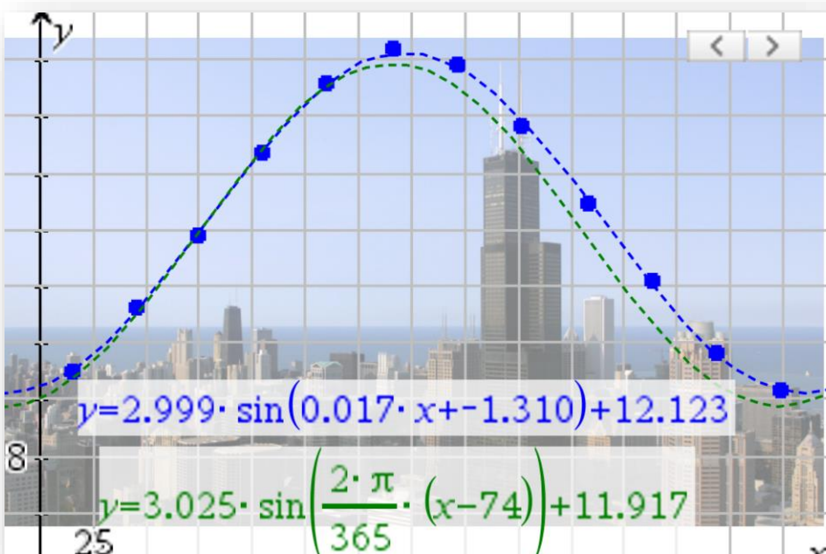
Using Data & Modeling to Take a Deep-Dive into the Patterns of Daylight



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Let the Sun Shine!!

If you think about the time that the sun rises and sets each day, there are various changes that occur throughout the year. It turns out these changes in daylight vary greatly depending on a city's location on our planet! In this activity, you will be working on tasks both individually and within a group to explore daylight hours for various world cities. You will complete the following tasks.

Individual Tasks

Task 1: Using your assigned city, collect data on the **Data Individual Worksheet** for the hours of daylight for the twelve indicated days throughout the year. Let x represent the day of the year (Ex: February 1 is when $x = 32$) and y represent the number of hours of daylight for the specified day of the year. To collect data, visit the website shown below and type in your city name. Scroll down the webpage and collect data from the yearly sun graph. Once you have collected data, answer the questions about your data set.

Daylight Website: https://www.timeanddate.com/sun/

Task 2: Graph your city's daylight data on the grid provided on the **Data Individual Worksheet.** Scale the axes appropriately based on the values within your data set.

Task 3: Using skills learned in previous trigonometry units estimate a sinusoidal regression model for your city's daylight data in the form $\boldsymbol{y = a \cdot \sin\big(b(x - c)\big) + d}$. Show all work on the **Data Individual Worksheet.** Once you have determined a sine regression model 'by hand', enter the daylight data into your graphing calculator and calculate a sinusoidal regression. Be sure to enter the daylight times as accurate as possible (Ex: 13:40 hours is $13 + 40/60$ hours). Round all values in your regression equation to three decimal places. Compare the numbers in your two regression equations and answer all questions that follow.

Group Tasks

Task 4: Working together as a group (with data for five different cities), plot the daylight data for all cities in your group on the same grid paper. Label and scale the axes appropriately based on the values within your collective data sets. Use a separate color for each city in the graph and include a key for the graph with your city names somewhere on the graph paper. Once your group graph is complete, compare the graphs for the group of five cities and answer all questions on the **Daylight Group Worksheet.**

Task 5: Working in your group, record the regression equations (calculator version) for each city on the '**Daylight Group Worksheet**' rounded to three decimal places. Compare the regression equations and answer all questions that follow.

***Task 6**: Follow instructions on the **Daylight Extension Worksheet** to investigate the rate of change of daylight at various times throughout the year.

Daylight Individual Worksheet
Task #1: Collecting/Analyzing Data

Name ______________________________________

City name ____________________________________

Date	Day #	Hours of Daylight	Date	Day #	Hours of Daylight
January 15	**15**	9:29	July 15	**196**	14:55
February 15	**46**	10:38	August 15	**227**	13:52
March 15	**74**	11:55	September 15	**258**	12:29
April 15	**105**	13:21	October 15	**288**	11:06
May 15	**135**	14:34	November 15	**319**	9:49
June 15	**166**	15:13	December 15	**349**	9:09

1. According to your data set, how long is the longest day of the year? When does the longest day occur?
On June 15, Chicago has 15 hours and 13 minutes of daylight.

2. According to your data set, how long is the shortest day of the year? When does the shortest day occur?
On December 15, Chicago has 9 hours and 9 minutes of daylight.

3. What is the difference between the longest and shortest (in terms of hours of daylight) day of the year?
15:13 – 9:09 = 6:04; 6 hours and 4 minutes

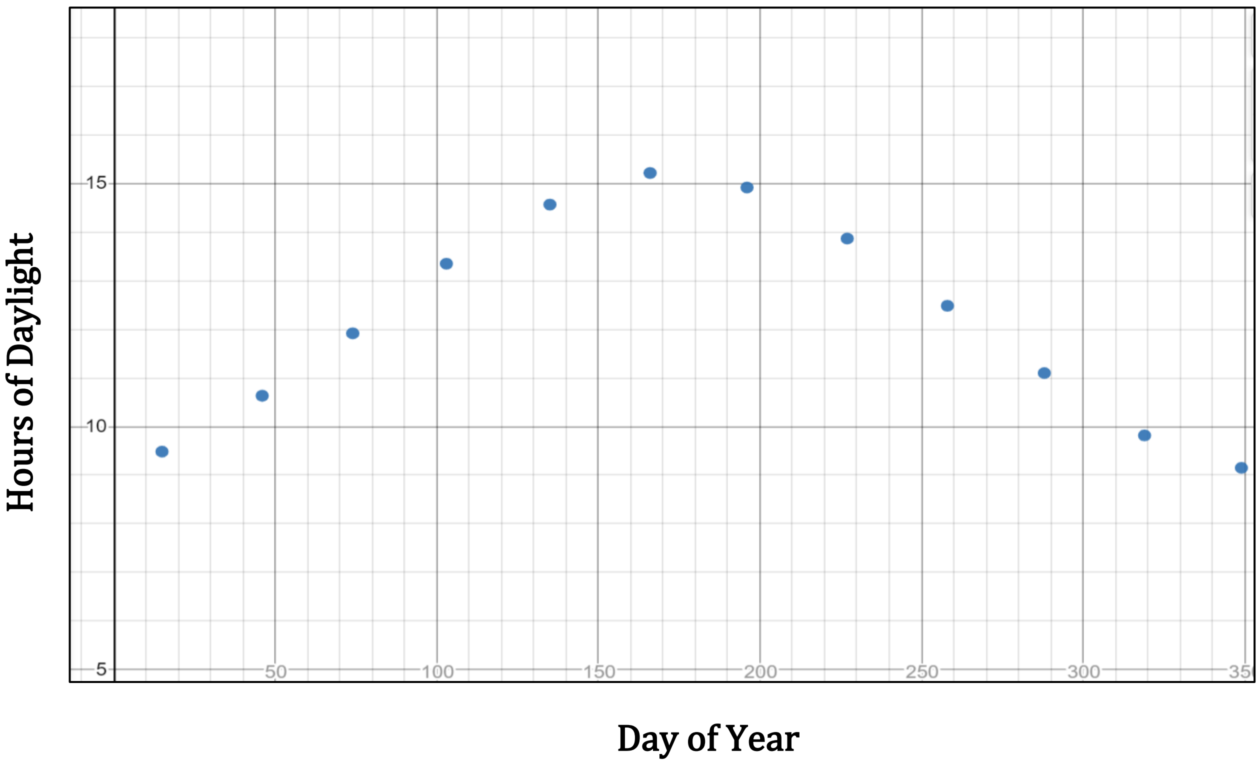
4. Analyze the <u>change</u> in daylight from month-to-month using your data. Is the change constant? If not, explain any observations about the change in daylight from month-to-month.
The change is not constant from month-to month; some months there is as little as 20 minutes of change and other months there is as much as 1 hour and 26 minutes of change. There are months when the daylight increases more per month (ex: March 15 to April 15 an increase of 1:17) and other months when the daylight increases by less (ex: December 15 to January 15 an increase of :20).

5. During which month(s) does it appear daylight changes at the fastest rate? March/April (1:26)

6. During which month(s) does it appear daylight changes at the slowest rate? Dec/Jan (:20)

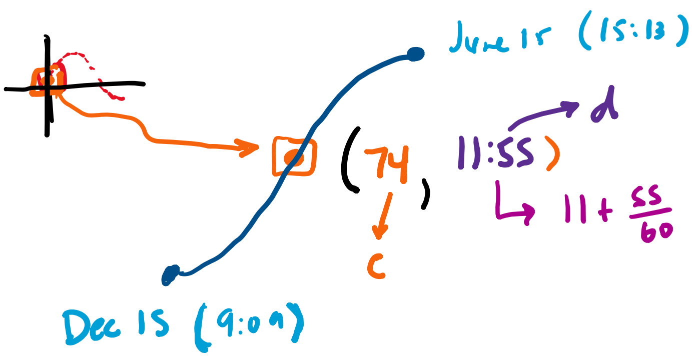
7. What type of function do you feel best matches your data? Explain your choice.
Because daylight increases and decreases and repeats the process of increasing and decreasing each year, a sine/cosine function will likely best match the data.

Task #2: Plotting the Data



Task #3: Determining Regressions Equations for the Data

8. Estimate a sinusoidal regression equation in the form $\boldsymbol{y = a \cdot \sin(b(x-c)) + d}$ 'by hand'. Use your calculator to approximate values in your regression, if necessary, to three decimal places.



$y = a \cdot \sin(b(x-c)) + d$

$d = 11.917$

$c = 74$

$a = \dfrac{15{:}13 - 9{:}09}{2} = 3.033$

$b = \dfrac{2\pi}{365} = .017$

Estimated Regression: $y = 3.033 \sin(.017(x-74)) + 11.917$

9. Explain how you found the values for $a, b, c,$ and d in your regression equation.

a: the 'amplitude' is half the difference between the low and high amounts of daylight. b: the 'dilation' factor is based on daylight having a period of 365 days. So, $b = 2\pi/365$. c/d: the 'phase shift' and 'translation' factors are based on the point where daylight is increasing and halfway between its min and max.

10. Rewrite your estimated regression equation from the previous task in the box below.

Estimated Regression: $y = 3.033 \sin(.017(x - 74)) + 11.917$

11. Determine a sinusoidal regression in the form $y = a \cdot \sin(bx + c) + d$ using your graphing calculator. Round all values to 3 decimal places.

Calculator Regression: $y = 2.999 \sin(.017x - 1.310) + 12.123$

12. Compare the values between your estimated and calculated regression equations. Are the values similar? If not, explain why some values are different in the two regression equations.

The values for a and b are very close. The value for d is within 0.2 hours. The value for c is different, but this is because the calculator determines the sine regression in a different form. If you divide 1.310/0.017 you get ≈77 which is only a few days off.

13. Use your calculator regression to predict the hours of daylight for today's date. Compare your answer to today's actual hours of daylight. (You will need to revisit the daylight website to find this information).

Prediction: 10.466 hrs

Oct 28 (Day 301)

Actual hours of daylight: 10:32

$0.466(60) \approx$ 10:27

14. What is the difference, in hours, between the maximum and minimum amount of daylight in a day? Explain how you can use your regression equation to find the difference between the maximum and minimum hours of daylight.

Difference between max/min: 5.998 Explanation: The difference between the high and the low can be determined by doubling the amplitude of the sine regression equation.

Daylight Group Worksheet
Task #4: Comparing Group Data

1. Look closely at your group's graph containing data plotted from all five cities. Discuss the similarities and differences observed.

Similarities: Each city has approximately the same hours of daylight (12 hours) at two days of the year (around March 15 and September 15). Each city has its max/min hours of daylight on the same days. Each city has data that is periodic over 365 days.
Differences: The variance in daylight is different for the various cities. The max/min hours of daylight for each city is different.

2. Which city in your group has the longest and shortest day of the year? Anchorage, USA

3. Which city in your group has the least variation in daylight during the year? Bangkok, Thailand

4. Which day number of the year is the longest and shortest? Is this the same for all locations?
Longest: June 15; Shortest: December 15; this is the same for all locations.

5. What do you think accounts for the similarities/difference explained above?
The distance each city is north of the Equator?!? (Ex: The more distance, the higher variation of daylight.)

6. If a location has 12 hours of daylight every day of the year, where do you think it is located?
Close to the Equator.

7. Suppose you collected data for cities from the Southern Hemisphere. Explain differences you would observe in the graph explored.
The cities would all have the same daylight (12 hours) as the cities in the Northern Hemisphere around March 15 and September 15. However, when Northern Hemisphere cities reach a max amount of daylight in June, Southern Hemisphere cities reach a min, and vice versa in December.

Task #5: Comparing Group Regression Equations

8. List each group member's city and regression equation (rounded to 3 decimal places).

City Name	Sinusoidal Regression Equation
Anchorage	$y = 6.611\sin(0.017x - 1.293) + 12.194$
Dublin	$y = 4.630\sin(0.017x - 1.305) + 12.146$
Chicago	$y = 2.999\sin(0.017x - 1.310) + 12.123$
Cairo	$y = 1.908\sin(0.017x - 1.329) + 12.125$
Bangkok	$y = 0.796\sin(0.017x - 1.350) + 12.110$

9. Discuss the similarities and differences observed in the sine regression equations in the table above. Then, explain why the similarities and differences occur.

Similarities The 'b' values are all 0.017 (since the data is periodic for all cities over 365 days). The 'c' and 'd' values are similar (since the 'transition point' or midpoint between low and high daylight hours occurs at the same time of the year).

Differences: The amplitudes of the various regressions are all different. The further the city is north of the Equator, the higher the amplitude.

10. Suppose the function below represents the hours of daylight, y, for a city in the world based on the day of the year, x (where $x = 1$ corresponds to January 1, $x = 32$ corresponds to February 1).

Function: $y = 1.5\sin(0.017x - 4.505) + 12.1$

Which hemisphere is this city located in? Show any work that helps support your answer and explain how you know.

Since $4.505/0.017 \approx 265$, this means the day when daylight is halfway between its min and max is increasing would be later in the year (around September). This would be in the Southern Hemisphere because daylight is decreasing in the Northern Hemisphere in September.

11. Northbrook's longest and shortest days (in terms of daylight) are approximately 9.1 and 15.3 hours, respectively. Is the city represented in the function closer to or further from the Equator than Northbrook? Show any work that helps support your answer and explain how you know.

The city in the function has a variance in daylight of 3 (double the amplitude, 1.5) hours. This is less variance than Northbrook experiences. Thus, the city must be closer to the Equator.

Daylight Extension
Task #6: Analyzing the Rate of Change in Daylight

In this task you will investigate the 'rate of change' for daylight for your given city. Follow the steps below to graph the sinusoidal regression equation determined on your calculator in Task #3 and draw a tangent line to your regression equation.

1. Insert a Graphs page into the same document containing the Lists & Spreadsheet page of daylight data. On the Graphs page, press tab and arrow up to $f1(x)$. The regression equation should appear. Press enter to graph the regression equation. Scale the axes appropriately to observe the regression equation for the domain of one year (365 days).

2. Press menu, select Geometry, then Points & Lines, and then select Point On. Plot a point on your regression equation.

3. Press menu, select Geometry, then Points & Lines, and then select Tangent. Draw a tangent line to the point plotted in the previous step. (See the definition of tangent line in the box below.)

A **tangent line** is a line that touches a function at one point. The slope of the tangent line represents the **instantaneous** rate of change for a function at that point.

4. Drag the point on your function to where x = 31 (January 31). Write down the slope of the tangent line and interpret its meaning.

Slope: 0.035 Interpretation: On January 31, daylight is increasing by 0.035 hours per day which corresponds to 2.1 mins per day.

5. Drag the point and tangent line on the function to the locations (based on amount of daylight/slope of the tangent line) in the table below. For each location, list the day of the year when the location occurs (convert the day number into month and day) and the slope of the tangent line on that day.

Location	Day of Year	Slope of Tangent Line
Maximum daylight	June 21 (day 172)	0
Minimum daylight	December 25 (day 359)	0
Most positive slope	March 16 (day 75)	0.05
Most negative slope	September 24 (day 267)	−0.05

6. Estimate the time, in minutes, for the change in daylight per day when the amount of change in daylight is at a maximum. Show work.

$0.05(60)$

Number of minutes: $\approx$ 3 mins per day