

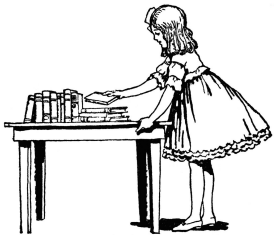
13. This month, Kami sold 70 figurines in 2 sizes. The large figurines sold for $12 each, and the small figurines sold for $8 each. The amount of money he received from the sales of the large figurines was equal to the amount of money he received from the sales of the small figurines. How many large figurines did Kami sell this month?

A. 20
B. 28
C. 35
D. 42
E. 50

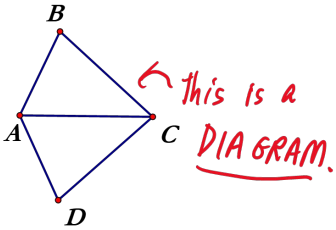
INTERPRETING WORDS

WHAT WE SAY VS. WHAT STUDENTS HEAR

"Put it in a table."



"Label the diagram."

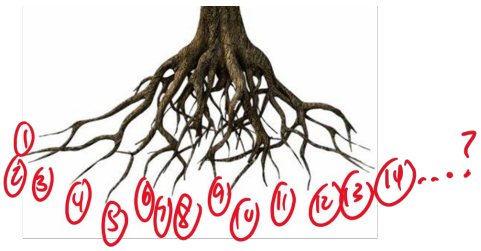


"Find x."

$3x + 4 = 17$

Here it is!

"List all roots."



WHAT DO DEVELOPING MATH LEARNERS THINK?

Determine the first four terms of the sequence.

$f(1) = 8,\ f(n) = f(n-1) + 4\ (n \geq 2)$

"F-one"?

"F times n"??

Our school has only 2 terms!

Where are the numbers?

WHAT DO FLUENT MATH LEARNERS THINK?

List first four numbers

Determine the first four terms of the sequence.

$f(1) = 8, f(n) = f(n-1) + 4 \ (n \geq 2)$

"f of 1 equals 8" — 1st term is 8

"f of n equals f of n-minus-1 plus 4" — for the nth term, take the previous term and add 4

"for n greater than or equal to 2"

$f(1) = 8$

$f(2) = 8 + 4 = 12$

$f(3) = 12 + 4 = 16$

$f(4) = 16 + 4 = 20$

$\boxed{8, 12, 16, 20}$

HOW DOES MATH ANXIETY AFFECT LEARNING?

- Feelings of fear and tension when doing math
- Physiological response that affects heart rate and neural activity
- *Students who feel anxious about math cannot learn.*
- Embarrassment stops language learners from making progress.

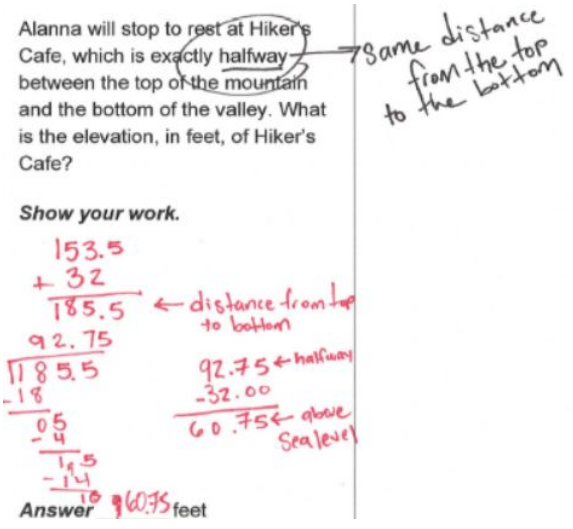
BUILDING MATHEMATICAL FLUENCY

How do we make students more confident and literate in mathematical language?

- Annotation (writing)
- Recitation (speaking)

WHAT IS ANNOTATION?

- DEFINITION: interacting with a text to understand and remember it
- More than underlining, copying, and highlighting
  - Original text → simpler English → picture → math symbols
- Annotation is a way to encourage students to process problems.



ANNOTATION: MODEL EXAMPLE

RESULT START

What is the image of $(-4,-5)$ after a dilation by a scale factor of 2 centered at the origin? ← (0,0)

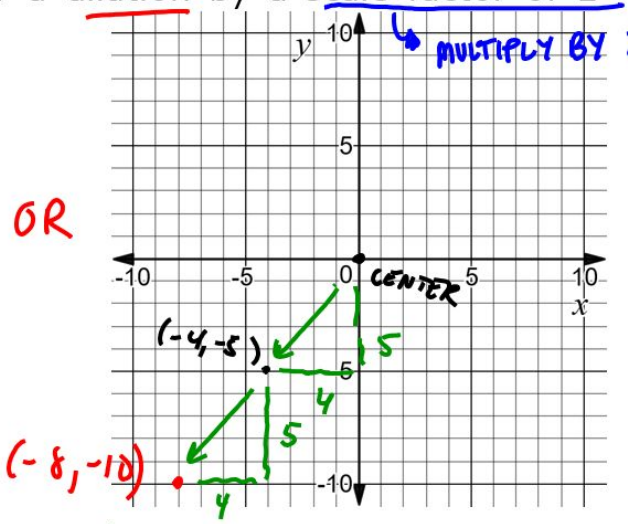
← make bigger or smaller

↳ MULTIPLY BY 2

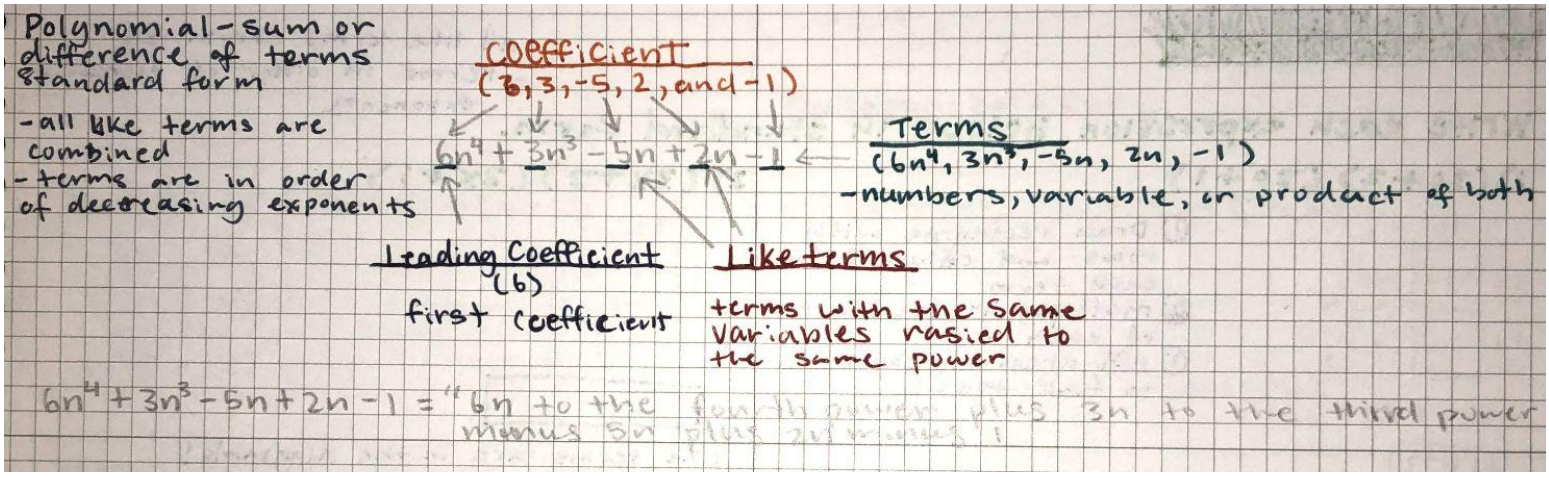
START: $(-4,-5)$

DILATE: $(-4(2),-5(2))$ OR

END: $(-8,-10)$



ANNOTATION: STUDENT EXAMPLE



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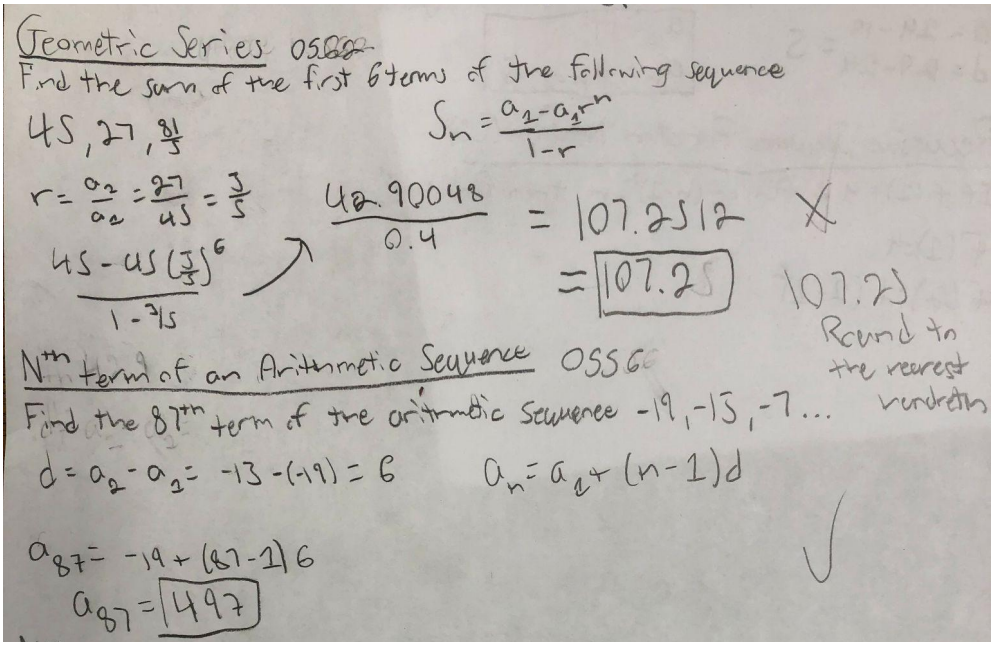
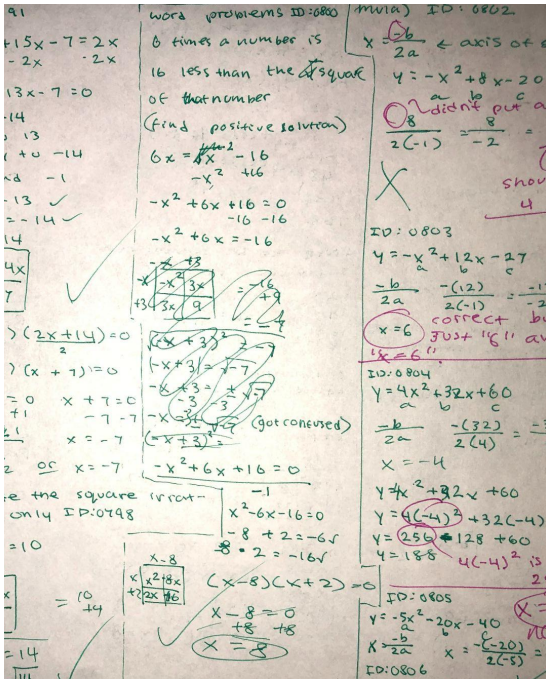




ANNOTATION ACTIVITY

On your handout, solve #1 and #2. Then annotate your solution with comments that you would want your students to write down. Work with a partner.

1. A contractor is building the base of a circular fountain. On the blueprint, the base of the fountain has a diameter of 18 centimeters. The blueprint has a scale of three centimeters to four feet. What will be the actual area of the base of the fountain, in square feet, after it is built? Round your answer to the nearest tenth of a square foot.
2. Express $\sqrt{48}$ in simplest radical form.

ANNOTATION #1

A contractor is building the base of a circular fountain. On the blueprint, the base of the fountain has a diameter of 18 centimeters. The blueprint has a scale of three centimeters to four feet. What will be the actual area of the base of the fountain, in square feet, after it is built? Round your answer to the nearest tenth of a square foot.

Scale - 3 cm = 4 ft

blueprint diameter - 18 cm

(1) Convert from blueprint to reality

Keep units lined up: $\frac{cm}{ft} \rightarrow \frac{3}{4} = \frac{18}{x}$

PROPORTION: product of means = product of extremes

$\frac{12}{3} = \frac{3x}{3}$

$24 = x$

(2) Find real area of circle

diameter of actual = 24 feet

$\frac{24}{2} = 12 \rightarrow$ radius

RADIUS = $\frac{\text{diameter}}{2}$

AREA of circle $\rightarrow A = \pi r^2$

$A = \pi(12)^2$

$A = 144\pi$

ROUND correctly (look at hundredths place)

$A = 452.389 \approx \boxed{452.4\ ft^2}$

ANNOTATION #2

DO NOT APPROXIMATE or calculator

Express in simplest radical form: $\sqrt{48}$

"square root of 48"

NO PERFECT SQUARES in RADICAND (number under square root symbol)

Find the LARGEST perfect square factor of 48

$= \sqrt{16 \cdot 3}$

perfect square

use Multiplication property of Radicals

$= \sqrt{16}\sqrt{3}$

simplify perfect square radical

$= \boxed{4\sqrt{3}}$

OR

FACTOR 48 into primes

48 → 6, 8 → (2)(3), 4 → (2)(2)

$\sqrt{48}$

$= \sqrt{2}\sqrt{2}\sqrt{2}\sqrt{2}\sqrt{3}$

$= 2 \cdot 2\sqrt{3}$

$= \boxed{4\sqrt{3}}$

PERFECT SQUARES (look for these factors):

$1 = 1^2$	$121 = 11^2$
$4 = 2^2$	$144 = 12^2$
$9 = 3^2$	$169 = 13^2$
$16 = 4^2$	$196 = 14^2$
$25 = 5^2$	$225 = 15^2$
$36 = 6^2$	$256 = 16^2$
$49 = 7^2$	$289 = 17^2$
$64 = 8^2$	$324 = 18^2$
$81 = 9^2$	$361 = 19^2$
$100 = 10^2$	$400 = 20^2$

WHAT DO YOU NOTICE? WHAT DO YOU WONDER?

7. If we want to represent the product of a number *represented by a letter, as* **a,** *by itself a certain number of times, instead of writing* **aa,** *or* **aaa,** *etc., as we might, we write* a^2, a^3, *etc.*

Thus b^4 means the same as $bbbb$. a^2 is read "a square;" a^3, "a cube;" b^4, "b fourth power;" x^5, "x fifth power," etc.

1. Read m^2. What does it mean? How otherwise could you write it?

8. The little figure *placed at the right and a little above the letter is one form of what is called an* **Exponent.**

Introduction to Algebra, Edward Olney (1878)

29. 1. How many days are 5 days, 4 days, and 3 days?

2. How many d's are $5d$, $4d$, $3d$, and $6d$?

3. How many c's are $6c$, $3c$, $5c$, and $2c$?

4. How many ab's are $3ab$, $2ab$, $4ab$, and $5ab$?

5. When no sign is prefixed to a quantity, what sign is it assumed to have?

6. When positive quantities are added, what is the sign of the sum?

7. If Henry owes one boy 3 cents, another 5 cents, and another 6 cents, how much does he owe?

8. If the sign $-$ is placed before each sum that he owes, what sign should be placed before the entire sum?

9. What sign will the sum of negative quantities have?

10. If a vessel sails $+5$ mi., $+8$ mi., $+9$ mi., and is driven back -4 mi., -2 mi., -6 mi., how far is she from the sailing port?

Elements of Algebra, William J. Milne (1894)

WHAT DO YOU NOTICE? WHAT DO YOU WONDER?

NOTATION AND NUMERATION

ORAL EXERCISE

1. What name do you give to 3 tens? to 10 tens?

2. What name do you give to 1000 thousands?

3. Looking at your fingers, do you see why people came to count as they do, by tens? Why was it?

4. Read aloud these numbers:

1,427	2,341	4,004	99,999	10,010
234,567	381,426	500,500	203,203	990,990
3,505,246	8,421,205	6,006,006	17,243,321	82,603,627

ORAL EXERCISE

1. State rapidly two one-figure numbers whose sum is 11; also two whose sum is 12; 13; 14; 15; 16; 17; 18.

2. Write upon the blackboard the series 7, 0, 8, 6, 9, 5, 4, 2, 1, 3, and read the numbers, each increased by 2; by 7.

Much drill work of this kind should be given. A short time daily devoted to such oral work is wisely spent.

Practical Arithmetic, David Eugene Smith (1905)

NUMBERS TO BE WRITTEN.

1. Twenty-three; twenty-four; twenty-five; twenty-six; twenty-seven; twenty-eight; twenty-nine.

2. Thirty-seven; forty-two; fifty-six; sixty-nine; seventy-three; eighty-seven; ninety-four.

3. Eighty-three; forty-five; ninety-nine; fifty-one; thirty-six: seventy-eight; sixty-two.

4. Fifty-five; ninety-three; eighty-one; sixty-seven; forty-nine; seventy-four; thirty-eight.

5. Seventy-six; forty-four; eighty-two; fifty-seven; thirty-five; ninety-one; sixty-three.

NUMBERS TO BE READ.

1.	71;	32;	53;	84;	65;	46;	97.
2.	58;	34;	79;	66;	41;	85;	92.
3.	75;	43;	88;	61;	59;	33;	95.
4.	39;	72;	54;	86;	47;	98;	64.
5.	68;	77;	31;	89;	52;	96;	48.

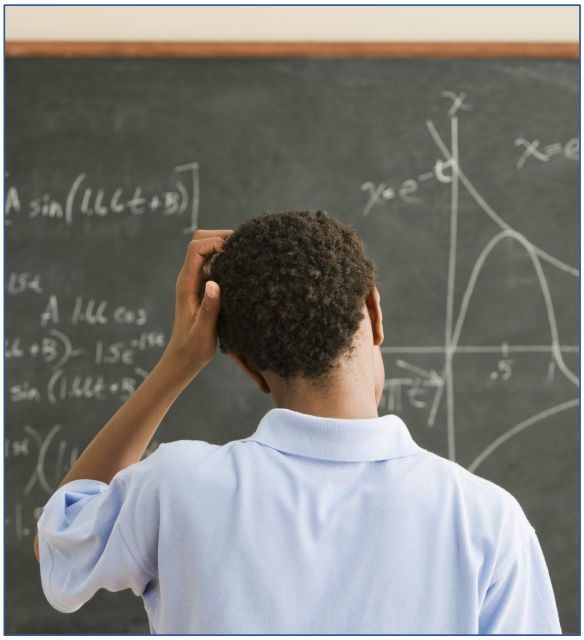
Ray's New Practical Arithmetic, Joseph Ray (1877)

WHAT IS RECITATION?

- **Recitation** was an important part of 19th-century education.
- Teachers called on students individually to recite passages or answer questions without notes, while other students worked independently.
- Recitation is still used today in some rural schools.

WHY DOES PRONUNCIATION MATTER?

- Students who struggle to pronounce a word struggle to interpret it.
- Practicing reading, writing, and speaking reduces students' math anxiety.



TEACHING PRONUNCIATION

Exercises

Write the pronunciation of each expression.

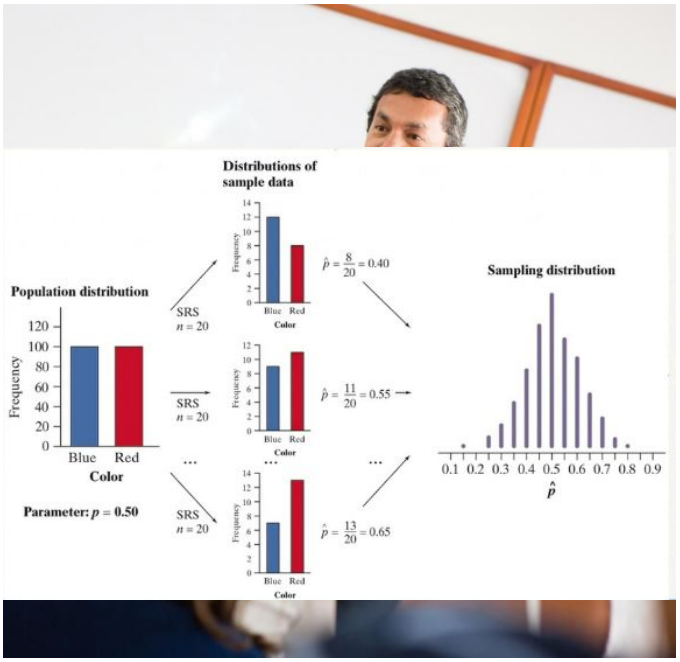
1. $-8 - (-12)$
2. $(+1) - (+3)$
3. $(+6) + (-4)$
4. $(+7)(+15)$

Source: Wong, B. & Bukalov, L., *Practical algebra: A self-teaching guide*, 3rd edition (Jossey-Bass, 2022).

Symbol	Pronunciation	Meaning
3 + 5	"three plus five"	addition
+3	"positive three"	positive
3 – 5	"three minus five"	subtraction
–5	"negative five"	negative
3 ± 5	"three plus or minus five"	plus or minus
±3	"positive or negative 3"	positive or negative
|3|	"the absolute value of three"	absolute value
3(5)	"three times five"	multiplication
(3)(5)		
3 · 5		
3 × 5		

CHORAL RESPONSE

- Students repeat or rephrase what teacher says
- Show vocabulary with picture
- Example: The **sampling distribution of a statistic** describes the values of the statistic in all possible samples of the same size from the same population.



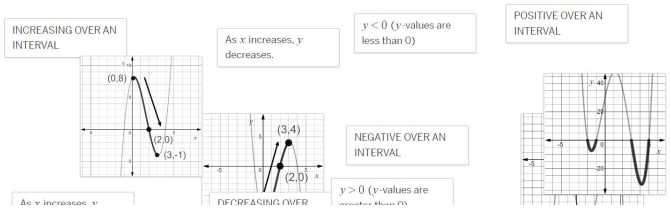
CARD SORTS

- Students organize cards into groups that are somehow connected.
- Can be done in Desmos
- Think about it: how does this stimulate student discussion?

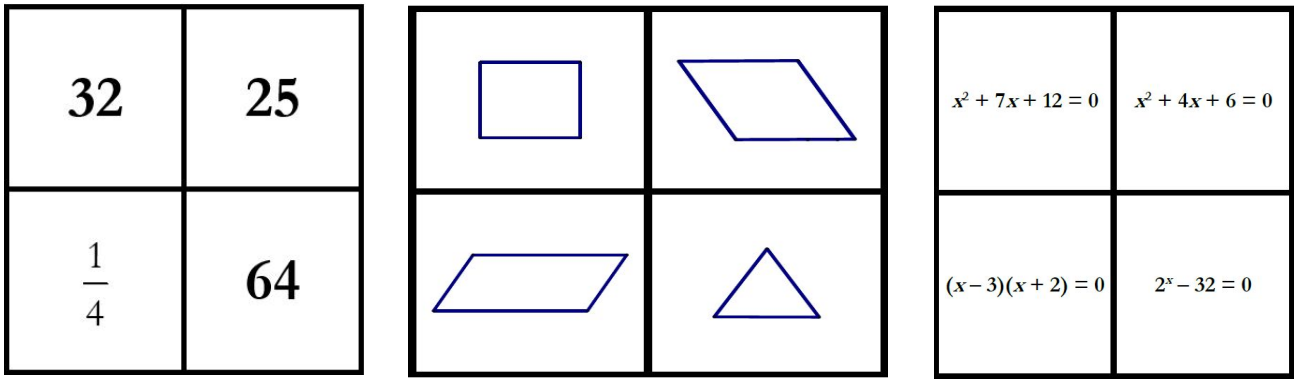
Go to student.desmos.com and type in:

AG7 QDC

https://student.desmos.com/join/ag7qdc



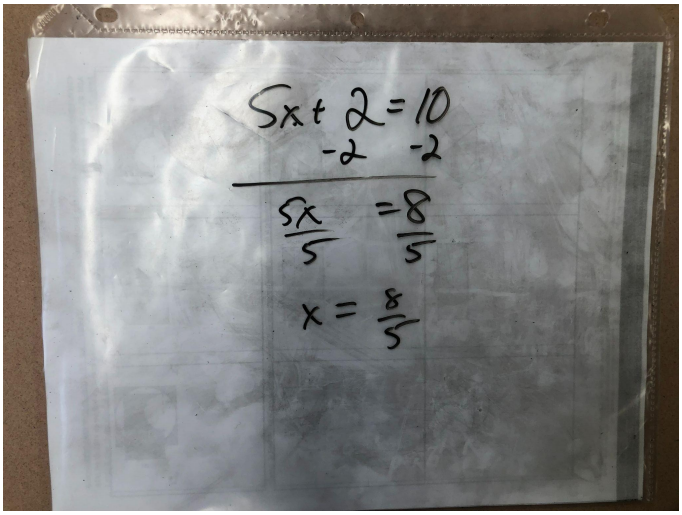
WHICH ONE DOESN'T BELONG?



- *Which One Doesn't Belong* (Stenhouse) by Christopher Danielson (http://wodb.ca)
- Open-ended introductory question: every box can be correct
- Can work at *any* level with almost *any* lesson
- Stimulates discussion for *all* students

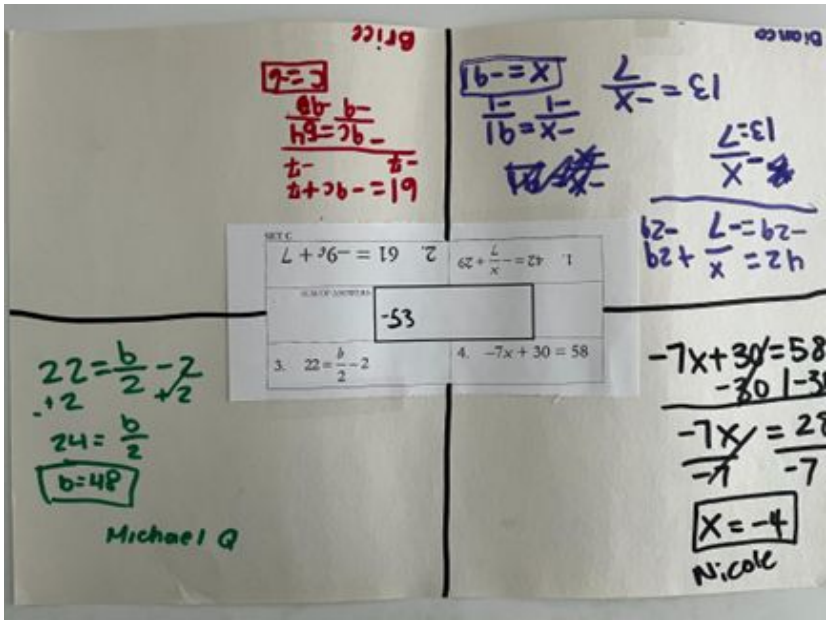
WHITEBOARDS

- Students work in pairs to plan their solution and write their work on whiteboards.
- Students can instantly share work with others.
- Can be done online with tools like Desmos.



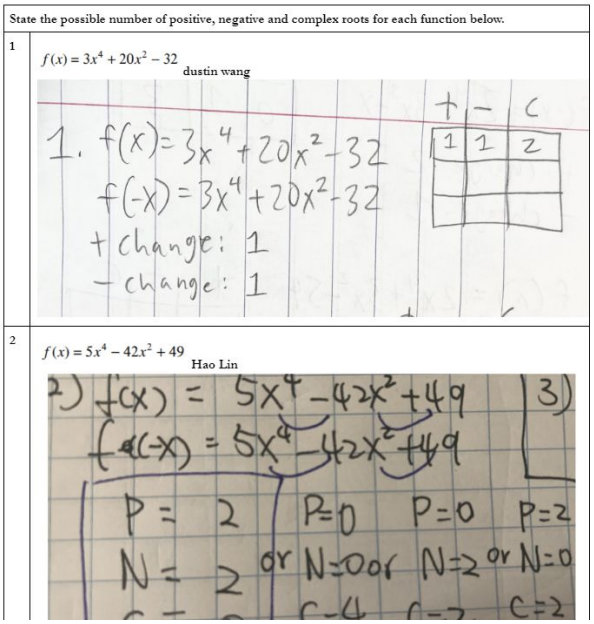
PLACEMATS

- Students work in groups of 4 to solve problems on a large sheet.
- Students write the sum of their solutions in the box in the middle.
- Check the work by verifying the group's sum.



HOMEWORK REVIEW

- How do you encourage mathematical conversations during homework review?
- Student video: https://drive.google.com/file/d/1jdE8iLPcCFiS3BIw4IYJmf9Lo8RM-Ufr/view?usp=drivesdk

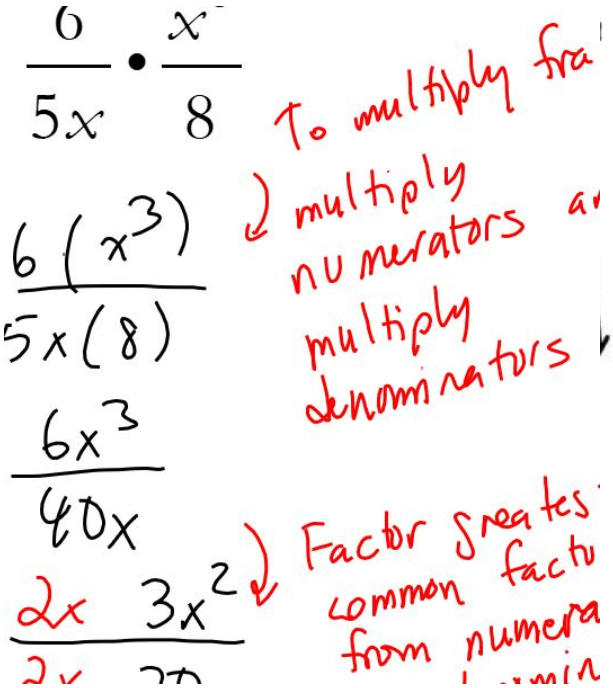


ORAL QUIZZES

- What questions would you ask?
- How and when would students answer?
- How would you assess students?
  - See the oral presentation rubric
- How do you find the time to quiz students?

CHALLENGES OF ANNOTATION AND RECITATION

- They take time away from practice.
- They require more planning.
- They can overwhelm students.
- They require a shift in mindset.
  - "Can I *read and the work* without help?"
  - "Can I *make sense* of the work without help?"

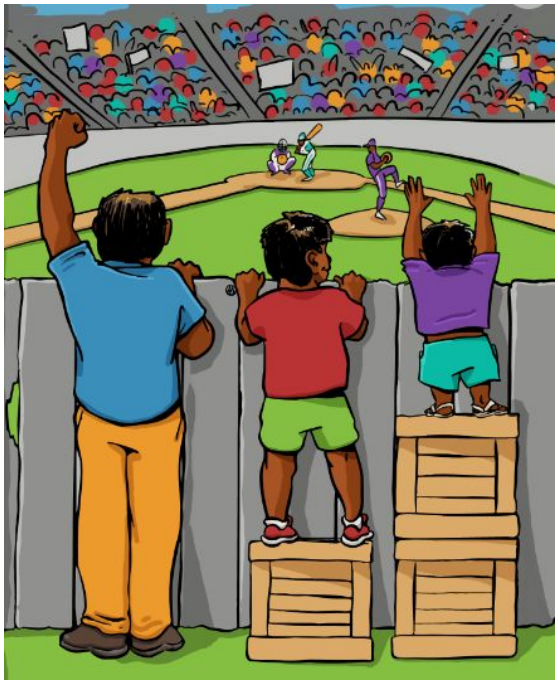


CONCLUSION: WHY USE ANNOTATION AND RECITATION?

- Explaining math by speaking and writing makes mathematics more understandable and more meaningful.
- When students can rephrase and summarize math in their own words, they are better able to remember it.

ANNOTATION, RECITATION, AND EQUITY

- Equity = redistributing resources to provide access to meet student needs
- Annotation and recitation empower students by improving their fluency in mathematical language.
- Improving students' mathematical language skills builds trust with students, which helps make math more accessible to them.



USEFUL RESOURCES

Wong, B. & Bukalov, L. *The math teacher's toolbox: Hundreds of practical ideas to support your students*. San Francisco, Jossey-Bass, 2020. http://bit.ly/math-teachers-toolbox

Wong, B. & Bukalov, L. *Practical algebra: A self-teaching guide (3rd ed.)*. San Francisco, Jossey-Bass, 2022. https://bit.ly/practicalalgebra

Lee, P. (2023, March 10). "A different approach to teaching annotation," *Edutopia*. https://www.edutopia.org/article/teaching-annotation-observation.

Sousa, D. A. *How the brain learns*. Thousand Oaks, Corwin, 2017.

THANK YOU!

- Questions or comments?
  - Bobson: mr@bobsonwong.com, http://bobsonwong.com, @bobsonwong (Twitter, Blue Sky, Mastodon)
- For more ideas, see:
  - *The Math Teacher's Toolbox*
    https://bit.ly/math-teachers-toolbox
  - *Practical Algebra: A Self-Teaching Guide* (3rd edition)
    https://bit.ly/practicalalgebra