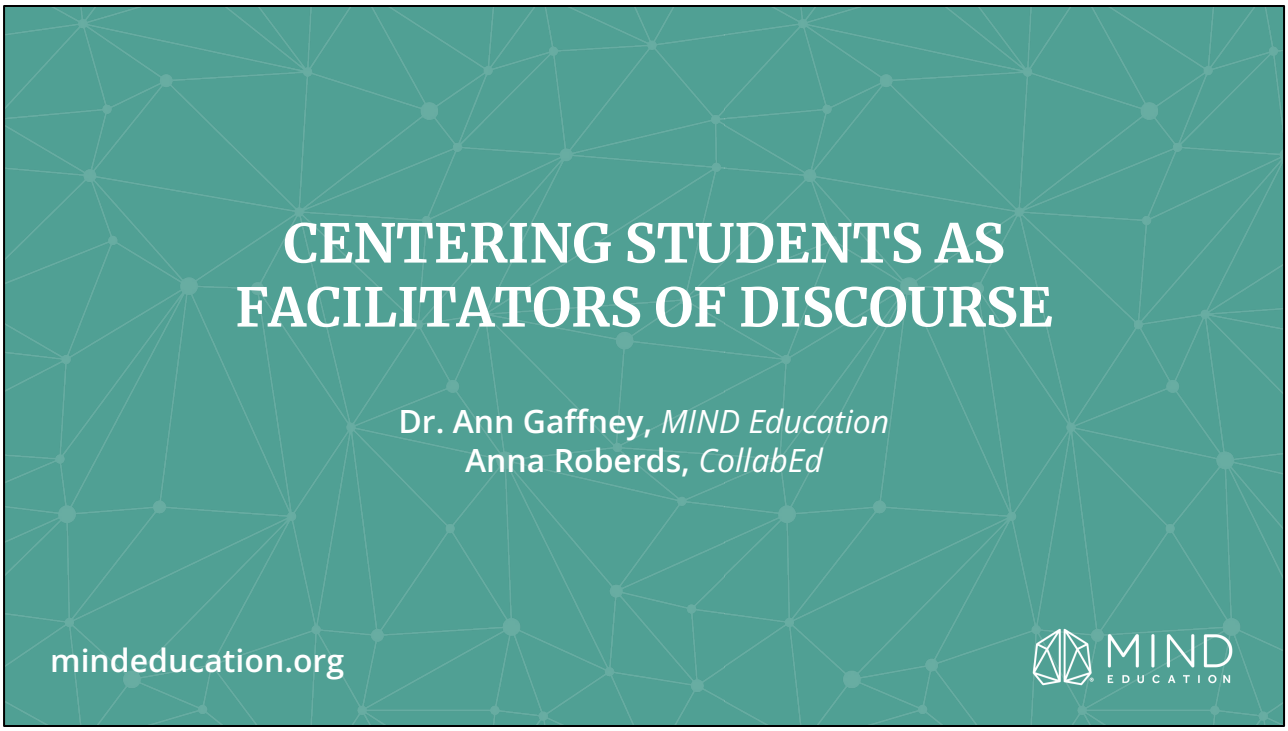
Dear NCTM Participant:
We are making these slides available on the NCTM app with our speaker notes, but please only use them for your personal use. We recognize the importance of being able to preview/review slides and our notes to enhance your learning. That said, these materials do represent our intellectual property and we ask for your help in ensuring that credit is given for our work and ideas.
Thank you!
Dr. Ann Gaffney & Anna Roberds



Materials:

- Paper copy of our speaking schedule and notes
- Bookmarks: print & digital (150: 1/person)
- Graphic Organizer: print & digital (150: 1/person)
- Manipulatives: Two-counter counters (50 groups' worth)
- Chart Paper (50: 1/group)
- Markers (60: 1/group including a few extras in case some are dead)
-

The Task

Roberds leads

LAUNCH

- You are the **students**!
- Experience structures that promote student-to-student discourse
- Afterwards, we will **discuss teacher moves** and **best practices** in the field

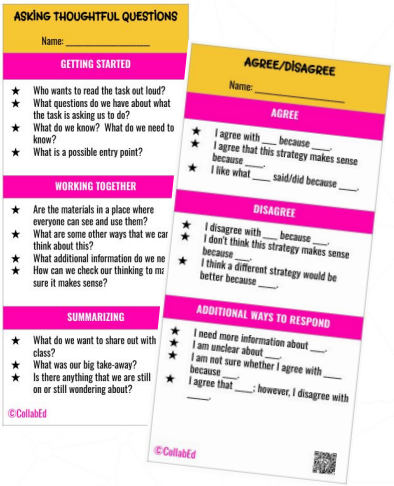
GROUP ROLES

Move yourself into groups of 3-4. Introduce yourself and decide your roles.

Writer I am in charge of managing communication between team members and between the team and the class.	**Team Captain** I am in charge of managing team participation. **note: combine with Reader in groups of 3*
Reader I am in charge of making sure the task is read aloud and monitoring my team's progress through the task.	**Resource Manager** I am in charge of managing the team's resources and encouraging communication, which includes asking **team questions**.

STRENGTHS
THAT WILL HELP YOUR GROUP TODAY...

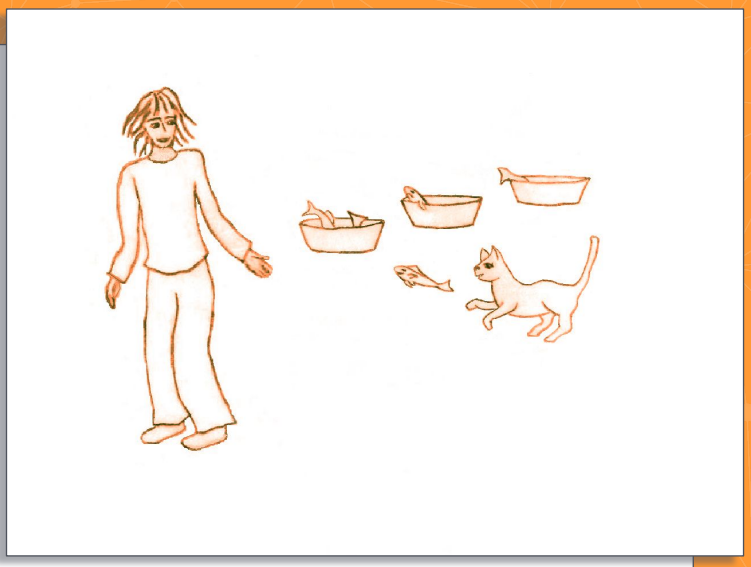
- Understand a complex, multi-step problem and develop a plan for how to solve it
- Use tools strategically
- Ask thoughtful questions that help your group think of things in new ways



Bookmark resource:
https://www.teacherspayteachers.com/Product/Group-Work-Collaborative-Learning-Bookmarks-10382166

THE TASK

A Fishy Story…



Special thanks to:

- Task: Natalya Vinogradova
- Images: Matthew Norris

Both used with permission.

Here's our task:
(click)
A Fishy Story
(click)
Three friends went deep sea fishing. As longtime friends, they decided they would pool all the fish they caught and split the number of fish equally at the end of the trip. The friends caught lots of fish and returned to the dock as it was getting dark. The friends decided to store all the fish in the freezer at the dock and split them up in the morning.
(click)
The first friend got up early the next morning, got some coffee and headed to the fish freezer. She split the number of fish equally into three groups. Finding one fish left over, she tossed the extra fish to the cat, took her share and went home to put the fish in her freezer.
(click)
Sometime later the second friend woke up and headed to the fish freezer. They split the number of fish they found there evenly into three groups and, finding one extra, tossed the extra to the cat. Then they took their share and headed home.
(click)
Finally, you guessed it, the third friend woke up. He evenly split the number of fish, tossed the one remaining fish to the cat, took his share, and headed home.

What mathematical questions might we ask about this situation? Turn and talk with your group.

[Share a few of what we heard while going around]

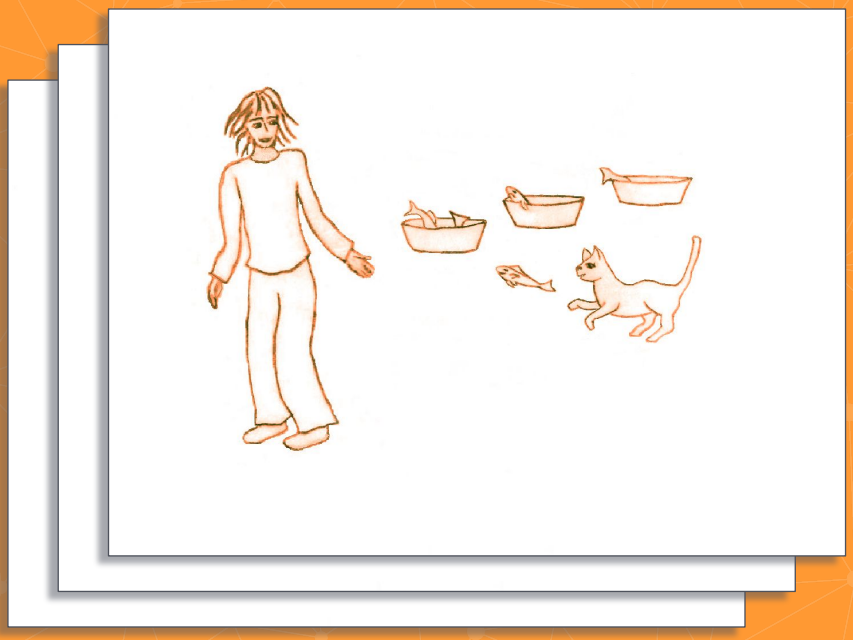
The question we are going to explore today is: How many fish could the friends have caught? We will have the next __ minutes to work on this task together.

THE TASK

A Fishy Story...

Special thanks to:
- Task: Natalya Vinogradova
- Images: Matthew Norris

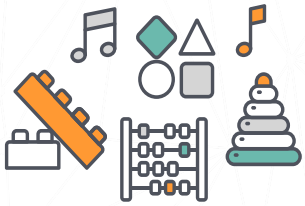
Both used with permission.



Stanford: 10 min
NCTM: 20 min

PARTICIPATION QUIZ

While you were working, we took notes on things that groups were doing that enhanced learning.



Math Tools

Asking Questions

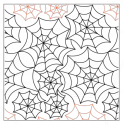
Middle Space

Time to put our TEACHER HATS back on

- Raise your hand if you were engaged in **productive struggle**
- Raise your hand if you **talked** while working
- Raise your hand if you **generated questions**
- Used **group facilitation skills**
- Raise your hand if you are **itching to keep working** :)

TRANSITION MOMENT: Ask teachers to put teacher hats on and put the task away

So, as much as we love the task and we know you are probably dying to keep working on it, we are actually going to focus on HOW this task was set up rather than WHAT you just worked on

DISCOURSE STRUCTURES	1:1 Teacher:Student 	Teacher-Led Discussion 	Student-Led Discussion 
TEACHERS 	• Asking students direct questions		…bserve, chart, …eedback,
STUDENTS 	• Responding to teacher, not talking to each other	• Building off of each other's thinking in facilitated way	ques… • Leading the discussions within groups

Not a bad place to be!

You might be here because...
- Newer to leading discussions
- Start of year
- Younger grades
- The nature of the task/situation

What you just felt, is the distinction between these different discourse structures.

We have outlined three different discourse structures:

- The first column represents the 1:1 Teacher to Student discourse. This functions like a wheel with the teacher at the center with each student as a separate spoke. The teacher is asking students questions and students are responding directly back to the teacher. This has the benefit of increasing student voices in the classroom; however students are not responding to each other.
- The second column is teacher-led discussion. You can imagine a web where a teacher is facilitating a discussion with students responding to each other. When I think of this structure, I think of math talks where the teacher is leading the talk and encouraging students to build off of each other's thinking.
- A lot of our current push towards increasing student engagement and student dialogue has lived in these first two columns [CLICK] and these are not a bad place to be! [read arrow]
- [CLICK] However, we believe that there should be an additional column added to this list of structures….and this is student-led discussions. This is like a multi-dimensional web where the students are the ones who are doing the difficult work of generating thoughtful questions and leading the discourse and the teacher is intentionally shifting into an active observer role where they are then giving students feedback about their collaboration skills and helping groups maintain focus on the learning goals, for example by emphasizing a comment or question that a student has voiced.

- I want you to pause for a moment to think about when you lead class discussions. What work goes into generating discussion questions either in the moment or in your prep work? [PAUSE] This is a really tough skill, but a super necessary one to be a critical thinker and someone who works with others. Our push is to start thinking about how we can support students so that they have a chance to learn and practice these skills in our classrooms.

If you take nothing else away from this presentation, it is this framework shift and the opportunity to be reflective about when and how you use these different structures.

THINK-PAIR-SHARE

What **teacher moves and structures** allowed you to:

- use your **peers as resources**
- **engage in deeper sense-making** opportunities

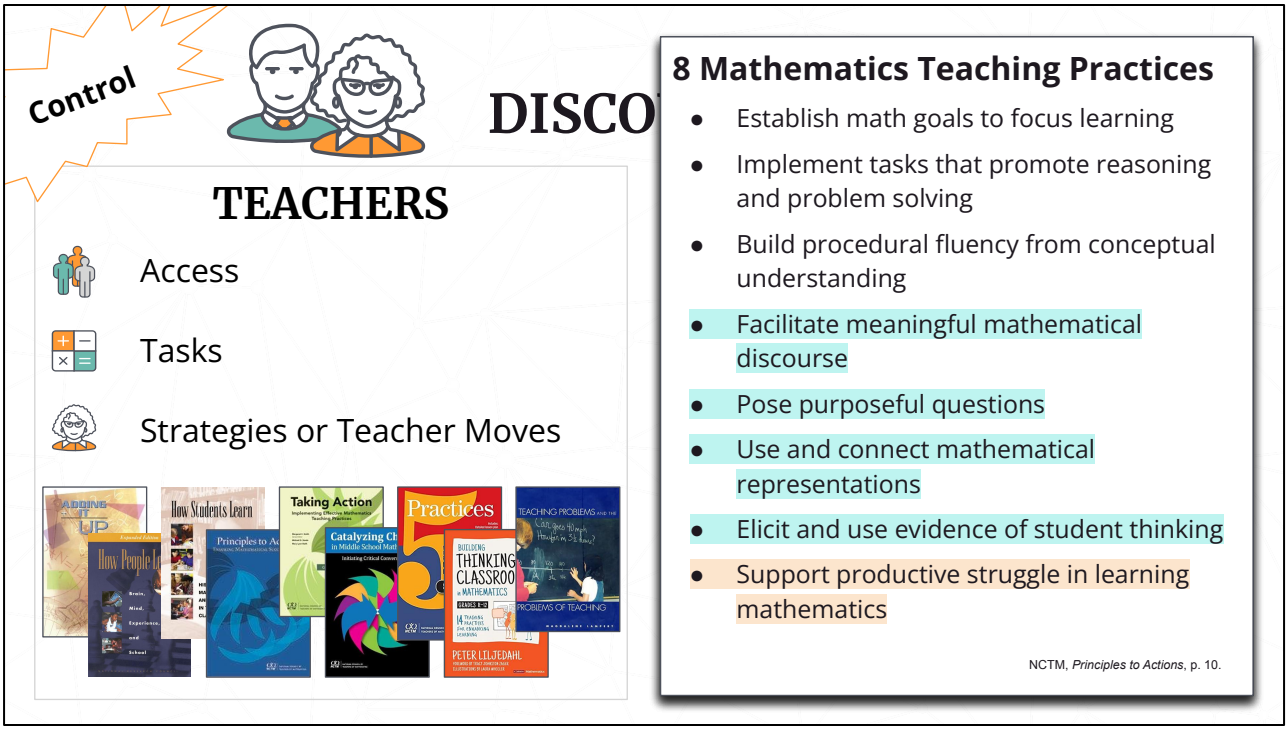
Teacher moves:

- Smith & Stein's moves:
  - **Set goals** for the activity to highlight effective discourse strategies and that positioned participants as knowers, doers, and sensemakers of mathematics
  - **Selected a task** manipulatives are particularly helpful as a status intervention
  - **Anticipated** responses and planned look fors and questions to:
    - **assess** current student thinking and
    - **advance** the thinking of the group;
  - **Monitored** the learning and made use of our questions;
  - **Selected** work to share and
  - **Sequenced** the discussion, in order to
  - **Connect** groups' thinking.
- **Called out strengths** that were not just getting the right answer BEFORE you began working, positioning those strengths as important
- **Designated roles**, provided some safety in structure for those who may need support participating (note: Liljedahl does not support roles and, in his small sample size of three, roles distracted students from thinking about the problem and instead kept them thinking about their roles).
- **Participation Quiz** - reinforced the behaviors we wanted to see in group work
- Set up with **shared materials, limiting resources** (marker)
- Suggested a "**peek around**" as needed

Student moves:

- Used middle space (and/or wall space?)
- Used manipulatives/drawing
- Asked questions of each other

Gaffney leads; Anna pass out graphic organizers subtly.



When looking across the research literature, discourse is one of those "givens" in high quality mathematics instruction, but we wanted to dig into why. Why is it important that we get students talking, and, in particular, why is it important that students control that dialogue?

In 2001 the National Research Council released, *Adding it Up*,
(click)
a compilation of the research on math teaching with a particular focus on the science of learning. In that book, the authors describe instruction as
(click)
"the product of interactions among the teacher, the students, and the mathematics" (p. 313). Both teachers and students contribute to those interactions. If we view teaching and learning as a series of interactions, it is becomes clear how discourse, or talking with each other, is one main component of how we interact and achieve learning. And, in fact, the National Research Council's book *How People Learn*,
(click)
and the corollary directed at teachers, *How Students Learn*, both emphasize the wealth of evidence in the research literature for the role that student discourse plays in developing students' metacognitive processes. This process of thinking about one's thinking and paying attention to feedback and the ways in which feedback enhances one's thinking on a topic, is what we at MIND Education call schema-building, and it's a really important part of effective, long-term conceptual learning (e.g., *How Students Learn*, pp. 236-242).

The vast majority of the educational research on discourse focuses on the **teacher side** of these interactions,
(click)
perhaps because that is the side **that we can control**.
(click)
We see this focus on the teacher side of discourse interactions in the series of NCTM publications on implementing high quality standards-based instruction:
(click)
Principles to Actions, *Taking Action*, and more recently, *Catalyzing Change*. These three publications use the 8 mathematics teaching practices
(click)
as a way to look at effective mathematics teaching, many of you have probably seen these. If we look at those practices, we can see more than half of them
(click)
are actually about discourse, and most of those
(click)
are about aspects of discourse that teachers firmly control, while the fifth is mediated between teachers and students.

And if we look at the research literature more broadly, we actually see many of the same themes appearing as the ones in this list. I want to talk briefly about three of those themes from the research literature that are on the teacher side of learning interactions:
(click)
Access, Tasks, and Strategies or Teacher Moves.

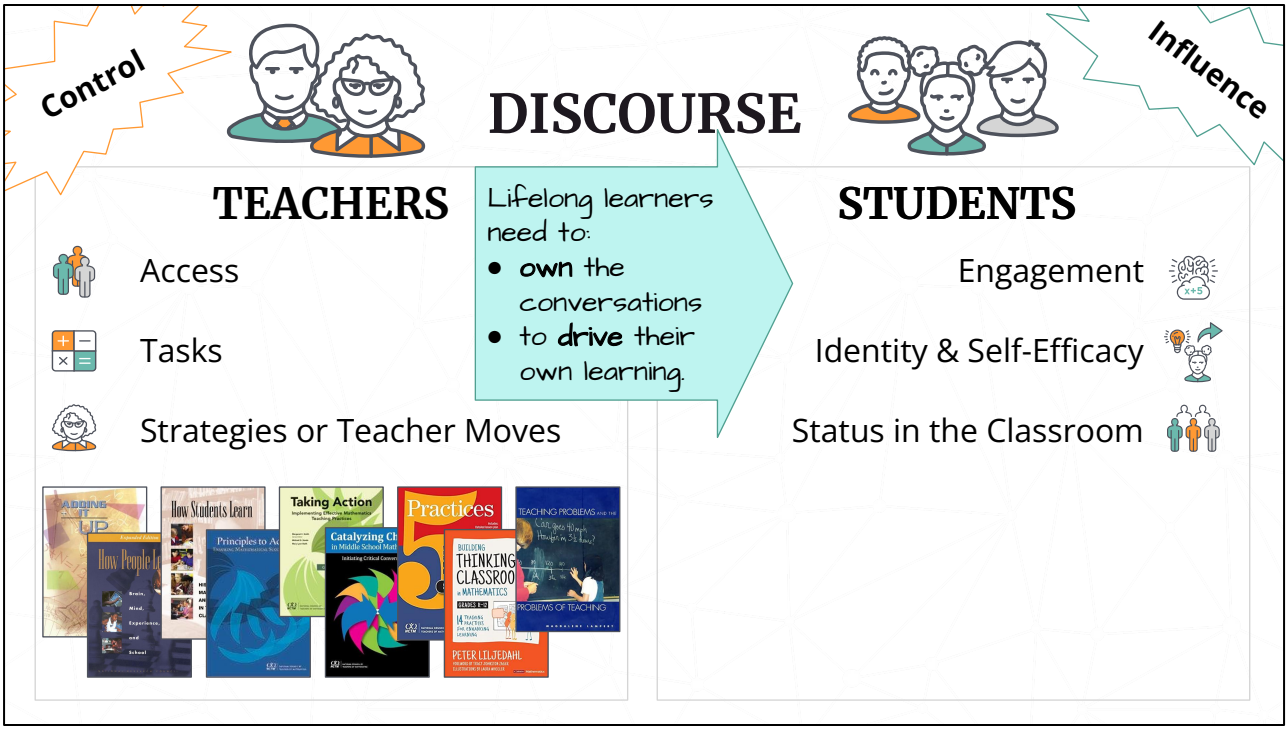
Access:
(click)
It is important that we provide access to high quality mathematics courses and opportunities to ALL students. We need to remove barriers to participation in mathematics discourse and need to advocate for ALL students' participation in discourse, not just students who are perceived as "good enough" in math to have something to talk about. ALL students have mathematical ideas to talk about, and participating in that discourse is in part what makes students "good" at math!

Tasks:
(click)
The literature speaks to how we choose and create tasks that give students something worthy to talk about – and there are plenty of presentations her this week where you can explore tasks. For our discussion today, we are going to assume that that is a given – you have chosen rich tasks to talk about.

Strategies or Teacher Moves:
(click)
This is a common theme in the research – how do we actually pull off discourse?

What are the teacher moves we make that make the discourse both happen, and drive learning. The biggest resources right now that really dig into those teacher moves are the *5 Practices* book and Liljedahl's *Building Thinking Classrooms*, both of which we recommend,, but both of which still really center the teacher, what teachers do to make discourse happen.

..and I don't mean to belittle the role that the teacher has in driving discourse.
(click)
One of my all-time favorite studies of teaching is Magdalene Lampert's *Teaching Problems and the Problems of Teaching*. At one point in the book, Dr. Lampert writes about all of the things she is thinking about when determining who to call on next – and it goes on for pages! Many of those things actually bring us to the student side of the interactions that make up learning.
(click)



She talks about how calling on one student versus another will reinforce their image of themselves as a mathematician, and how having a particular student share their strategy positions that student as knowledgeable. Dr. Lampert talks about how teachers **influence** the student interactions that happen in our classrooms.
(click)
In order to help our students gain the skills they need to be successful lifelong learners, we need to help them learn how to own the conversations and drive their own learning forward.
(click)
This means not only thinking about those teacher aspects of discourse that we can control, but shifting the focus to the student side where we influence how students participate in the discourse in our classrooms.
(click)
We can control who is our classes, the tasks we use, and the teacher moves we make, but we can only influence what our students do within the environment we create.

The research on this student-controlled side of discourse focuses on how teachers influence
(click)
student engagement, student's identities and self-efficacy, and status in the classroom.

Before we dig into each of them as a full group, we'd like to give you an opportunity to

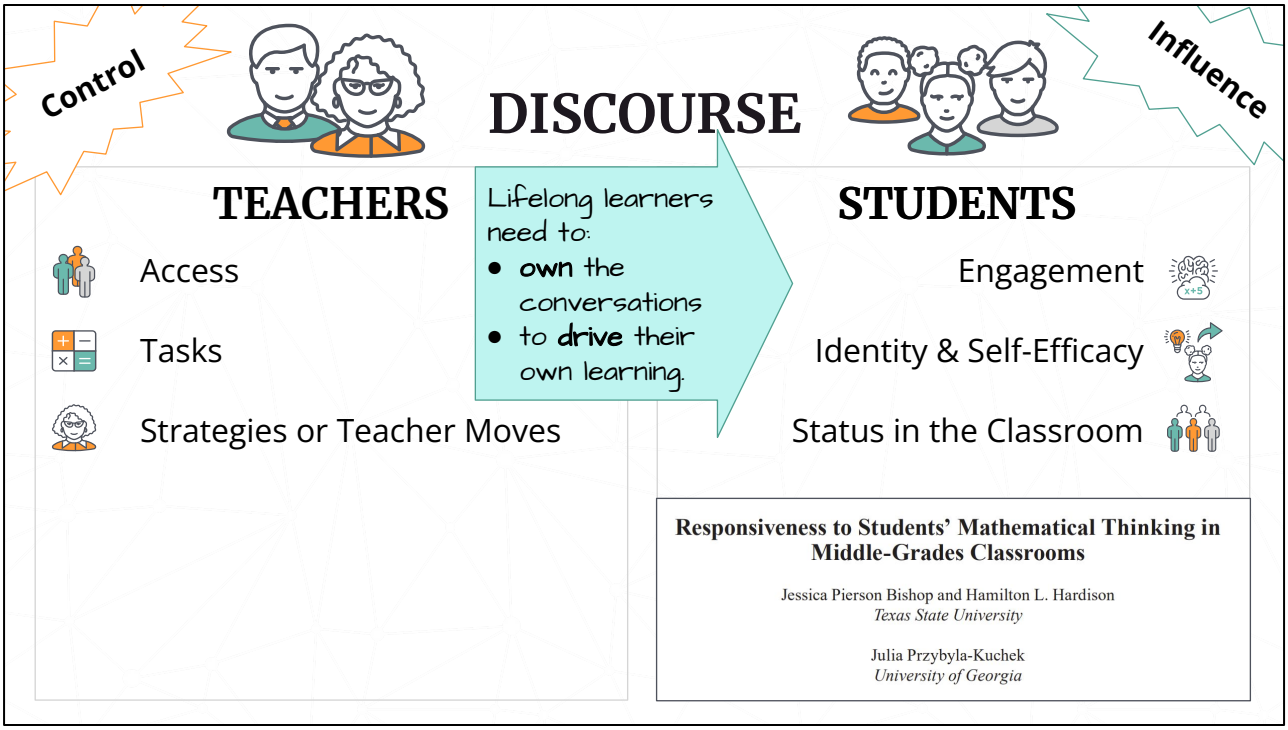
think about and discuss these topics together.
(click)

GROUP TASK: What did we just experience?

	What do you already know about what this means and looks like in classrooms?	How did this show up in our task today? What impacts did that have?
Engagement 		
Identity & Self-Efficacy 		
Status in the Classroom 		

In your group, the same group you did math with earlier, work together to fill out he “what did we just experience?” side of the graphic organizer on your tables. Be sure to give yourselves some think time before talking together about it. We’re going to give you about 5 minutes total in your groups to do and discuss this before coming back together.

[Don’t debrief on this slide.]



Let's review each of these areas together or just a little bit.

Engagement:
(click)
I don't want to spend a lot of time today on engagement, but just want to highlight one fallacy that I often run across when talking about student engagement. Teachers often think that they need to have some sort of hook that is separate from the mathematics in order to entice students to pay attention and then hope that engagement transfers to the math task. This is a schema that we need to change. As most of you know, rich mathematics tasks are in-and-of themselves interesting. When students are given a rich task, that task itself is the hook, and they are engaged – because mathematics IS engaging.

Identity & Self-Efficacy:
(click)
Identities are the ways in which define ourselves and present ourselves to the world. Self-efficacy refers to how capable we think we are at something. Needless to say, if we think we are capable at something, we have a more positive associated identity. So if we believe we are good at mathematics, we are more likely to hold a positive mathematical identity and see ourselves as a mathematician. Teachers can influence students' self-efficacy and identity by by sharing the ways in which we see their mathematical brilliance – and not just the brilliances that they already see as mathematical, but also those that they may not see, but that help them learn mathematics – all those strategies we highlighted earlier in the participation quiz are

great examples of this.

Status in the Classroom:
(click)
Status in some ways can be thought of the identities that others assign to us. So we may have an identity of ourselves as a capable mathematician, but others may not recognize our mathematical brilliance. Women, people of color, English language learners, and students with learning differences unfortunately experience this way too often. We, as teachers, need to intervene. We need to use our status in the classroom to influence students' status in the classroom; we need to position our students as capable and brilliant mathematicians. You saw some of those strategies during our task, for example, when we called out strengths that would be helpful during the group work, or when we suggested you do a "peek around" to view others' thinking as they worked. Much of our thinking in this area is influenced by work on Complex Instruction out of Stanford University. Complex Instruction shares a similar focus in some ways to UDL or Universal Design for Learning in that both of these ideas and sets practices work on eliminating barriers. In UDL we think of removing barriers due to learning differences, while with Complex Instruction we work on removing barriers due to differences in status. There are lots of aspects of status: we can have academic status – the ways people assign worthiness based on our perceived academic ability, such as seeing us as "smart;" we can have social status – are we seen as "cool" or as a "geek" and how does that change how people treat us; status based on bias or stereotypes. On our resources slide later on we provide a link to resources on Complex Instruction which contain many more strategies that allow you to highlight and influence students' status in the classroom.
(click)

I just want to share one more piece of research with you, and it comes from the research on teacher responsiveness.
(click)
Jessica Bishop and others developed what they called the Mathematically Responsive Instruction Framework and studied the ways in which teachers responded to student thinking in their middle school mathematics classrooms by doing things such as "asking probing questions, revoicing student ideas, or attaching terminology or notation to a student's idea" (p. 16), and then further how they engaged others by explicitly "directing students to engage significantly with their peer's mathematical ideas" (p. 16).

They found many really interesting things, but the finding I want to highlight here is that they discovered that when teachers are both responsive to student thinking and encourage their students to be responsive to each other's thinking, students began to do so – spontaneously! That is when teachers said things like, "Anna, what do you think about Deshawn's strategy?" or "How might Anika's strategy help you with this next problem?", their students learned, from the teacher modeling, that they were allowed and encouraged to respond to each other – and so they did! And not only did

they respond to each other more, but they began to ask the same types of responsive questions that teacher would ask. So it's not just that we improve the student-to-student discourse in our classrooms when we invite students to respond to each other and ask each other questions, but that by inviting them we see students spontaneously facilitating discourse even when we don't invite them. This is how we prepare them to be lifelong learners.

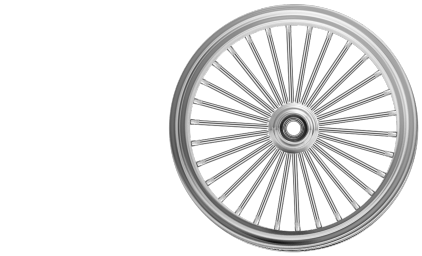
We'd love to know what other things you came up with when discussing with each other, but in the interest of time, we'd like to work our way through the rest of our slides and hold this conversation for the end during our Questions, Thoughts, and Comments time.

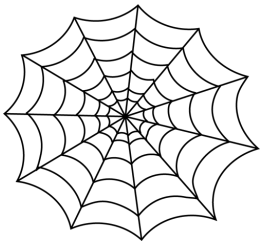
Our Vision

Roberds leads

Our Vision:

More Complex Webs





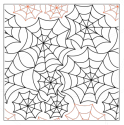
Our vision of the future is one where classrooms are full of complex webs. Just to recap what we mean when we say this:

Most often in traditional classrooms we see a wheel model
(click)
where students demonstrating their understanding via computation and 1:1 teacher questioning.
(click)
The web model is where students explain their thinking and teachers facilitate whole class discussions designed to elicit and illuminate that thinking.
(click)
We hope the next step is a future full of complex webs, where students learn the skills to ask questions and be the facilitators of discussions that deepen their own thinking and the thinking of those they are working with. In order to be lifelong learners, they will need these skills!

GROUP TASK: What next?

	How do the **web models** encourage more of each category?	Where do you already see this happening? What's **going well**?	What **supports** are you going to call on when this gets hard?
Engagement 			
Identity & Self-Efficacy 			
Status in the Classroom 			

Take just a few moments with whomever you turned and talked a few moments ago and use the "What's Next" side of your graphic organizer to gather ideas together about how you might move forward in pursuit of this goal. We'll put up that other slide with the descriptions of the wheel and webs for you to refer to as needed while you work.

	**1:1 Teacher:Student** 	**Teacher-Led Discussion** 	**Student-Led Discussion** 
TEACHERS 	• Asking students direct questions	• Leading a class discussion • Generating discourse questions	• Observe, chart, provide feedback, and manage the focus within the discussion
STUDENTS 	• Responding to teacher, not talking to each other	• Building off of each other's thinking in facilitated way	• Generating discourse questions • Leading the discussions within groups

[To refer to while working on the What's Next graphic organizer.]

ANNOTATED BIBLIOGRAPHY

Bishop, Jessica P., Hamilton L. Hardison, and Julia Przybyla-Kuchek. "Responsiveness to Students' Mathematical Thinking in Middle-Grades Classrooms," *Journal for Research in Mathematics Education* 53, no. 1 (2022): 10-40. https://doi.org/10.5951/jresematheduc-2020-0188

The key take-away from this article, for our purposes, is that when teachers are more responsive to their students' thinking and the more they invite their students to respond to each other's thinking, the more their students spontaneously respond to each other's thinking.

Lampert, Magdalene. *Teaching Problems and the problems of Teaching*. New Haven, CT: Yale University Press, 2001.

This amazing book is a year-long case study of Lampert's own math teaching. It highlights the depth of thought that goes into the practice of teaching mathematics well, from the perspective of someone who is living it – and really gets it. Reading this book provides a powerful way to think about one's own teaching.

Liljedahl, Peter. *Building Thinking Classrooms in Mathematics, Grades K-12: 14 Teaching Practices for Enhancing Learning.* Thousand Oaks, CA: Corwin Press, 2021.

One of the newest, biggest books in math pedagogy, this book gives you lots to think about. We don't agree with all of it, but do agree that it's a must-read.

Gaffney leads

We've put together an annotated bibliography of the work we used or talked about today. You can find it in our slides on the NCTM app.

National Council of Teachers of Mathematics (NCTM). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM, 2014.

A statement of NCTM's vision for mathematics education in the era of the Common Core Standards and a resource for how to achieve that vision, *Principles to Actions* defines and describes the eight teaching practices referenced in this presentation that are essential in order to provide a high-quality mathematics education for all students.

National Council of Teachers of Mathematics (NCTM). *Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations*. Reston, VA: NCTM, 2020.

NCTM's publication on how to achieve equitable classrooms, this middle school volume of the *Catalyzing Change* series digs into what equity looks like in middle school mathematics classrooms.

National Research Council. *Adding It Up: Helping Children Learn Mathematics.* Jeremy Kilpatrick, Jane Swafford, and Bradford Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press, 2001. https://nap.nationalacademies.org/catalog/9822/adding-it-up-helping-children-learn-mathematics

A seminal work, this research tome sums up the research literature on the teaching and learning of mathematics through the late 1990s. It's definition of mathematical proficiency, for example, is still widely used as one of the most comprehensive definitions in the field.

National Research Council. *How People Learn: Brain, Mind, Experience, and School,* Expanded Ed. John D. Bransford, Ann L. Brown, and Rodney R. Cocking (Eds.). Committee on Developments in the Science of Learning and Committee on Learning Research and Educational Practice, Commission on Behavioral and Social Sciences and Education. Washington, DC: National Academy Press, 2000. https://nap.nationalacademies.org/catalog/9853/how-people-learn-brain-mind-experience-and-school-expanded-edition

This book is a compilation of the research from the fields of education, psychology, and neuroscience on, quite literally, how people learn. It a dense research tome, best reda in chunks, and is for those who want to really know the science behind classroom practices.

National Research Council. *How Students Learn: History, Mathematics, and Science in the Classroom*. Committee on How people Learn, A Targeted Report for Teachers, M. Suzanne Donovan, and John D. Bransford (Eds.). Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press, 2005. https://nap.nationalacademies.org/catalog/10126/how-students-learn-history-mathematics-and-science-in-the-classroom

This book is a companion to *How People Learn* and is edited for an intended audience of classroom teachers. It includes more classroom examples to flesh out the science.

Roberds, Anna. *Group Work: Collaborative Learning Bookmarks*. CollabEd, 2023. https://www.teacherspayteachers.com/Store/Collabed-With-Anna

These are the bookmarks you used in our session. If you want to use these with students, here is where you can buy the download.

Schwartz, Katrina. “How A Strengths-Based Approach to Math Redefines Who Is 'Smart',” KQED News: MInd/Shift, May 23, 2016. https://www.kqed.org/mindshift/45012/how-a-strengths-based-approach-to-math-redefines-who-is-smart

A quick overview article that provides a good snapshot of how and why complex instruction practices are so important in equitable classrooms. This is a good intro/hook to equitable classroom work in math classrooms.

Smith, Margaret, Michael Steele, and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics, 2017.

This book illuminates NCTM’s eight effective mathematics teaching practices and provides professional learning activities to help teachers dig into and make switches towards better use of those practices.

Smith, Margaret S., and Mary Kay Stein. *5 Practices for Orchestrating Productive Mathematics Discussions*, 2nd Ed. Reston, VA: National Council of Teachers of Mathematics, 2018.

This comprehensive work is a step-by-step manual for planning, implementing, and thinking about productive classroom math discussions. We used the ideas herein when planning how to orchestrate our task discussion in this session and use these ideas as part of our daily classroom practice.

Stanford University. “Complex Instruction: Building Equitable Classrooms.” October 24, 2023. https://complexinstruction.stanford.edu/

This website provides a wealth of resources on Complex Instruction. It is our go-to for equitable classroom practices and ways to assign status in the classroom – in mathematics and beyond.

Celebrate YOUR own expertise as math educators!

There is so much brilliance and passion in this room, and we hope you also take time to recognize and celebrate that in yourselves.

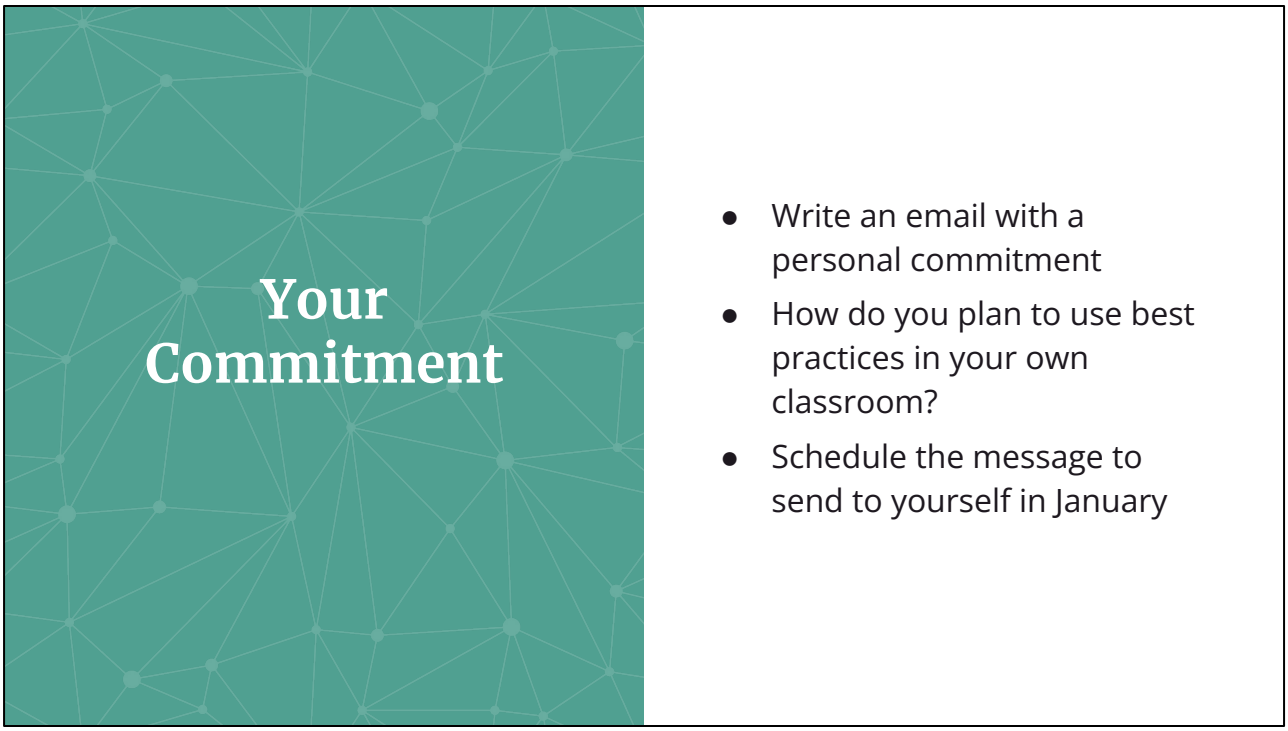
Presidential Award for Excellence in Mathematics and Science Teaching (PAEMST)

www.paemst.nsf.gov

Math Equity in Design (MEND)

agaffney@mindeducation.org

Gaffney



Gaffney: 1-2 minutes then they can work on it while we then field questions.

Questions, Thoughts, Comments?

OUR MISSION

Ensuring all students are mathematically equipped to solve the world's most challenging problems.

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