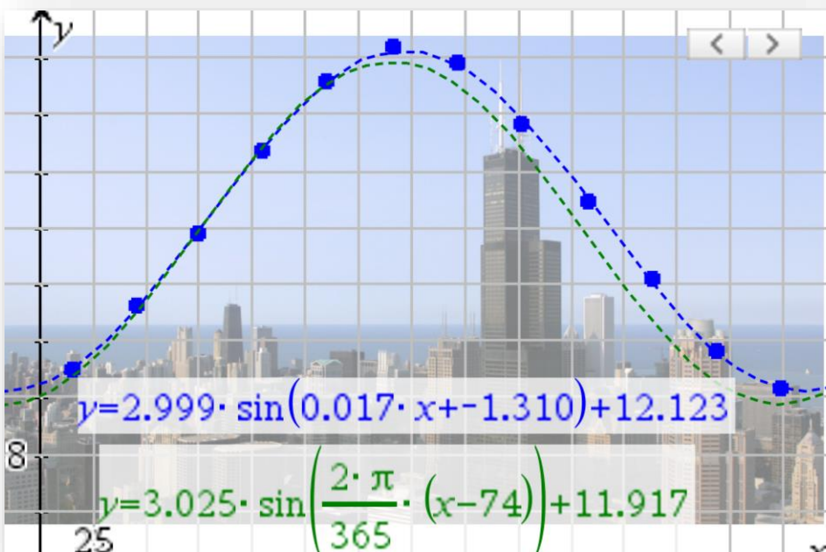
Using Data & Modeling to Take a Deep-Dive into the Patterns of Daylight



Scott Knapp
Glenbrook North High School
sknapp@glenbrook225.org
Twitter: @scottknapp

2023 NCTM Conference, Washington, D.C.

Let the Sun Shine!!

If you think about the time that the sun rises and sets each day, there are various changes that occur throughout the year. It turns out these changes in daylight vary greatly depending on a city's location on our planet! In this activity, you will be working on tasks both individually and within a group to explore daylight hours for various world cities. You will complete the following tasks.

Individual Tasks

Task 1: Using your assigned city, collect data on the **Data Individual Worksheet** for the hours of daylight for the twelve indicated days throughout the year. Let x represent the day of the year (Ex: February 1 is when $x = 32$) and y represent the number of hours of daylight for the specified day of the year. To collect data, visit the website shown below and type in your city name. Scroll down the webpage and collect data from the yearly sun graph. Once you have collected data, answer the questions about your data set.

Daylight Website: https://www.timeanddate.com/sun/

Task 2: Graph your city's daylight data on the grid provided on the **Data Individual Worksheet.** Scale the axes appropriately based on the values within your data set.

Task 3: Using skills learned in previous trigonometry units estimate a sinusoidal regression model for your city's daylight data in the form $\boldsymbol{y = a \cdot \sin(b(x - c)) + d}$. Show all work on the **Data Individual Worksheet.** Once you have determined a sine regression model 'by hand', enter the daylight data into your graphing calculator and calculate a sinusoidal regression. Be sure to enter the daylight times as accurate as possible (Ex: 13:40 hours is $13 + 40/60$ hours). Round all values in your regression equation to three decimal places. Compare the numbers in your two regression equations and answer all questions that follow.

Group Tasks

Task 4: Working together as a group (with data for five different cities), plot the daylight data for all cities in your group on the same grid paper. Label and scale the axes appropriately based on the values within your collective data sets. Use a separate color for each city in the graph and include a key for the graph with your city names somewhere on the graph paper. Once your group graph is complete, compare the graphs for the group of five cities and answer all questions on the **Daylight Group Worksheet.**

Task 5: Working in your group, record the regression equations (calculator version) for each city on the '**Daylight Group Worksheet**' rounded to three decimal places. Compare the regression equations and answer all questions that follow.

***Task 6**: Follow instructions on the **Daylight Extension Worksheet** to investigate the rate of change of daylight at various times throughout the year.

Daylight Individual Worksheet
Task #1: Collecting/Analyzing Data

Name ____________________________________

City name __________________________________

Date	Day #	Hours of Daylight	Date	Day #	Hours of Daylight
January 15	**15**		July 15	**196**	
February 15	**46**		August 15	**227**	
March 15	**74**		September 15	**258**	
April 15	**105**		October 15	**288**	
May 15	**135**		November 15	**319**	
June 15	**166**		December 15	**349**	

1. According to your data set, how long is the longest day of the year? When does the longest day occur?

____

2. According to your data set, how long is the shortest day of the year? When does the shortest day occur?

____

3. What is the difference between the longest and shortest (in terms of hours of daylight) day of the year?

____

4. Analyze the <u>change</u> in daylight from month-to-month using your data. Is the change constant? If not, explain any observations about the change in daylight from month-to-month.

____

____

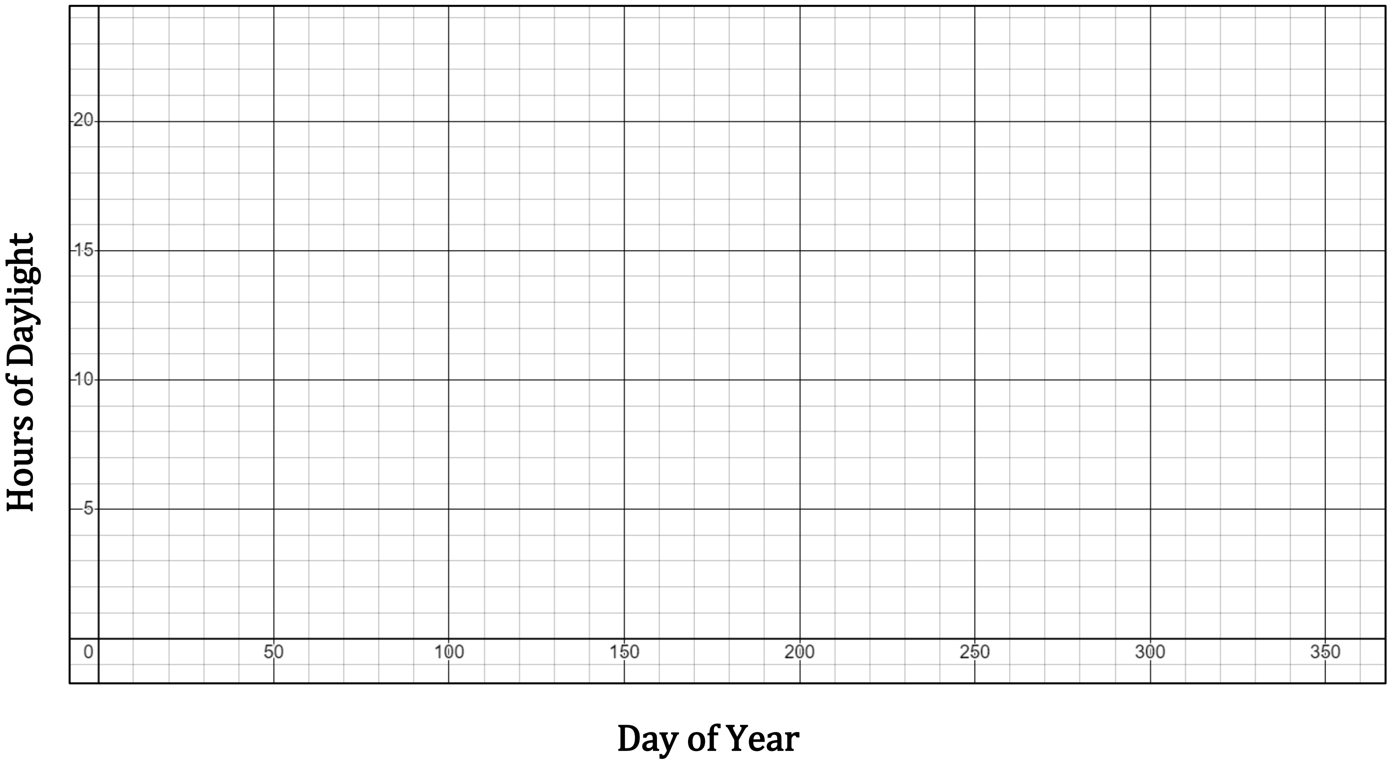
5. During which month(s) does it appear daylight changes at the fastest rate? ____________________

6. During which month(s) does it appear daylight changes at the slowest rate? ____________________

7. What type of function do you feel best matches your data? Explain your choice.

____

Task #2: Plotting the Data



Task #3: Determining Regressions Equations for the Data

8. Estimate a sinusoidal regression equation in the form $\boldsymbol{y} = \boldsymbol{a} \cdot \mathbf{sin}(\boldsymbol{b}(\boldsymbol{x} - \boldsymbol{c})) + \boldsymbol{d}$ 'by hand'. Use your calculator to approximate values in your regression, if necessary, to three decimal places.

<u>Estimated</u> Regression: ____

9. Explain how you found the values for $a, b, c,$ and d in your regression equation.

____

____

____

10. Rewrite your estimated regression equation from the previous task in the box below.

<u>Estimated</u> Regression: ____

11. Determine a sinusoidal regression in the form $\boldsymbol{y = a \cdot \sin(bx + c) + d}$ using your graphing calculator. Round all values to 3 decimal places.

<u>Calculator</u> Regression: ____

12. Compare the values between your estimated and calculated regression equations. Are the values similar? If not, explain why some values are different in the two regression equations.

____

____

____

____

13. Use your calculator regression to predict the hours of daylight for <u>today's</u> date. Compare your answer to today's actual hours of daylight. (You will need to revisit the daylight website to find this information).

Prediction: ____________________ Actual hours of daylight: ____________________

14. What is the difference, in hours, between the maximum and minimum amount of daylight in a day? Explain how you can use your regression equation to find the difference between the maximum and minimum hours of daylight.

Difference between max/min: ________________ Explanation: ____________________

____

Daylight Group Worksheet
Task #4: Comparing Group Data

1. Look closely at your group's graph containing data plotted from all five cities. Discuss the similarities and differences observed.

Similarities: ______________________________

____

____

Differences: ______________________________

____

____

2. Which city in your group has the longest and shortest day of the year? ______________________________

3. Which city in your group has the least variation in daylight during the year? ______________________________

4. Which day number of the year is the longest and shortest? Is this the same for all locations?

____

5. What do you think accounts for the similarities/difference explained above?

____

____

6. If a location has 12 hours of daylight every day of the year, where do you think it is located?

____

7. Suppose you collected data for cities from the Southern Hemisphere. Explain differences you would observe in the graph explored.

____

____

Task #5: Comparing Group Regression Equations

8. List each group member's city and regression equation (rounded to 3 decimal places).

City Name	Sinusoidal Regression Equation

9. Discuss the similarities and differences observed in the sine regression equations in the table above. Then, explain why the similarities and differences occur.

Similarities:____

Differences:____

10. Suppose the function below represents the hours of daylight, y, for a city in the world based on the day of the year, x (where $x = 1$ corresponds to January 1, $x = 32$ corresponds to February 1).

Function: $y = 1.5\sin(0.017x - 4.505) + 12.1$

Which hemisphere is this city located in? Show any work that helps support your answer and explain how you know.

11. Northbrook's longest and shortest days (in terms of daylight) are approximately 9.1 and 15.3 hours, respectively. Is the city represented in the function closer to or further from the Equator than Northbrook? Show any work that helps support your answer and explain how you know.

Daylight Extension
Task #6: Analyzing the Rate of Change in Daylight

In this task you will investigate the 'rate of change' for daylight for your given city. Follow the steps below to graph the sinusoidal regression equation determined on your calculator in Task #3 and draw a tangent line to your regression equation.

1. Insert a Graphs page into the same document containing the Lists & Spreadsheet page of daylight data. On the Graphs page, press tab and arrow up to $f1(x)$. The regression equation should appear. Press enter to graph the regression equation. Scale the axes appropriately to observe the regression equation for the domain of one year (365 days).

2. Press menu, select Geometry, then Points & Lines, and then select Point On. Plot a point on your regression equation.

3. Press menu, select Geometry, then Points & Lines, and then select Tangent. Draw a tangent line to the point plotted in the previous step. (See the definition of tangent line in the box below.)

A **<u>tangent line</u>** is a line that touches a function at one point. The slope of the tangent line represents the **<u>instantaneous</u>** rate of change for a function at that point.

4. Drag the point on your function to where x = 31 (January 31). Write down the slope of the tangent line and interpret its meaning.

Slope: ________________ Interpretation: ____

____

5. Drag the point and tangent line on the function to the locations (based on amount of daylight/slope of the tangent line) in the table below. For each location, list the day of the year when the location occurs (convert the day number into month and day) and the slope of the tangent line on that day.

Location	Day of Year	Slope of Tangent Line
Maximum daylight		
Minimum daylight		
Most positive slope		
Most negative slope		

6. Estimate the time, in minutes, for the change in daylight per day when the amount of change in daylight is at a <u>maximum</u>. Show work.

Number of minutes: ____________________