A Meaningful Path to Lasting Multiplication Fact Fluency

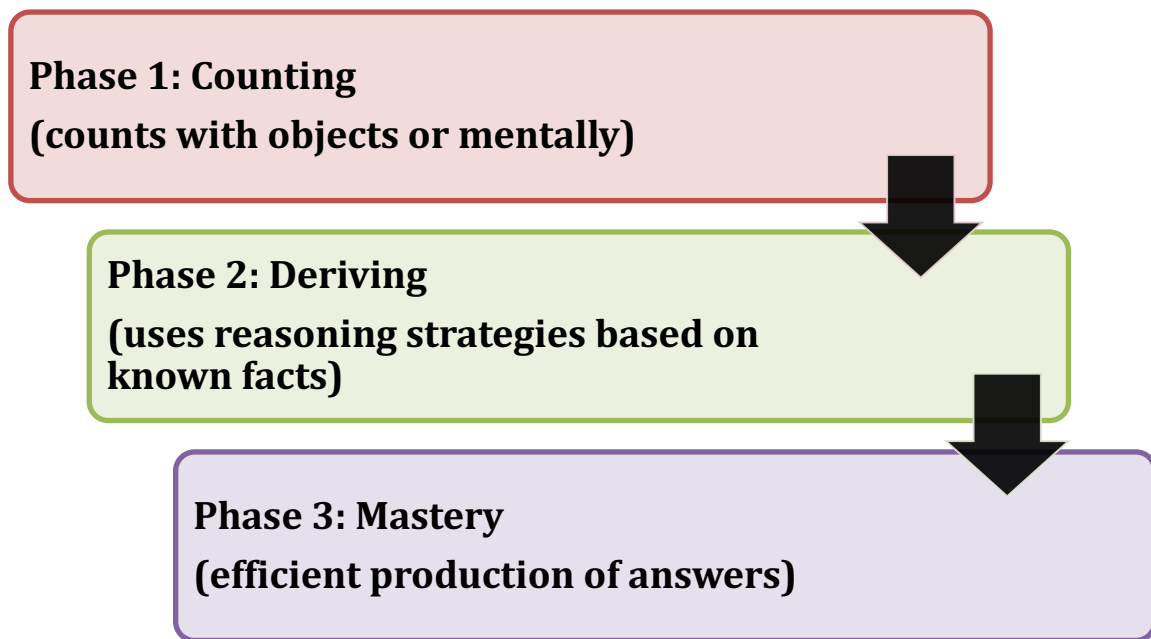Gina Kling
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Procedural Fluency is skill in carrying out procedures flexibly, accurately, efficiently and appropriately.

(NCTM, 2014; CCSSO, 2010; NRC, 2001)

Phases of Basic Fact Mastery (Baroody, 2006)

Phase 1: Counting

(counts with objects or mentally)

Phase 2: Deriving

(uses reasoning strategies based on known facts)

Phase 3: Mastery

(efficient production of answers)

Indicators for each phase for the example 6 x 4 = ?

Phase 1: The child uses fingers to help keep track of his counts as he either counts by ones or skip counts to find the product.
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Phase 2: The child uses a known fact such as 5 x 4 and thinks "5 x 4 = 20 and 1 more group of 4 will make 24. So 6 x 4 = 24.."

Phase 3: The child answers "24" with little/no hesitation.

Key Games for Developing Fact Fluency

Multiplication Draw*

Provide partnerships with a die labeled with 2, 2, 5, 5, 10, 10 and a set of number cards 0–10. Players take turns rolling the die and drawing a card from the deck to generate two multiplication factors, and record the corresponding fact and product. After 5 turns each partner finds the sum of his/her products, and the player with the largest sum wins the round.

Multiplication Tetris***

This game requires a deck of cards with numbers 1-10 (Ace = 1), grid paper, and colored pencils or crayons. The teacher draws two cards, and children draw a rectangle with those dimensions somewhere on their grid papers, recording a number sentence representing the area of their rectangle inside the rectangle. For example, if the teacher draws 5 and 7, children fit a 5-by-7 rectangle on their grid paper and record "5 x 7 = 35" on the inside of their rectangle. Children can rotate rectangles as they try to fit them, but rectangles cannot overlap or move after they are placed. The object is to stay in the game as long as possible by having room on your grid paper to fit a given rectangle; if there is not enough room to fit the rectangle on your page, you are out of the game.

Multiplication Top-It*

Using a deck of cards with numbers 0–10 (Ace = 1, Jack = 0), partners begin by splitting the deck roughly in half so that they each have their own deck of cards. Then each partner turns over 2 cards and calls out the product of the numbers. Players check each other's products, and the player with the largest product wins the round and takes all of the cards.

Salute!*

Played in a group of three with a deck of cards (omitting face cards and using aces as ones, jacks as zeros). Two players draw a card without looking at it and place it on their foreheads facing outward (so the others can see it). The player with no card tells the sum. The other two players determine the value of their cards. Once both players have done so, they look at their cards and then players rotate roles before starting the next hand. Adaptations include restricting cards used (e.g., addition facts using only the numbers 1 through 5) or focusing on multiplication/division.

Product Pile-Up*

Using a deck of number cards 0–10 and a group of 3-4 players, each player is dealt 8 cards. The first player to the left of the dealer begins by playing two cards and saying out loud the product of those two cards. Then next player must play two cards that have a product *higher than the cards just played*, and play continues around the group in this manner. If a player cannot top the product just played, he or she draws two cards from the deck and tries again to create a higher product. If he or she is still unable to do so, the player says "pass" and play moves to the next player. If all players have to pass, then the player who last laid cards gets to start a new hand by playing any two cards he or she wishes. The object is to use up all cards in your hand, and whoever plays all of his or her cards first is the winner.

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Squares Bingo (Bay-Williams and Kling, 2019**)

Questions to Ask While Playing Facts Games

Questing is important to encourage good mathematical thinking during game play:
- *How did you figure it out?*
- *Can you say out loud how you thought about it in your head?*
- *Is there another way you could figure it out?*
- *Can you think of another fact that strategy would work well for?*
- *If someone didn't know the answer to ____, how would you tell them to figure it out?*

Multiplication Draw Record Sheet

	Round 1	**Round 2**	**Round 3**
1st draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
2nd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
3rd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
4th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
5th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
Sum of products:	___	___	___

	Round 1	**Round 2**	**Round 3**
1st draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
2nd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
3rd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
4th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
5th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
Sum of products:	___	___	___

	Round 1	**Round 2**	**Round 3**
1st draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
2nd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
3rd draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
4th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
5th draw:	___ × ___ = ___	___ × ___ = ___	___ × ___ = ___
Sum of products:	___	___	___

TRIOS** ~ Board A

5	50	40	35	15
10	50	25	45	15
30	40	20	35	5
35	15	10	45	25
20	45	5	40	30

How to Play TRIOS

1. You need a die or card deck with numbers 1-10.
2. Roll the die/flip a card and multiply that number by 5.
3. Place one of your counters on any square with that product. Take turns.
4. You want to try to cover three spaces in a row to make a Trio. Each Trio earns you five points. New Trios can only intersect by one number.
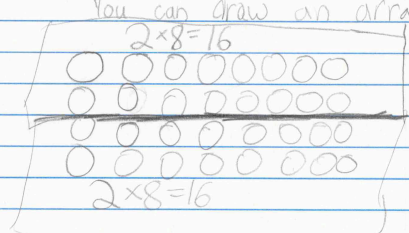5. Highest score wins.

TRIOS** ~ Board B

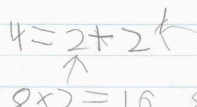| 25	5	45	10	40
15	10	30	10	50
35	35	20	5	20
20	40	50	40	15
45	30	50	25	30

How to Play TRIOS

1. You need a die or card deck with numbers 1-10.
2. Roll the die/flip a card and multiply that number by 5.
3. Place one of your counters on any square with that product. Take turns.
4. You want to try to cover three spaces in a row to make a Trio. Each Trio earns you five points. New Trios can only intersect by one number.
5. Highest score wins.

3rd Grade Responses to:
If your friend didn't know the answer to 8 x 4, how would you tell him to figure it out?

Do 10×4=40, then take away 2 groups of 4 because you have 8 to many 40-8=32. The product is 32.

Or, You can draw an array
2×8=16
○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○
○ ○ ○ ○ ○ ○ ○ ○ ○
2×8=16
16+16=32

8×4=?

4=2+2

8×2=16   8×2=16

     16
    +16
     32

If you break four into two and two you can multiply eight by two twice eight times two equalss sixteen so add sixteen and sixteen that is thirty-two and then you have your answer

Well first do you know what 5×4=. Yes, 20. Now what's 3×4=, 12. Ok. What is 20+12=. 32. Now, if you add the 5 in 5×4 and you add the 3 in 3×4 that will equal 8. Do not change the 4 in 5×4 and 3×4. Your number sentance will be 8×4. So whats 5×4 again? 20. What's 3×4? 12. 20+12=, 32. so 8×4 8×4=32.

Developmental Sequence for Multiplication Fact Strategies

Foundational Facts*	
1. 2s, 5s, and 10s (begin these in late second grade)	Use story problems involving equal groups and arrays, skip counting, and repeated addition to learn these facts.
2. 0s*, 1s, Multiplication squares (2 x 2, 3 x 3, etc.)	
Derived Fact Strategies	
3. Adding a group	Start with a nearby 2s, 5s, or 10s fact and add a group to derive the unknown fact. *Ex: I don't know 6 x 8, so I think **5** x 8 = 40 and add one group of 8 to get 48.*
4. Subtracting a group	Start with a nearby 2s, 5s, or 10s fact and subtract a group to derive the unknown fact. *Ex: I don't know 9 x 6, so I think **10** x 6 = 60 and subtract one group of 6 to get 54.*
5. Halving and doubling	Look for an even factor. Find the fact for half of that factor, then double it. *Ex: I don't know 6 x 8, so I think 3 x 8 = 24 and double that to get 48.*
6. Near squares (adding or subtracting a group to a square)	Look for a nearby square. Find that fact and add on/subtract off the extra group. *Ex: I don't know 7 x 6. I use 6 x 6 = 36 and add one more 6 to get 42.*
7. Breaking apart	Break apart one of the factors into a convenient sum of known facts, find the two known facts and combine the products. *Ex: I don't know 7 x 6. I break the 7 into 2 and 5, because I know 2 x 6 and 5 x 6. Then I add 12 and 30 to get 42.*

* Note: 0s are foundational, but are not typically used for derived fact strategies

Kling, G. & Bay-Williams, J. (May 2015). Three Steps to Mastering Multiplication Facts. *Teaching Children Mathematics,* 21(9), 548-559.

Observational Notes

3rd grade boy   **Mid-November**

Phase 1
Phase 2
Phase 3

3rd grade girl   **Mid-November**

Phase 1
Phase 2
Phase 3

Additional Resources for Developing and Assessing Basic Fact Fluency

Gina Kling & Jennifer Bay-Williams (2021). Eight Unproductive Practices in Developing Fact Fluency. *Mathematics Teacher: Learning and Teaching PK-12*, 114(11), 830-838.
> *Overview of common pitfalls in teaching and assessing basic facts as well as suggestions for fluency-focused alternatives.*

Jennifer Bay-Williams & Gina Kling (2019). Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention. Association for Supervision and Curriculum Development (ASCD) and National Council of Teachers of Mathematics (NCTM), Alexandria, VA.
> *A book completely devoted to developing and assessing addition, subtraction, multiplication, and division facts in meaningful, effective ways! Provides detailed descriptions of each basic fact strategy, many games for each phase of basic facts learning, and a variety of fluency-focused assessment tools.*

Gina Kling & Jennifer Bay-Williams (2018). Games and Tools for Teaching Multiplication Facts. Quick Reference Guide. Association for Supervision and Curriculum Development (ASCD), Alexandria, VA.
> *A multi-panel glossy guide with suggestions for activities, games, and assessment techniques designed to encourage multiplication and division fact fluency.*

Jennifer Bay-Williams & Gina Kling (2017). Games and Tools for Teaching Addition Facts. Quick Reference Guide. Association for Supervision and Curriculum Development (ASCD), Alexandria, VA.
> *A multi-panel glossy guide with suggestions for activities, games, and assessment techniques designed to encourage addition fact fluency.*

Gina Kling & Jennifer M. Bay-Williams (2015). Three Steps to Mastering Multiplication Facts. *Teaching Children Mathematics, 21*, 548-559.
> *Contains a summary of multiplication fact strategies as well as directions for a variety of multiplication facts games.*

Jennifer M. Bay-Williams & Gina Kling (2015). Developing Fact Fluency. Turn Off Timers, Turn Up Formative Assessment. In NCTM *Annual Perspectives in Mathematics Education (APME) 2015: Assessment to enhance learning and teaching.* Chris Suurtamm, (Ed.) National Council of Teachers of Mathematics, Reston, VA.
> *Contains examples of different types of fact assessments that can be used across the three phases of fact mastery.*

Jennifer M. Bay-Williams & Gina Kling (2014). Enriching Addition and Subtraction Fact Mastery through Games. *Teaching Children Mathematics, 21*: 238-247.
> *Contains game directions in the main article as well as detail on how to use the games to help children progress through the three phases of fact mastery.*

Gina Kling & Jennifer M. Bay-Williams (2014). Assessing Basic Fact *Fluency*. *Teaching Children Mathematics, 2:* 488-497.
> *Contains examples of different types of fact assessments that can be used to more accurately capture the four components of fluency.*

Gina Kling (2011). Fluency with Basic Addition. *Teaching Children Mathematics, 18:* 80-88.
> *Contains a summary of basic addition strategies as well as game directions, including games focused on developing foundational fact fluency.*

You are welcome to contact me with any questions or ideas you have to share about basic fact fluency. Thank you for participating in this session! ☺