WHAT DO YOU NOTICE: Strategies for Inquiry with Technology

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Dropbox Folder: http://bit.ly/InquiryStrategies

Inquiry Strategies enable students to use critical thinking to build understanding of math topics. These techniques provide entry points into problem-solving, encourage engagement and sense-making, and can make the math learning deeper and more durable.

A. What do you notice, what do you wonder?

- This strategy is a way to get students involved in a mathematical context before being asked to do something with it.
- **HOW:** Present a graph, equation, problem scenario; give students individual think time to note down what they notice and wonder about the context.
- Sometimes students will attend to mathematical features of the situation that aren't part of the intended lesson objectives—don't be afraid to "take the scenic route" to discuss important math.
- Sources:
  - Search for #NoticeWonder
  - NCTM website: Click on Classroom-Resources>Notice-and-Wonder or this link
  - Annie Fetter's Ignite: Ever Wonder What They'd Notice? https://youtu.be/a-Fth6sOaRA
  - Max Ray-Riek blog: Noticing & Wondering in High School also in our Dropbox

Example 1:

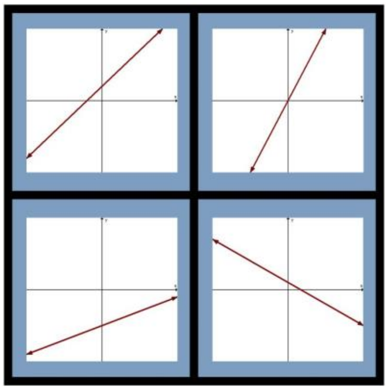
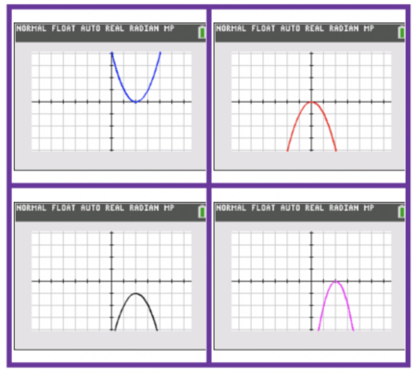
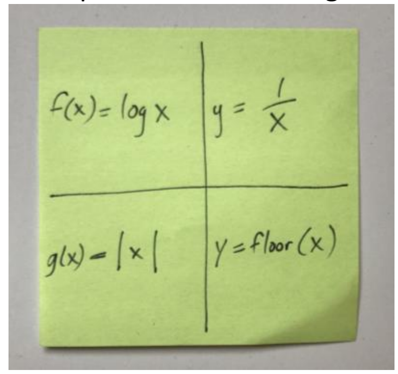
1. On your calculator, graph:	2. On your calculator, graph:
Y1= x^2	**Y1=** x^3
Y2= x^4	**Y2=** x^5
Y3= x^6	**Y3=** x^7
What do you observe?	What do you observe?

Example 2:
Graph the following 3 functions on an appropriate window. What do you notice? What do you wonder?

$y = 2x^2 - 8$ $\qquad$ $y = x^2 - 16$ $\qquad$ $y = \dfrac{2x^2 - 8}{x^2 - 16}$

B. Which One Doesn't Belong?

- This strategy has students generate a reason why each one of the four choices doesn't belong, and justify why their choice is valid. A well-constructed WODB will have good reasons why each of the 4 options does not belong with the other 3.
- **HOW:** Present 4 items, give students individual think time to note down what they think doesn't belong and why. Orchestrate discussion about why EACH of the 4 items might not belong.
- Provides access for all and encourages mathematical thinking, communicating, and justifying (and supports Mathematical Practices 1, 2, 3, 6).
- Sources:
  - Search for #WODB
  - Website http://wodb.ca/ maintained by Mary Bourassa @MaryBourassa
  - Lots of blog posts, including Jennifer Wilson and Mashup Math and ATMIM
  - Change it up: give blank grid, 1 clue at a time, students revise items. From Fawn Nguyen.

<table>
<tr><th>Example 3: source WODB.ca</th><th>Example 4: source Karen Campe</th><th>Example 5: source @tangentz1</th></tr>
<tr><td></td><td>
</td><td>
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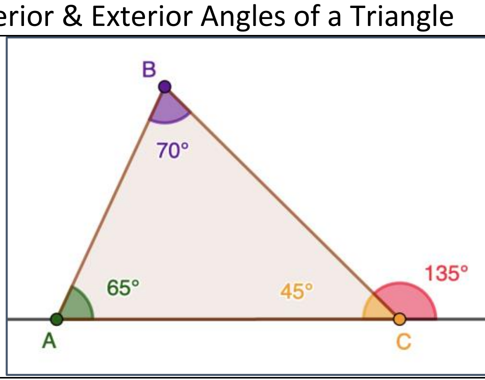
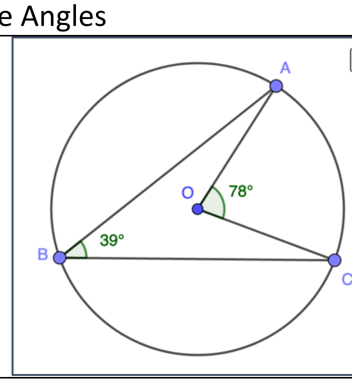
C. Action-Consequence-Reflection: What changes, what stays the same?

- This strategy asks students to perform a mathematical action, observe a math consequence, and reflect on the result, making mathematical meaning.
- Categories of this technique include using graphs/sliders, dynamic tables, and looking for invariants.
- **HOW:** Have students engage in a mathematical <u>action context</u>, ask themselves "what changes, what stays the same?, and <u>record</u> observations, reflections, predictions, conclusions.
- Key components: Require students to record, Ask good questions, Summarize results with class.
  - What will happen if…?
  - What must I change to make … happen?
  - How is … affected by …?
  - What changes, what stays the same?
  - When will … be true?
  - Why does this happen?
- Sources:
  - karendcampe.wordpress.com Reflections & Tangents Blog Action-Consequence Advantage.
  - GeoGebra book Action-Consequence and North American GeoGebra Journal Article.
  - "Table Techniques" Article Mathematics Teacher May 2019, and Teacher Guide.
  - Karen's 7 for 7 talk: The Power of the Action-Consequence-Reflection Cycle Video.

<u>Example 6</u>: Use Sliders to graph the Quadratic Function $Y = Ax^2 + Bx + C$.
How does each parameter affect the graph?

<u>Example 7</u>: Use Sliders to graph the Exponential Function $Y = A^X$.
- What happens as A increases from 2 to 10, incrementing by 1?
- What happens if A = 1 or A = 0? Why?
- What happens as A increases from 2 to 3, incrementing by 0.1? Can you estimate value of *e*?

<u>Example 8 & 9</u>: Searching for <u>Invariants</u> (something about a mathematical situation—a measurement, calculation, shape, or location—that stays the same while other parts of the situation change)

<table>
<tr><th>Interior & Exterior Angles of a Triangle</th><th>Circle Angles</th></tr>
<tr><td>
</td><td>
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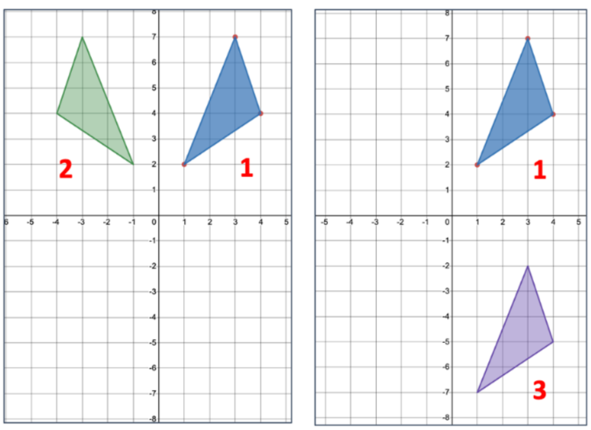
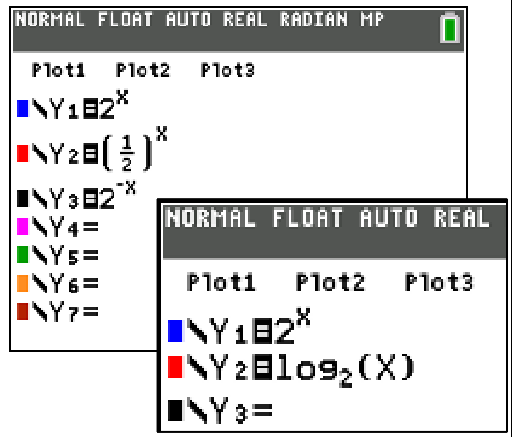
D. Same and Different (Compare and contrast)

- This strategy asks students to compare and contrast features of two mathematical situations. They may require different solution strategies, be similar *except* for one feature, or have mathematically meaningful nuances to notice.
- **HOW:** Present two math situations, have students examine and note how they are the same and how they are different.
- Powerful when Ss must choose among various solving techniques (systems of equations, solving quadratic equations, simplifying exponents & radicals, right triangles, calculus integration).
- Sources:
  - Karen's Reflections & Tangents Blog Same and Different and Same & Different Calculus Edition
  - Search for #SameDifferent
  - https://www.samebutdifferentmath.com/ from Sue Looney (@LooneyMath)
  - https://samedifferentimages.wordpress.com/ & https://minimallydifferent.com/
  - Same Surface Different Deep (SSDD) problems from Craig Barton @mrbartonmaths

Example 10: Comparing Coordinate Transformations



Example 11: Exponential Functions



E. Two Truths and A Lie (Find the Fiction)

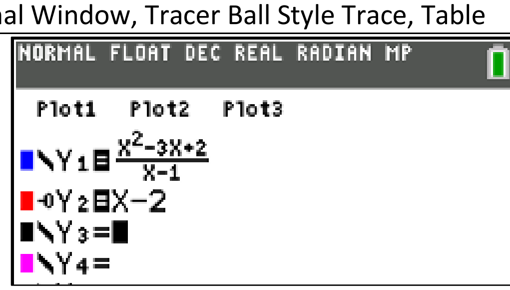
- This strategy asks Ss to distinguish between true and false math statements. These can focus on common misconceptions, uncover a deeper property, or build a mathematical argument.
- **HOW:** Examine/CREATE 3 statements about a math concept, only two of which are true. Identify the wrong statement and be able to explain why or defend your position. Be sure to give individual think time before sharing.
- Use as a warmup to begin discussion, have Ss create these in groups or gallery walk to review a unit, or share on an electronic platform (Desmos Challenge Creator or Google Slides).
- Sources:
  - Jon Orr blog: https://mrorr-isageek.com/better-questions-two-truths-one-lie/
  - Sara Carter template: https://mathequalslove.net/two-truths-and-a-lie-template/
  - Desmos Two Truths activities: HERE (or use search box)

Example 12: Rational Function Graph — ZoomDecimal Window, Tracer Ball Style Trace, Table

The graph of the rational function

$$y = \frac{x^2 - 3x + 2}{x - 1}$$

A. Has Y-intercept (0, –2)
B. Has no vertical asymptotes
C. Is the same as the graph of $y = x - 2$



F. Conjectures & Counterexamples

- This strategy asks Ss to examine/create a conjecture statement and disprove it with one or more counterexamples. Variation on "Always-Sometimes-Never".
- **HOW**: Create a false claim or "always" statement for a situation that is sometimes/never true. Students generate counterexamples and discuss details of math categories and definitions.
    - Can you convince me?
    - How many counterexamples can you find?
    - When is the original true? (What is the "sometimes"?)
- Supports skills for testing hypotheses, experimentation, constructing viable arguments, and communicating about mathematical properties. Can also generate conjectures from rich tasks and "Notice & Wonder" situations.
- Sources:
    - Dan Finkel MathForLove Counterexamples lesson and blog post
    - Paul Gafni post https://paulgafni.com/conjectures/
    - Always Sometimes Never collection from Algebra's Friend https://asnmath.blogspot.com/
    - Sarah Carter's blog https://mathequalslove.net/always-sometimes-or-never-resources-for/

Example 13: Numeric	Example 14: Algebraic
a) Prime numbers are always odd b) I claim that X^2 is greater than X c) The sum of 4 even numbers will always be divisible by 4	a) Graphs of lines always pass through the first quadrant b) $(X + 1)^2 = X^2 + 1$ c) The vertex of a parabola is its maximum value
Example 15: Geometry	Example 16: Functions
a) A line and a circle will have 2 points of intersection b) The supplement of an angle is obtuse c) Any quadrilateral inscribed in a circle must be a square	a) A polynomial of degree 3 will cross the X-axis 3 times [or n^{th} degree crosses n times] b) The graph of a "parent function" passes through the origin. c) I claim $Y = a \bullet b^X$ increases as X increases