What Mathematics Do Students Need for College? A Data-Informed Discussion

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Session Goals

In this session, participants will:

1. Examine data from a survey of university faculty representing different majors about the necessity of a large range of mathematical and statistical topics, including mathematical and statistical practices.

2. Analyze the mathematical and statistical priorities for students in different clusters of majors based on that data.

3. Discuss those priorities in light of the broader purposes of mathematics described in Catalyzing Change and other documents.

Agenda

1. Introduction and context for the discussion
2. Design of a survey of university faculty
3. An interactive exploration of the results of the survey, including implications for different majors
4. A discussion of next steps and how the findings might be used to further critical conversations about mathematics locally and more broadly

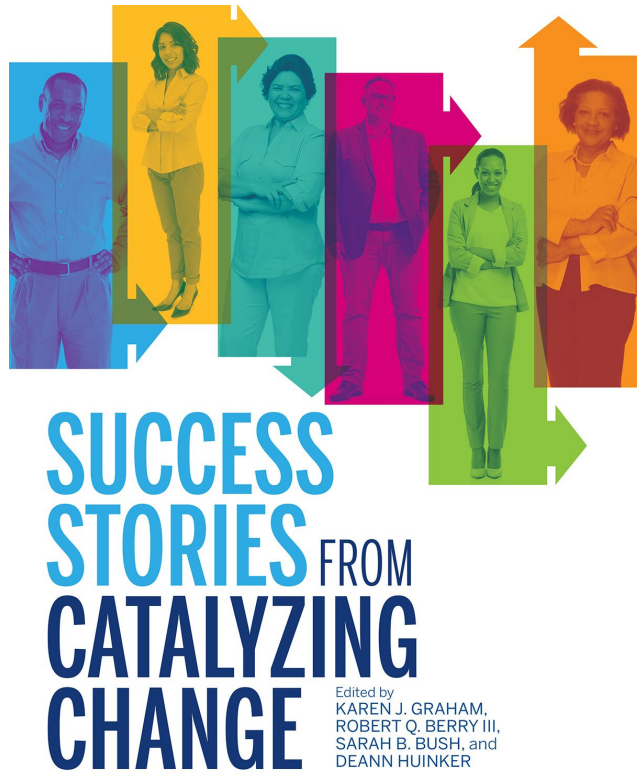
Revision of the *Alabama Course of Study: Mathematics*

The committee faced a number of issues as they began to revise the document, including:

1. There was not a clear pathway through or definition of the "required" mathematics content needed by all students.
2. Schools were allowed to spread both Algebra I and Geometry over two years.
3. Eighth grade algebra was creating problems when some students were not making the intended progress.
4. The high school courses had too many standards to teach at the necessary level of rigor.
5. Statistics and data analysis were not meaningfully included in the high school mathematics curriculum.
6. The majority of post-Algebra II courses did not appear to lead anywhere or serve any purpose other than providing students a fourth credit in mathematics.

Catalyzing Change in High School Mathematics helped guide our response to these issues

Learn more in our chapter from *Success Stories from Catalyzing Change*



Focus for This Talk

The previous Course of Study had too many post-Algebra II courses that did not appear to lead anywhere or serve any purpose other than providing students a fourth credit in mathematics.

Purposes for Teaching High School Mathematics

Each and every student should [study high school mathematics] in order to:

- expand professional opportunities,
- understand and critique the world, and
- experience the joy, wonder, and beauty of mathematics. (NCTM, 2018, p. 9)

Our Solution

The subcommittee focused on the ***professional needs*** students may have, then used a backwards design process, beginning by identifying potential postsecondary pathways, careers, and opportunities.

The team engaged community stakeholders, future employers, and community college and university professors to determine what mathematics would be necessary for students to be successful in whatever pathway they chose.

Our assumption was that ***any course we developed*** should also address to goals of:

- understanding and critiquing the world, and
- experiencing the joy, wonder, and beauty of mathematics.

“Specialized” Mathematics Courses

- ***Precalculus*** -- designed for students entering a major in college for which calculus is required, focusing on prerequisites for studying calculus
- ***Mathematical Modeling*** -- designed for students entering a major in college which involves mathematics but not necessarily calculus
- ***Applications of Finite Mathematics*** -- designed for students entering a major in college which does not involve mathematics or entering a technical field of study

Each of these courses is designed to prepare students for success in the postsecondary study of mathematics, careers, and **their lifelong use and enjoyment of mathematics.**

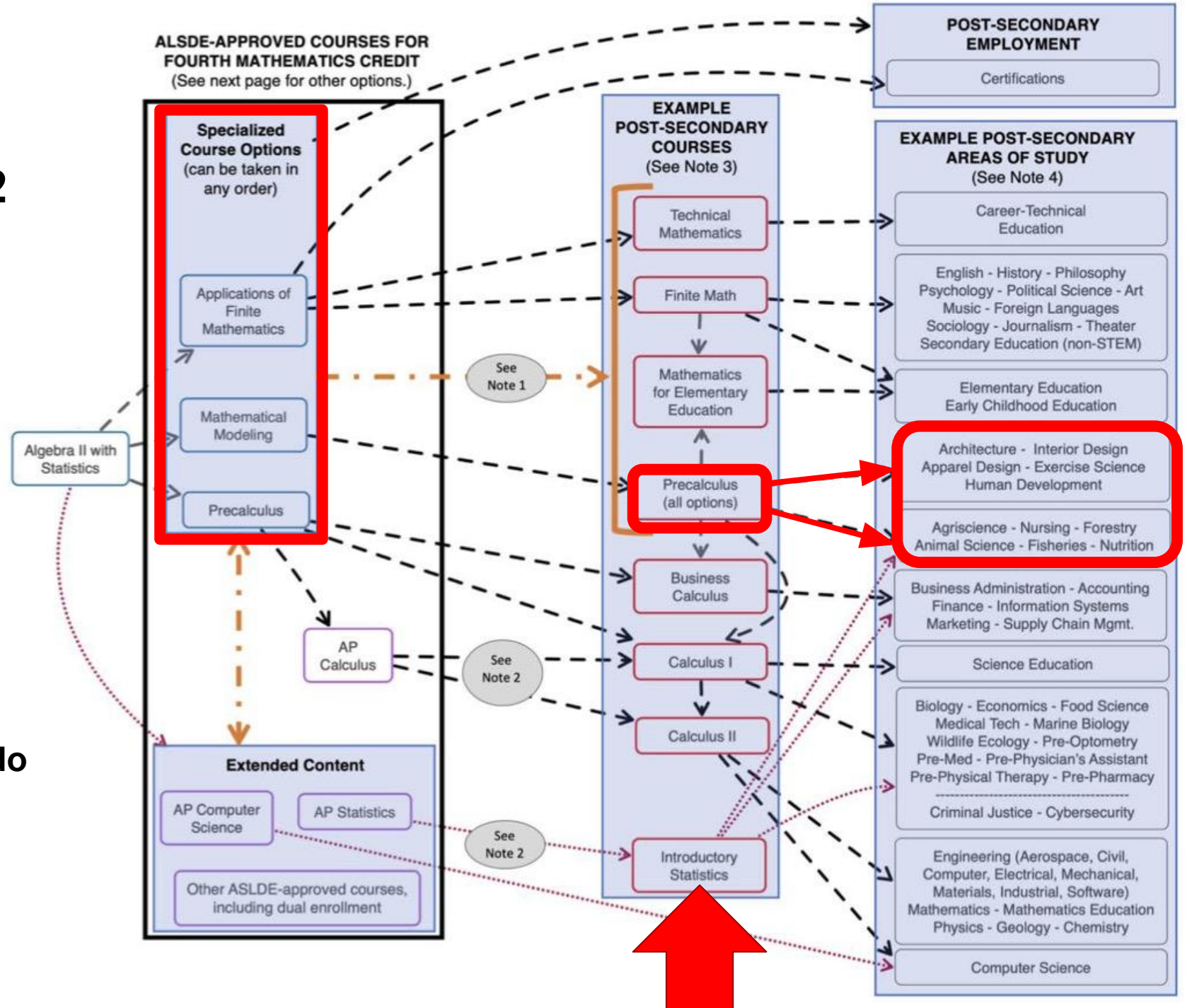
Note that AP Calculus and AP Statistics (and selected other courses) can also be taken for a credit in mathematics.

Course of Study,

Appendix B, Chart 2: Pathways through K-12 Mathematics to Postsecondary

The Big Question:

- Our courses are based on the introductory mathematics course required by various majors.
- But are they really the right courses for those majors?
- **For example, if these majors do not require calculus, why do they need PRE-calculus?**



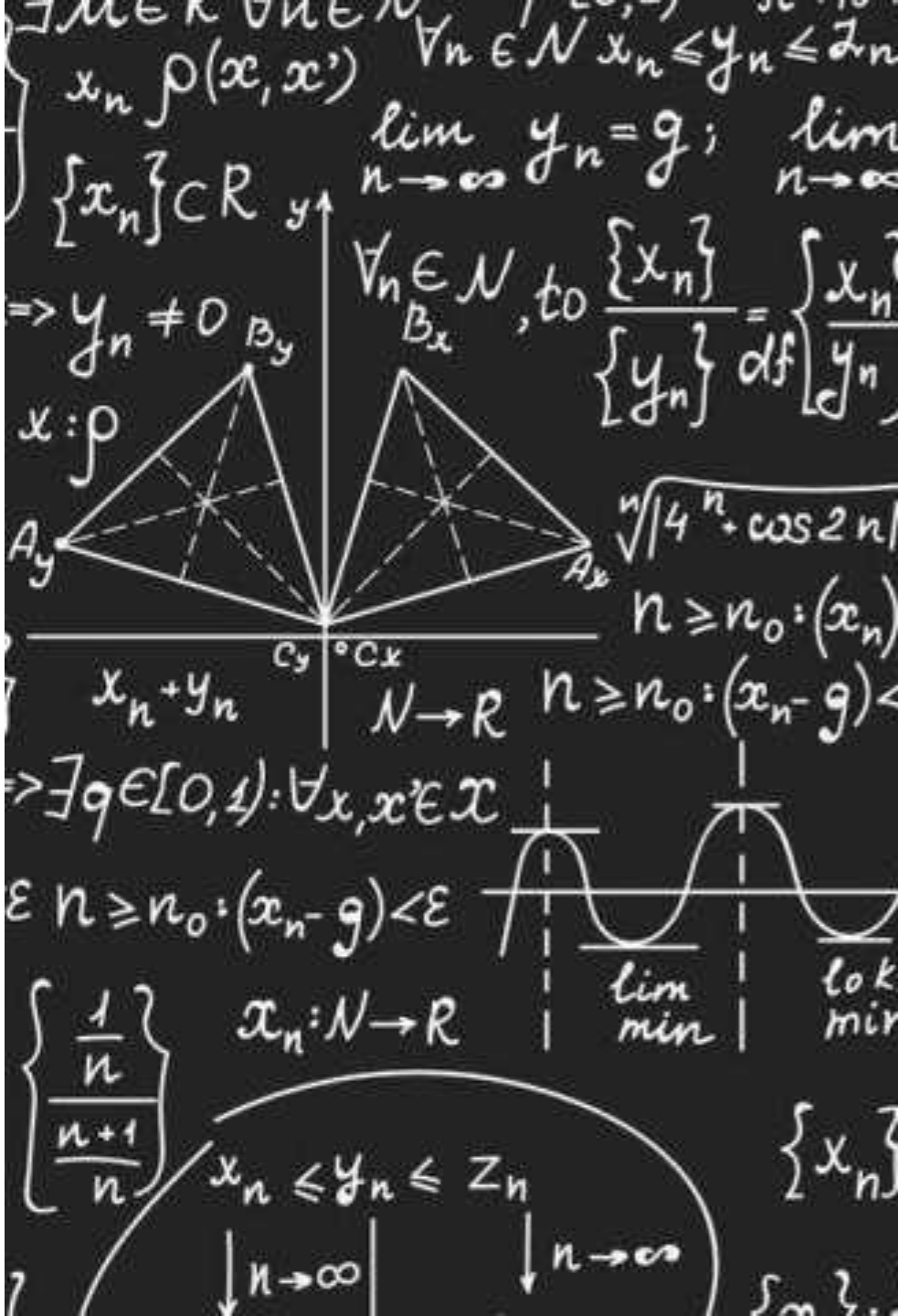
Meanwhile, back at the ranch…

ALABAMA GATEWAY MATH OPTIONS GROUP

A working group of post-secondary institutions in Alabama was tasked with re-evaluating the state's mathematics general education requirement.

Based on past research, the group hypothesized that the content of a typical College Algebra course might not align well with non-STEM majors' mathematical needs. However, consensus on what content should be addressed remained elusive.

Consequently, a subcommittee was formed to gather comprehensive data on what mathematics students across different majors should know.



Purpose of the Survey

To identify the mathematical needs of students in non-mathematics intensive majors.

Survey Procedure

Step 1. Academic leaders at all two- and four-year IHEs in Alabama were asked to provide a contact name for each degree/certificate program in ACHE's list of degree **for which calculus is not a prerequisite**.

Step 2. A survey was sent out to each contact asking them to rate the importance of a wide range of mathematics concepts, skills, and processes for students in their program.

Structure of the Survey

Respondents asked to rate how important 14 areas of mathematics are for students in their program.

- Areas were compiled by looking at syllabi of existing courses, courses in other states, and MAA and ASA standards.

If they rated an area as important or very important, they were asked additional details about that area of mathematics.

Mathematical areas

- Students will understand properties of **functions** (domain, range, compositions, inverses) and use algebraic, symbolic, graphical, and numerical techniques to model related contexts and solve related equations.
- Students can analyze the structure of **equations or inequalities** to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.
- Students can use properties of **triangles, circles, and/or trigonometry** to solve problems.
- Students should explain and apply basic notions of **symbolic logic** and analyze arguments by means of symbolic propositional logic.
- Students should understand and be able to use the mathematical **language of sets** and use this language in different fields of mathematics and in the solving of real-life situations.
- Students can use and create mathematical models involving growth and decay in solving real-world problems involving **personal finance**.

Which of the following mathematical practices are needed by students in your degree (or program)?

- Interpreting quantitative information
- Strategically evaluating, inferring and reasoning
- Manipulating mathematical expressions and computing quantities
- Communicating mathematical ideas in various forms
- Make sense of quantitative problems and persevere in solving them.
- Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Look for patterns and relationships and make generalizations.

Example

What best describes the importance of the
statement about students in degree (or pr
Students will understand properties of
(domain, range, compositions, inverses
algebraic, symbolic, graphical, and nur
techniques to model related contexts a
equations.

- ○ Definitely needed
- ○ Somewhat needed
- ○ Not needed

Please mark the topics needed for students in your degree (or program), .

- ☐ Find the domain of a function
- ☐ Find the range of a function
- ☐ Use compositions of functions
- ☐ Find the inverse function
- ☐ Use the algebraic representation of a function to
- ☐ Sketch the graph of a function by hand using pro
- ☐ Sketch the graph of a function using technology
- ☐ Find numerical approximations of solutions using calculator, computer programs, etc.)

Which types of functions are needed for students in your degree (or program), .

- ☐ Absolute value functions: $f(x) = |x|$
- ☐ Linear functions: $f(x) = 3x - 2$
- ☐ Exponential functions: $f(t) = 10,000 \cdot (1.07)^t$
- ☐ Logarithmic functions: $f(x) = 2 \cdot \log_2(4 - x)$
- ☐ Quadratic functions: $f(x) = -x^2 + \frac{1}{4}x - \frac{2}{3}$
- ☐ Polynomial functions: $f(x) = (x - 3)(2x + 5)(4 - x)(x^2 + 1)$
- ☐ Rational functions: $f(x) = \frac{x - 2}{(x^2 + 4)(3x + 1)}$
- ☐ Radical functions: $f(x) = \sqrt{x}$

Statistical areas

- Students should become critical **consumers** of statistically-based results reported.
- Students should be able to recognize **statistical questions** and design appropriate statistical studies.
- Students should be able to produce and interpret **data visualizations, numerical summaries, and statistical models**.
- Students should recognize and be able to explain the central role of **variability and randomness** in the field of statistics.
- Students should demonstrate an understanding of, and ability to use, basic ideas of **statistical inference**, both hypothesis tests and interval estimation, in a variety of settings.
- Students should be able to interpret and draw conclusions from standard output from mathematical and statistical **software packages**.
- Students should demonstrate an awareness of **ethical issues** associated with sound statistical practice.
- Students can use tools and techniques involving the **theory of probability** to understand the nature of chance and to quantify variation.

Responses

1,047 surveys were sent (not counting incorrect contacts, not giving permission, etc.)

298 completed responses (meaning they hit the “complete” button, many skipped questions)

Overall response rate: 28.4%

Response rates by institution varied greatly, from 0% to 60%

- 33% for 4-years
- 22% for 2-years

Meta-majors

Colleges create meta-majors to group majors with similar interests and prerequisite courses, facilitating students' choices in their first year (Waugh, 2016).

To help account for the potential variations in mathematical and statistical needs across non-calculus majors, we examined these needs within five meta-majors that do not require calculus:

- Data Sciences
- Natural Sciences (not physics, chemistry, or math)
- Social Sciences (including education and public safety)
- Business (includes marketing, and management fields)
- Humanities (languages, arts, communication)

Degrees by Meta-major

	Percentage of Degrees in AL
Calculus-based STEM	12%
Data Sciences	4%
Natural Sciences (not physics, chemistry, or math)	20%
Social Sciences (including education and public safety)	25%
Business (includes marketing, and management fields)	22%
Humanities (languages, arts, communication)	15%
Other	1%

Data Analysis

- We set the null hypothesis to be that half of the programs would report that their students need a given topic and half would not.
  - Simplified to Yes/No
- We then computed a two-tailed test of the null hypothesis with a level of significance of $\alpha=.05$.
  - If a proportion of programs in a meta-major is less than 50% and the corresponding two-tailed p-value is less than 0.05, then we will say that that topic is not needed by most programs in that meta-major.
  - If the proportion of programs in a meta-major is greater than 50% and the corresponding two-tailed p-value is less than 0.05, then we will say that the topic is needed by most programs in that meta-major.

Results for Mathematical Topics

Find some people near you, then....

PREDICT: Which of these topic areas will be needed by students in different meta-majors? (Mark those that you think are a high need.)

	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Properties of Function	☐	☐	☐	☐	☐
Equations/ Inequalities	☐	☐	☐	☐	☐
Geometry/ Trigonometry	☐	☐	☐	☐	☐
Logic	☐	☐	☐	☐	☐
Set Theory	☐	☐	☐	☐	☐
Financial Math	☐	☐	☐	☐	☐

Functions

Students will understand properties of functions (domain, range, compositions, inverses) and use algebraic, symbolic, graphical, and numerical techniques to model related contexts and solve related equations.	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
	94%	87%	70%	73%	29%
Function Types					
Absolute value functions	75%	66%	33%	50%	13%
Linear functions	81%	71%	54%	63%	13%
Exponential functions	69%	66%	32%	47%	4%
Logarithmic functions	50%	47%	11%	30%	2%
Quadratic functions	44%	24%	20%	33%	4%
Polynomial functions	44%	24%	20%	37%	4%
Rational functions	44%	29%	13%	33%	4%
Radical functions	56%	21%	13%	37%	4%

Students will understand properties of functions (domain, range, compositions, inverses) and use algebraic, symbolic, graphical, and numerical techniques to model related contexts and solve related equations.	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
	94%	87%	70%	73%	29%
Function Properties					
Find the domain of a function	56%	34%	28%	37%	6%
Find the range of a function	63%	47%	37%	50%	4%
Use compositions of functions	56%	37%	22%	43%	8%
Find the inverse function	50%	29%	19%	43%	2%
Use the algebraic representation of a function to model contexts	88%	50%	43%	50%	6%
Sketch the graph of a function by hand using properties of the function	56%	42%	37%	53%	8%
Sketch the graph of a function using technology	69%	55%	35%	50%	15%
Find numerical approximations of solutions using technology	81%	79%	59%	67%	17%

Equations and Inequalities

Students can analyze the structure of equations or inequalities to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		94%	74%	63%	80%	35%
	Finding solutions to linear equations and inequalities.	81%	66%	50%	57%	27%
	Finding solutions for systems of linear equations and inequalities.	50%	34%	30%	40%	6%
	Finding solutions for systems of non-linear equations and inequalities.	31%	24%	20%	30%	8%
	Linear Programming (linear optimization)	56%	18%	9%	20%	2%

Geometry and Trigonometry

Students can use properties of triangles, circles, and/or trigonometry to solve problems.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		63%	58%	43%	23%	27%
	Pythagorean Theorem	50%	47%	28%	17%	23%
	Definitions of trigonometric functions on right triangles	38%	18%	6%	10%	6%
	Law of Sines	38%	32%	6%	7%	15%
	Law of Cosines	38%	29%	6%	7%	13%
	Area of a triangle	50%	47%	35%	20%	23%
	Area of a circle	63%	47%	37%	17%	23%
	Circumference of a circle	63%	45%	37%	17%	25%
	Other properties of circles	31%	32%	15%	17%	15%

Logic

Students should explain and apply basic notions of symbolic logic and analyze arguments by means of symbolic propositional logic.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		75%	45%	39%	43%	35%
	Logic statements, quantifiers, and negations	69%	21%	33%	27%	21%
	Truth tables	63%	5%	9%	13%	8%
	Logical equivalence and DeMorgan's Laws	31%	11%	4%	7%	13%
	Valid arguments and fallacies	50%	32%	30%	33%	33%

Set Theory

Students should understand and be able to use the mathematical language of sets and use this language in different fields of mathematics and in the solving of real-life situations.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		81%	53%	48%	53%	23%
	Describe memberships of sets, including the empty set, using proper notation, and decide whether given items are members and determine the cardinality of a given set.	44%	21%	19%	20%	4%
	Describe the relations between sets regarding membership, equality, subset, and proper subset, using proper notation.	50%	21%	22%	33%	8%
	Perform the operations of union, intersection, complement, and difference on sets using proper notation.	44%	24%	13%	30%	6%
	Be able to draw and interpret Venn diagrams of set relations and operations and use Venn diagrams to solve problems.	56%	40%	35%	27%	10%
	Recognize when set theory is applicable to real-life situations, solve real-life problems, and communicate real-life problems and solutions to others.	50%	40%	37%	37%	19%

Financial Mathematics

Students can use and create mathematical models involving growth and decay in solving real-world problems involving personal finance.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		63%	29%	30%	60%	38%
	Organize and display financial information using arithmetic sequences to represent simple interest and straight-line depreciation.	50%	11%	20%	43%	25%
	Organize and display financial information using geometric sequences to represent compound interest and proportional depreciation, including periodic (yearly, monthly, weekly) and continuous compounding.	56%	16%	17%	40%	19%
	Compare and contrast housing finance options including renting, leasing to purchase, purchasing with a mortgage, and purchasing with cash.	38%	16%	17%	40%	29%
	Investigate growth and reduction of credit card debt using spreadsheets, including variables such as beginning balance, payment structures, credits, interest rates, new purchases, finance charges, and fees.	44%	16%	15%	43%	29%
	Investigate the advantages and disadvantages of various means of paying for an automobile, including leasing, purchasing by cash, and purchasing by loan.	38%	11%	17%	37%	29%

Summary of Mathematical Topics

	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Properties of Function	Needed (Linear and Exponential Only)	Needed (Linear and Exponential Only)	Needed (Linear and Exponential Only)	Needed (Linear and Exponential Only)	Not Needed
Equations/ Inequalities	Unknown (Numerical Only)	Unknown (Numerical Only)	Unknown (Numerical Only)	Unknown (Numerical Only)	Not Needed
Geometry/ Trigonometry	Unknown (Not likely)	Unknown (Not likely)	Not Needed	Not Needed	Not Needed
Logic	Unknown (Not likely)	Not Needed	Not Needed	Not Needed	Not Needed
Set Theory	Somewhat Needed	Not Needed	Not Needed	Not Needed	Not Needed
Financial Math	Unknown	Not Needed	Not Needed	Unknown	Not Needed

Conclusions for Mathematical Topics?

How did the results compare to your predictions?

What conclusions can you draw?

Results for Mathematical Practices:

PREDICT: Which of these practices will be needed by students in different meta-majors? (Mark those that you think are a high need.)

	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Interpreting quantitative information	☐	☐	☐	☐	☐
Strategically evaluating, inferring and reasoning	☐	☐	☐	☐	☐
Manipulating mathematical expressions and computing quantities	☐	☐	☐	☐	☐
Communicating mathematical ideas in various forms	☐	☐	☐	☐	☐
Make sense of quantitative problems and persevere in solving them.	☐	☐	☐	☐	☐
Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.	☐	☐	☐	☐	☐
Look for patterns and relationships and make generalizations.	☐	☐	☐	☐	☐

Mathematical Practices

	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Interpreting quantitative information	94%	90%	93%	93%	60%
Strategically evaluating, inferring and reasoning	88%	84%	82%	80%	69%
Manipulating mathematical expressions and computing quantities	75%	37%	32%	47%	13%
Communicating mathematical ideas in various forms	69%	47%	46%	70%	23%
Make sense of quantitative problems and persevere in solving them.	81%	45%	57%	70%	23%
Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.	81%	63%	74%	70%	69%
Look for patterns and relationships and make generalizations.	94%	84%	91%	73%	83%

Conclusions for Mathematical Practices?

How did the results compare to your predictions?

What conclusions can you draw?

Results for Statistics

PREDICT: Which of these practices will be needed by students in different meta-majors? (Mark those that you think are a high need.)

	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Data Analysis Consumers	☐	☐	☐	☐	☐
Statistical Questioning	☐	☐	☐	☐	☐
Data Displays and Computations	☐	☐	☐	☐	☐
Variability	☐	☐	☐	☐	☐
Statistical Inference	☐	☐	☐	☐	☐
Software	☐	☐	☐	☐	☐
Statistical Ethics	☐	☐	☐	☐	☐
Probability	☐	☐	☐	☐	☐

Data Analysis and Statistics Topics

- **Data Analysis Consumers**
  - Students should become critical consumers of statistically-based results reported.
- **Statistical Questions**
  - Students should be able to recognize statistical questions and design appropriate statistical studies.
- **Data Displays and Computations**
  - Students should be able to produce and interpret data visualizations, numerical summaries, and statistical models.
- **Variability**
  - Students should recognize and be able to explain the central role of variability and randomness in the field of statistics.
- **Statistical Inference**
  - Students should demonstrate an understanding of, and ability to use, basic ideas of statistical inference, both hypothesis tests and interval estimation, in a variety of settings.
- **Statistical Software**
  - Students should be able to interpret and draw conclusions from standard output from mathematical and statistical software packages.
- **Statistical Ethics**
  - Students should demonstrate an awareness of ethical issues associated with sound statistical practice.
- **Probability**
  - Students can use tools and techniques involving the theory of probability to understand the nature of chance and to quantify variation.

Data Analysis Consumers

Students should become critical consumers of statistically-based results reported.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		100%	100%	100%	100%	88%
	Identify and define variables	100%	95%	87%	83%	50%
	Classification of a variable as categorical, continuous, nominal, or ordinal.	81%	68%	74%	73%	25%
	Identify cases or observational units of a study and the corresponding generalized population.	81%	90%	80%	80%	56%
	Interpret displays of data and statistical analyses to understand the reasonableness of the claims being presented.	100%	92%	98%	97%	85%

Statistical Questions

Students should be able to recognize statistical questions and design appropriate statistical studies.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		94%	95%	89%	93%	56%
	Writing statistical questions	69%	63%	57%	63%	23%
	Distinguishing between experimental and observational studies	94%	87%	80%	67%	35%
	Applying various sampling methods	69%	90%	61%	73%	31%
	Distinction between probabilistic sampling techniques and non-probabilistic techniques	69%	58%	48%	60%	25%

Data Displays and Computations

Students should be able to produce and interpret data visualizations, numerical summaries, and statistical models. (no meta-major desired computations by hand)		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		100%	97%	96%	100%	65%
	Create dot plots	69%	76%	65%	53%	35%
	Create histograms and bar charts	100%	84%	80%	83%	54%
	Create pie charts	94%	84%	76%	87%	54%
	Create stem-and-leaf plots	38%	53%	44%	23%	17%
	Create box plots	63%	61%	44%	33%	21%
	Create line charts	81%	84%	78%	93%	42%
	Create scatter plots	81%	90%	61%	63%	19%
	Compute linear regression	94%	55%	50%	67%	10%
	Create frequency tables	69%	68%	74%	73%	23%

Data Displays and Computations

Students should be able to produce and interpret data visualizations, numerical summaries, and statistical models.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		100%	97%	96%	100%	65%
	Compute means and/or medians	100%	84%	87%	90%	27%
	Compute proportions	69%	68%	61%	77%	29%
	Compute standard deviations and/or variances	94%	79%	67%	83%	17%
	Compute interquartile ranges	69%	45%	41%	40%	13%
	Compute chi-square values	75%	45%	46%	43%	13%
	Compute ANOVA analyses	75%	50%	43%	40%	13%
	Compute multiple regression	81%	34%	44%	50%	13%

Variability

Students should recognize and be able to explain the central role of variability and randomness in the field of statistics.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		88%	87%	89%	80%	40%
	Role of sample size	88%	87%	85%	73%	35%
	Properties and applications of Normal distributions	88%	76%	78%	67%	25%
	Properties and applications of Uniform distributions	63%	58%	39%	50%	21%
	Central Limit Theorem	63%	40%	43%	53%	15%
	Have a mathematical understanding of "random" (rather than haphazard or unplanned)	81%	74%	65%	63%	27%
	Role of random assignment in comparative experiments	63%	63%	63%	40%	17%
	Difference between correlation and causation	88%	84%	85%	77%	38%

Statistical Inference

Students should demonstrate an understanding of, and ability to use, basic ideas of statistical inference, both hypothesis tests and interval estimation, in a variety of settings.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		88%	82%	87%	93%	42%
	Generating and using simulations with software	50%	45%	35%	53%	10%
	Hypothesis Testing	81%	66%	76%	77%	25%
	Confidence Intervals	75%	74%	63%	77%	13%
	p-values	69%	74%	63%	67%	17%

Statistical Software

Students should be able to interpret and draw conclusions from standard output from mathematical and statistical software packages.	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
	100%	84%	78%	97%	33%
SPSS	50%	45%	50%	43%	10%
R	44%	42%	9%	17%	2%
Graphing calculators	44%	26%	9%	23%	4%
SAS	38%	26%	7%	3%	2%
Spreadsheets (i.e. Excel)	94%	82%	57%	73%	17%
LISREL	0%	0%	0%	3%	0%
MATLAB	38%	21%	2%	3%	0%
Maple/Mathematica	0%	0%	0%	0%	0%
Stata	13%	0%	17%	0%	2%
StatKey	0%	0%	0%	0%	0%
Other	19%	5.3%	4%	10%	2%

Statistical Ethics

Students should demonstrate an awareness of ethical issues associated with sound statistical practice.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		94%	100%	91%	97%	79%
	Application of proper data collection principles of such as human subjects review and informed consent.	69%	68%	85%	43%	63%
	The difference between using statistical methods to inform decisions and abusing data to justify foregone conclusions	94%	95%	80%	90%	63%
	Understand influence and role of confounding variables.	75%	79%	72%	53%	33%
	Understanding impact of multiple testing on false positive rates.	69%	66%	52%	40%	23%

Probability

Students can use tools and techniques involving the theory of probability to understand the nature of chance and to quantify variation.		Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
		94%	84%	72%	77%	33%
	Calculate probabilities by applying probability laws and theoretical results.	88%	53%	52%	50%	17%
	Use the rules of probability to compute probabilities of compound events in a uniform probability model.	69%	24%	20%	33%	13%
	Calculate expected values and use them to solve problems	75%	61%	30%	63%	23%
	Identify an appropriate probability distribution for a given discrete or continuous random variable and use its properties to calculate probabilities.	44%	32%	28%	43%	15%
	Understand independence and conditional probability and use them to interpret data.	56%	29%	35%	47%	21%
	Derive probability distributions of functions of random variables.	44%	32%	13%	23%	13%
	Apply results from large-sample theory and the Central Limit Theorem to approximate a sampling distribution.	50%	26%	30%	27%	13%

Data Analysis and Statistics Topics

	Data Sciences	Natural Sciences	Social Sciences	Business	Humanities
Data Analysis Consumers	Needed	Needed	Needed	Needed	Interpret displays and analyses
Statistical Questioning	Needed	Needed	Needed	Needed	Not likely
Data Displays and Computations	Needed (with technology)	Needed (with technology)	Needed (with technology)	Needed (with technology)	Not needed to create
Variability	Normal Distributions Correlation/Causation	Normal Distributions Correlation/Causation	Normal Distributions Correlation/Causation	Normal Distributions Correlation/Causation	Not likely
Statistical Inference	Needed	Needed	Needed	Needed	Not likely
Software	Spreadsheets+	Spreadsheets	Spreadsheets	Spreadsheets	Not needed
Statistical Ethics	Needed	Needed	Needed	Needed	Needed
Probability	Needed	Some	Some	Some	Not needed

Conclusions for Statistics?

How did the results compare to your predictions?

What conclusions can you draw?

Discussion Questions

1. What are the implications of this work for what should be included in higher education general education mathematics courses?
2. What are the implications of this work for what is emphasized in high school mathematics?
3. How can we maintain a proper balance among the goals for study mathematics, both at the high school and post-secondary levels?
    - expanding professional opportunities,

    as well as

    - understanding and critiquing the world and
    - experiencing the joy, wonder, and beauty of mathematics.
4. How can we expand communication between high school and post-secondary mathematics instructors to better meet the needs of our students?