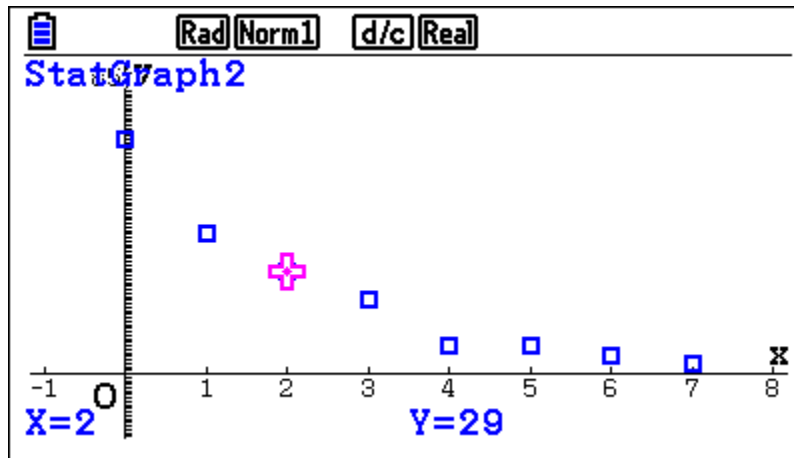


## Exploring Functions and Connecting Algebraic Representations through Hands-On Data Collection



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# Common Core State Standards for Mathematics Addressed

## Standards for Mathematical Practice

### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation.

## Interpreting Functions

### **B. Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

### **C. Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

## Building Functions

### **A. Build a function that models a relationship between two quantities**

Write a function that describes a relationship between two quantities.

## Linear, Quadratic, and Exponential Models

### **Construct and compare linear, quadratic, and exponential models and solve problems**

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**Modeling** (*Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.*)

1. Modeling links classroom mathematics and statistics to everyday life, work, and decision-making.
2. In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO<sub>2</sub> over time.
3. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that is empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

## Statistics and Probability

### **Connections to Functions and Modeling.**

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

### **Summarize, represent, and interpret data on two categorical and quantitative variables**

1. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
2. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.  
*Use given functions or chooses a function suggested by the context.*

Algebra  
The Cheerios Lab #1

Name(s):

Period:

Date:

Materials: small cup of Cheerios, jar lids of various sizes, centimeter ruler

Can you find a relationship between the number of Cheerios that lie flat around the outside of a lid and the diameter of the lid? The goal of this lab is to determine if there is a mathematical model that represents this relationship.

**Investigation**

- A) Measure the diameter of the jar lid in centimeters.
- B) Make a ring of Cheerios around the lid. Do not use broken pieces!
- C) Count the number of Cheerios used for your ring.
- D) Record the diameter and the corresponding number of Cheerios in the table.

Diameter of Lid (in cm.)	Number of Cheerios

**Analysis**

1. Let  $x$  represent diameter of the jar lid, and let  $y$  represent the number of Cheerios. Use an appropriate viewing window and make a scatter plot of the data. What do you notice about the graph?
2. Describe the shape of the graph. Do the points appear to be linear?
3. Use your calculator to find an appropriate model to represent the relationship for the data. Write your equation in the space below with all values rounded to the nearest hundredth. Write the  $r$  and  $r^2$  values obtained from your calculator to the nearest ten thousandth.  
Equation: \_\_\_\_\_  $r =$  \_\_\_\_\_  $r^2 =$  \_\_\_\_\_
4. Store your equation on the [Y=] screen. Graph your equation over the scatterplot. How well does your equation fit your data?
5. What is the real world meaning of the  $y$ -intercept for your equation? Do you think this is possible? Explain!

**TURN OVER!**

6. What is the real world meaning of the slope for your equation? Do you think this is possible? Explain!

7. Use your equation to find the number of Cheerios that are needed to make a ring around a trash can lid that has a diameter of 55 cm.

8. Use your equation to find the diameter of a jar/can lid that you can put 400 Cheerios around.

9. What is a reasonable domain for this model? Explain!

Algebra  
The Cheerios Lab #2

Name(s):

Period:

Date:

Materials: small cup of Cheerios, jar lids of various sizes, centimeter ruler

Can you find a relationship between the number of Cheerios that lie flat inside a lid and the diameter of the lid? The goal of this lab is to determine if there is a mathematical model that represents this relationship.

**Investigation**

- A) Measure the diameter of the jar lid in centimeters.
- B) Fill the lid with Cheerios one layer deep. Do not use broken pieces!
- C) Count the number of Cheerios in the lid.
- D) Record the diameter and the corresponding number of Cheerios in the table.

Diameter of Lid (in cm.)	Number of Cheerios

**Analysis**

1. Let  $x$  represent diameter of the jar lid, and let  $y$  represent the number of Cheerios. Use an appropriate viewing window and make a scatter plot of the data. What do you notice about the graph?

2. Describe the shape of the graph. Do the points appear to be linear or quadratic?

3. Use your calculator to find an appropriate model to represent the relationship for the data. Write your equation in the space below with all values rounded to the nearest hundredth.

Equation: \_\_\_\_\_

4. What are the  $r$  and  $r^2$  value obtained from your calculator rounded to the nearest ten thousandth? What do these numbers represent?

$r$ =\_\_\_\_\_  $r^2$ =\_\_\_\_\_

5. Store your equation on the [Y=] screen. Graph your equation over the scatterplot. How well does your equation fit your data?

**TURN OVER!**

6. What is the real-world meaning of the y-intercept for your equation? Do you think this is possible? Explain!

7. Use your equation to find the number of Cheerios that are needed to fill a trash can lid that has a diameter of 55 cm.

8. If 400 Cheerios were needed to fill the jar lid, what is the diameter of the jar lid?

9. What is a reasonable domain for this model? Explain!

# THE KNOTS LAB

60 Points

NAME(S):

PERIOD:

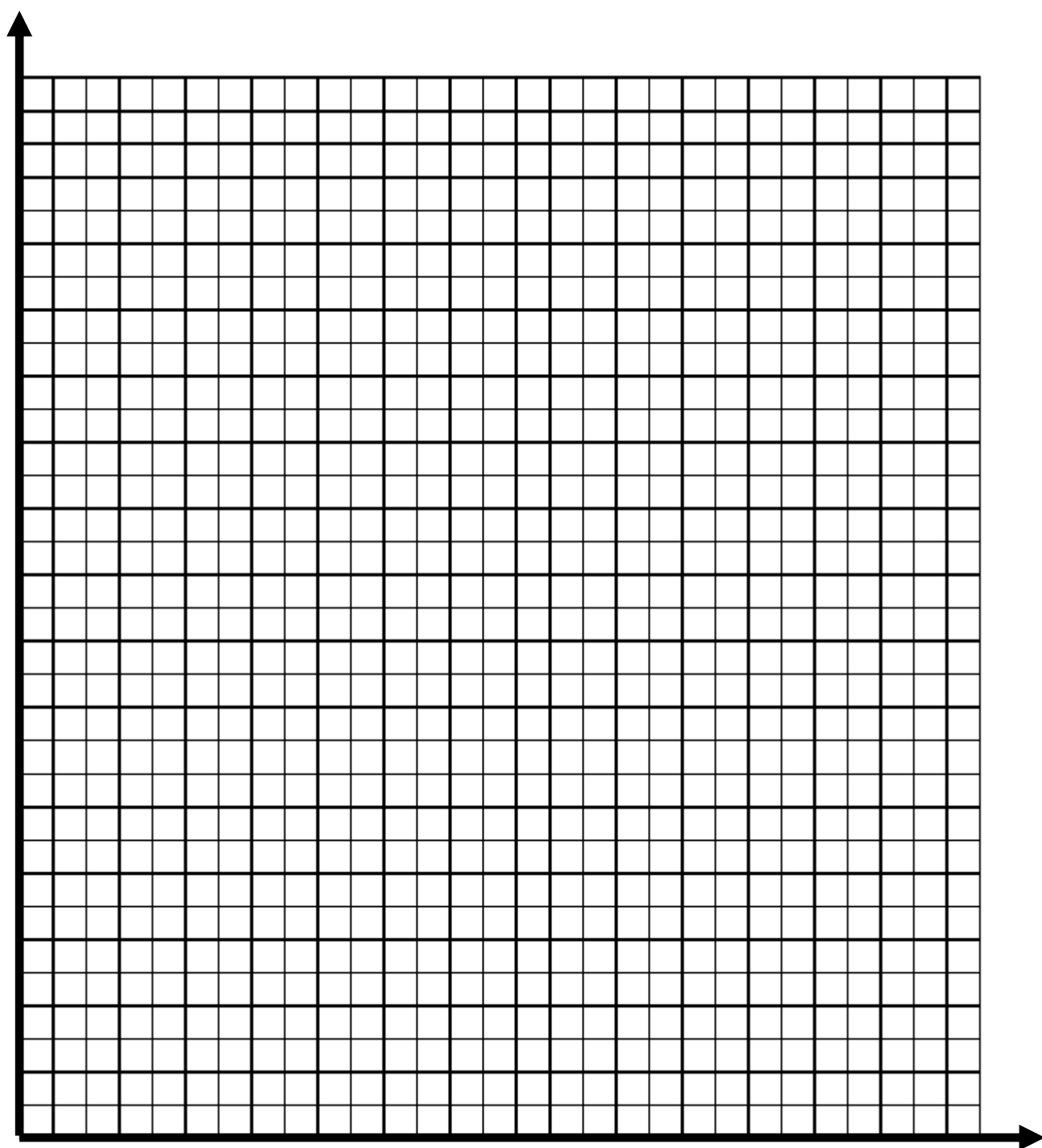
DATE:

1. You will collect data showing how the length of the rope changes as you tie more knots in the rope. Before your group begins, discuss what you expect to find out. Write your groups prediction about what you think will happen in the space below. (2 pts.)

2. Measure the length of the rope before you tie any knots, and record the length in the table below. Tie one knot in the rope, measure the new length, and record it in the data table. Continue tying knots in the rope, measuring and recording the data until you have six or seven knots. [HINT: Tie the knots as close to each other as possible so that you can get the requisite number of knots.] (4 pts.)

Number of Knots	Length of Knotted Rope (cm)
0	
1	
2	
3	
4	
5	
6	
7	

3. Graph your data, the length of the rope is a function of the number of knots. Be sure to write a title on your graph, draw the axes, label your axes and provide a scale! What pattern does the data seem to form? Answer below! (16 pts.)





4. Find the equation of the best-fit line that represents length of the rope as a function of the number of knots. Use the linguine to help you “eyeball” the line. Identify the points your group used to find the equation of this line in slope-intercept form. Show all steps and work. Round all coefficients to the nearest hundredth. **Then graph the line accurately on your scatterplot! (10 pts.)**

The points are:\_\_\_\_\_

The equation of the line is:\_\_\_\_\_

5. What is the slope of your line and what is its real-world meaning? (4 pts.)

6. What is the y-intercept of your line and what is its real-world meaning? (4 pts.)

7. Does the thickness of the rope itself have anything to do with the results? Explain! (4 pts.)

8. Does the type of knot have anything to do with the results? Explain! (4 pts.)

9. Theoretically, what is the maximum number of knots you could tie in your rope? Explain your answer! (3 pts.)

10. What is the x-intercept of your line and what is its real-world meaning? (3 pts.)

11. A rope is 563 cm. long. Each knot changes the length of the rope by 8.5 cm. Write an equation of a line in slope-intercept form that can be used to determine the length of the rope for any given number of knots. (3 pts.)

12. Using your calculator, create a scatterplot and find the line of best fit. What is the equation of the line of best fit determined by the calculator? How close is your equation of best fit line to the line determined by the calculator? Can you explain for any discrepancy if there is any? (3 pts.)

Algebra  
"Double Your Pleasure" Lab

Name(s):

Period:

Date:

**Conducting the Experiment:**

1. Put 4 M&Ms in a cup.
2. Pour the candies onto a sheet of paper. Count the number of M's showing. **Add TWICE this number of M&Ms to your pile.** Record the new total of M&Ms. This constitutes one trial.
3. Return the M&Ms to the cup. Repeat step 2 until you don't have enough to add the appropriate amount.
4. Record your results in a table.

TRIAL NUMBER	NUMBER OF M&Ms	CLASS DATA	MEAN

**Analyzing the Data:**

5. Collect data for the entire class.
6. Find the mean number of M&Ms for the class for each trial number.

7. Make a graph of the mean number of M&Ms versus the trial number. Let  $x$  = the trial number.

8. Discuss the characteristics of your resulting graph.

9. Predict how many trials it would take to have more than 500 M&Ms. Explain how you made your prediction.

10. How would your graph be affected if you started with 10 M&M's instead of 4?

Algebra  
Radioactive Decay Simulation

Name(s):

Period:

Date:

Materials: paper plate, small cup of M&M's, plastic baggie

The particles that make up the atoms of radioactive elements are unstable. In a certain time period, the particles change so that the atoms will become a different element. This process is called **radioactive decay**. The M&M's represent atoms. Each time you repeat the procedure counts as one year.

Investigation

1. Pour the M&M's onto the paper plate so that the candies are one layer thick. Inspect the candies. Eat those candies that *do not* have an "M" showing on one side (Look closely at the yellow ones because the "M" is hard to see.). For our purposes, the candies we use **must** be marked. Count the marked M&M's. Write this number in your table as your starting value.
2. Put all the marked candies into the cup and dump them onto the plate so that the candies are one layer thick.
3. Count all the M&M's with the "M" showing and put them into the baggie. These represent atoms that have decayed, so they are safe for you to eat later.
4. Count the remaining M&M's and record this number in your table. Put them back into the cup.
5. REPEAT STEPS 2,3, and 4 UNTIL ALL THE M&M'S ARE REMOVED. Use as many trials as you need to complete this task.

Time (years)	Number of M&M's Removed	Number of M&M's Remaining	Ratio
0			
1			
2			
3			
4			
5			
6			
7			

Analysis

A. Let  $x$  represent elapsed time in years, and let  $y$  represent the number of M&M's remaining. Use an appropriate viewing window and make a scatter plot of the data. What do you notice about the graph?

B. Calculate the ratio of M&M's remaining between successive years. That is, divide the number of M&M's remaining after year one by the number of M&M's remaining after zero years; then divide the number of M&M's remaining after two years by the number of M&M's remaining after one year; and so on. How do the ratios compare?

C. Choose one representative ratio. Call this ratio  $r$ . This ratio represents the fraction of M&M's remaining. Explain how and why you made your choice.

D. At what rate did your M&M's decay?

E. Write an exponential equation that models the relationship between time elapsed and the number of M&M's remaining.

F. Graph the equation with the scatter plot. How well does it fit the data?

G. In your equation, what is the real world meaning of your value for  $a$ ? Explain!

H. In your equation, what is the real world meaning of your value for  $b$ ? Explain!

I. Using your calculator, find the best fit exponential model, accurate to three decimal places. Write this equation for the best fit exponential model. Graph it. How well does it fit the data?

K. Consider the connection among the numbers in **your** model. How are these numbers related to the number of M&M's you started with?

L. How many years did it take until about half of the original number of M&M's remained?

M. Theoretically, what do you think your value for  $r$  should have been?

N. Can you account for any difference between your experimental value and the theoretical value for  $r$ ?

Algebra  
Radioactive Decay Simulation (Sector)

Name(s):

Period:

Date:

Materials: paper plate with a marked central angle, small cup of M&M's

The particles that make up the atoms of radioactive elements are unstable. In a certain time period, the particles change so that the atoms will become a different element. This process is called **radioactive decay**. The M&M's represent atoms. Each time you repeat the procedure counts as one year.

Investigation

A. Count all the M&M's in your cup. This is your starting amount.

B. Determine the best way to drop the M&M's so they are randomly distributed on the plate. Determine a plan for handling M&M's that miss the plate. Additionally, make a plan for handling those M&M's that land on the lines of your plate. Count all the M&M's.

C. Drop the M&M's so they are randomly distributed on the plate. Those candies that fall in the designated area have decayed and are now safe. Remove them. **You may eat them if you wish!** The remaining M&M's are still radioactive! Count them, and record the number in the table. Scoop them up and drop them randomly over the plate again. **Year 0 is the starting amount.**

D. Repeat step C at least seven times. Record the data pairs (*time, M&M's remaining on the plate*) in the table.

Time (Years)	Number of M&M's Remaining	Ratio
0 (Starting Amount)		
1		
2		
3		
4		
5		
6		
7		

E. Now, after you have completed seven or more trials, end the experiment by counting all the remaining candy that is still radio- active. **Eat the leftover M&M's at your own risk!!!**

Analysis

1. Let  $x$  represent elapsed time in years, and let  $y$  represent the number of M&M's remaining. Use an appropriate viewing window and make a scatter plot of the data. What do you notice about the graph?

2. Calculate the ratio of M&M's remaining between successive years. That is, divide the number of M&M's after year one by the number of M&M's after zero years; then divide the number of M&M's after two years by the number of M&M's after one year; and so on. How do the ratios compare?

3. Choose one representative ratio. Explain how and why you made your choice.

4. At what rate did your M&M's decay?

5. Write an exponential equation that models the relationship between time elapsed and the number of M&M's remaining.

6. Graph the equation with the scatter plot. How well does it fit the data?
7. In your equation, what is the real world meaning of your value for  $a$ ? Explain!
8. In your equation, what is the real world meaning of your value for  $b$ ? Explain!
9. Using your calculator, find the best fit exponential model, accurate to three decimal places. Graph it. How well does it fit the data?
10. Consider the connection among the numbers in your model. How are these numbers related to the number of M&M's you started with and the angle measure on your plate?
11. What theoretical model could you have written without conducting the simulation?
12. Describe the difference between paper plates for the equations  $y = 360(0.72)^x$  and  $y = 360(0.82)^x$ ?

Bonus: What do you think would happen if you changed the angle to an angle whose vertex isn't at the center of the plate?

# THE BUCKET BRIGADE

NAME(S):

PERIOD:

DATE:

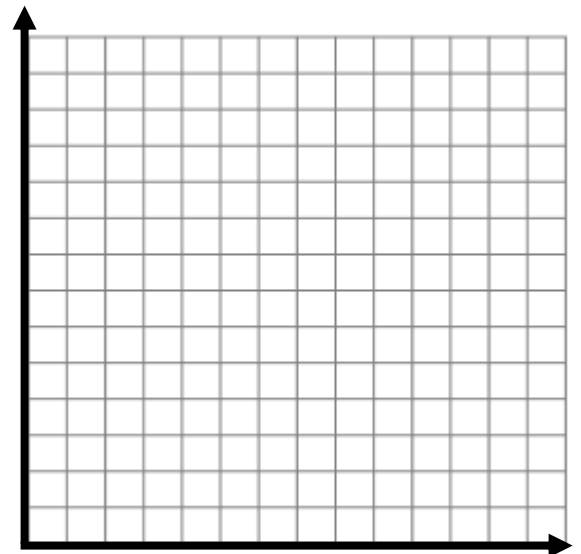
1. Line up a bucket brigade. Everyone should stand in single file. The line may spread around the room. Spread out so that there is an arm's length between every two people.
2. Record the number of people in the line. Starting at one end of the line, pass a bucket as quickly as you can to the other end. Record the total passing time from picking up the bucket to setting it down at the very end.
3. Now have one or two people sit down, and close the gaps in the line. Repeat the bucket passing. Record the new number of people and the new passing time.

4. Continue the bucket brigade until you have collected seven to ten data points in the form (*number of people, passing time in seconds*). Record your data in the table.
5. Let  $x$  represent the number of people, and let  $y$  represent the time in seconds. Plot the data on the graph below. Be sure to write a title on your graph, label your axes, and provide a scale!
6. Find the equation of the best-fit line that represents length of time as a function of the number of people. Use the linguistic to help you "eyeball" the line. Identify the points your group used to find the equation of this line in slope-intercept form. Show all steps and work. Round all coefficients to the nearest hundredth. **Then graph the line accurately on your scatterplot!**

Number of people	Time in seconds

The points are: \_\_\_\_\_

The equation of the line is: \_\_\_\_\_





## Questions

1. What is the slope of your line and what is its real-world meaning?
2. What is the y-intercept of your line and what is its real-world meaning?
3. What is a reasonable domain for this function? Why?
4. How will the graph change if the bucket was heavier or lighter? Explain!
5. What is the x-intercept of your line and what is its real-world meaning?
6. Use your function to determine how long it would take if 100 people passed the bucket.
7. Use your function to determine how long it would take if everyone in the class passed the bucket.
8. Use your function to determine how many people would be in a line if it took one minute to pass the bucket.
9. Using your calculator, create a scatterplot and find the line of best fit. What is the equation of the line of best fit determined by the calculator? How close is **your** equation of best fit line to the line determined by the calculator? Can you explain for any discrepancy if there is any?

Adapted from Discovering Algebra, 3<sup>rd</sup> Edition, Kendall Hunt Publishing Company, Dubuque, Iowa. 2014

Functions, Statistics, and Trigonometry  
Radioactive Decay Simulation 40 Points

Name(s):

Period:

Date:

Materials: paper plate with a marked central angle, protractor, small cup of M&M's, zip lock bsg

Investigation

D. Determine the best way to drop the M&M's so they are randomly distributed on the plate. Determine a plan for handling M&M's that miss the plate.

E. Drop the M&M's so they are randomly distributed on the plate. Those candies that fall in the designated area have decayed and are now safe. Remove them, count them, and record the number in the table. **You may eat them if you wish!** The remaining candy is still radioactive! Scoop these up and drop them randomly over the plate again.

C. Repeat #'s 1 and 2 at least seven times. Record the data pairs (*time, candy removed from the plate*) in the table.

D. Now, after you have completed seven or more trials, end the experiment by counting all the remaining candy that is still radioactive. Record the three values – *time, candy removed from the plate, candy remaining on the plate*. Eat the leftover M&M's at your own risk!!! Or save them for later in the zip lock bag.

E. Calculate the values for the *candy remaining on the plate* by adding the numbers going up from the bottom.

Time	Candy removed from plate	Candy remaining on plate
<b>Starting total</b>		
<b>1</b>		
<b>2</b>		
<b>3</b>		
<b>4</b>		
<b>5</b>		
<b>6</b>		
<b>7</b>		

Analysis

F. Now find an appropriate window and make a scatter plot for the data in the third column (*time, radioactive M&M's*). Use the techniques you learned in class to find an exponential model (of the form  $y=ab^x$ ) that fits the data, accurate to three decimal places. Graph this equation on your scatterplot. Describe the accuracy of your model. Show how you found your model in the space below. Circle your equation! (9 pts.)

G. In your equation, what is the real world meaning of your value for a? Explain! (3 pts.)

H. In your equation, what is the real world meaning of your value for b? Explain! (3 pts.)

I. Using your calculator, find the best fit exponential model, accurate to three decimal places. (4 pts.)

J. What is your value of  $r^2$  (accurate to three decimal places)? What does this tell you about the relationship between your best fit curve and your scatterplot? Explain below! (4 pts.)

K. Consider the connection among the numbers in your model. How are these numbers related to the number of M&M's you started with and the angle measure on your plate? (5 pts.)

L. What theoretical model could you have written without conducting the simulation? (4 pts.)

M. Describe the difference between paper plates for the equations  $y = 360(0.72)^x$  and  $y = 360(0.82)^x$ ? (4 pts.)

N. Make a conjecture about what would happen if you changed the angle to an inscribed angle or an angle whose vertex isn't at the center. (4 pts.)