



# Breaking Down the Gatekeepers

## Models and Visual Strategies for Integers, Fractions and Decimals

NCTM Washington DC  
October, 2023



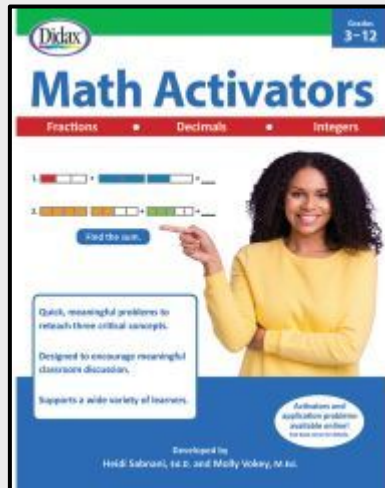
# About Us



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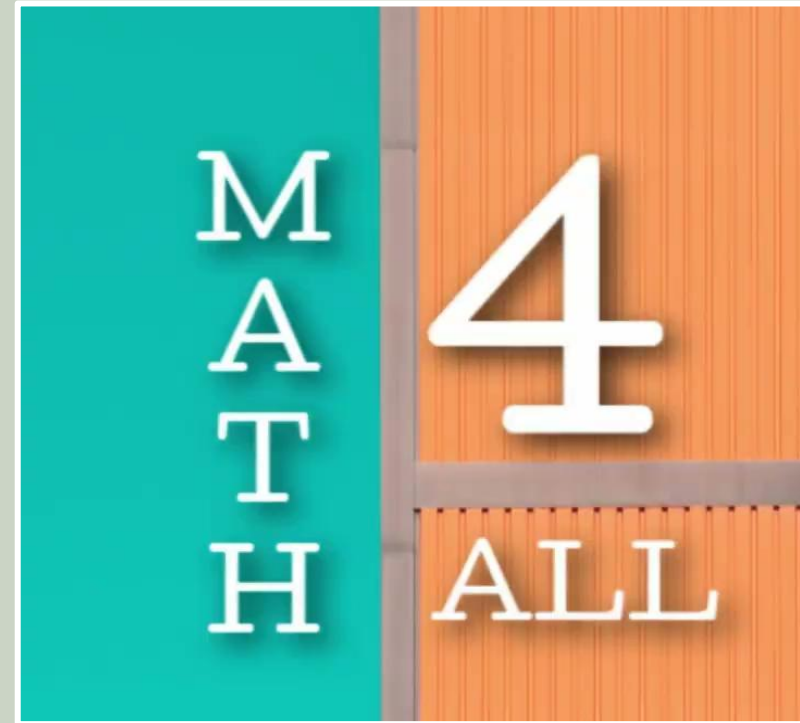
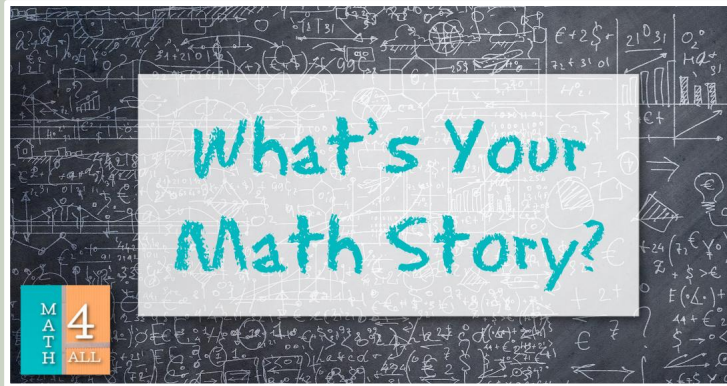


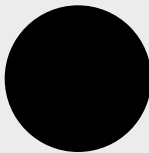
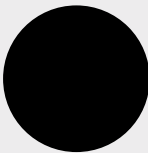
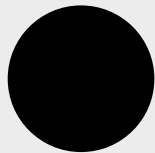
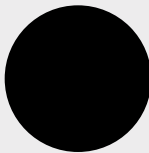
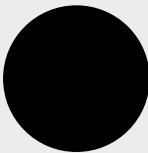
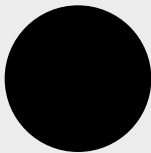
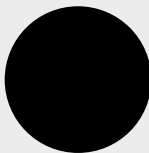
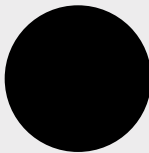
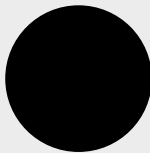
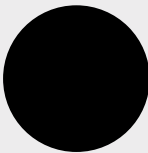
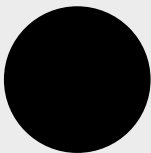
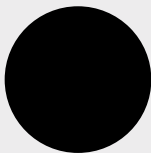
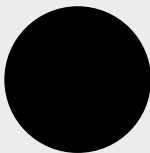
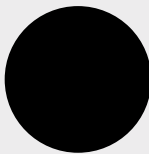
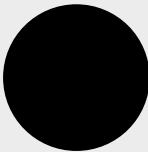
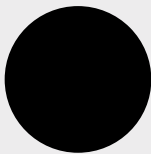
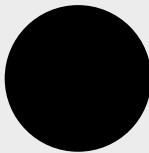
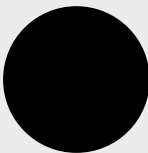
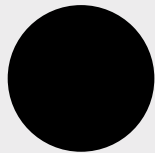
Heidi Sabnani, EdD  
Consultant & Author



# Math-4-All

[youtube.com/@math4allshow](https://youtube.com/@math4allshow)  
[@math4allshow](https://twitter.com/math4allshow)







# Gatekeepers

Fractions, decimals, and integers are areas that are consistently hard for kids. High school teachers ask for materials to help students with these areas.

In order to get into higher levels of math these areas must be solid or the math becomes too clunky.



# Centering Student Thinking

These activators can be used at the beginning of instruction or as an intervention. In small groups, concepts can be explored more deeply.

The goal is to represent student thinking in order to make connections for all students.



# Centering Student Thinking

Consider how we VISUALLY represent student's thoughts and make connections explicit between these representations.



# Consider these problems

$$1 \frac{1}{5} - \frac{4}{5} =$$

$$1 \frac{3}{4} - 1 \frac{5}{8} =$$





# Pictorial Representations make difficult concepts accessible

$$1 \frac{1}{5} - \frac{4}{5} =$$

$$1 \frac{3}{4} - 1 \frac{5}{8} =$$

Find the difference.

1.  -  = \_\_\_\_

2.  -  = \_\_\_\_



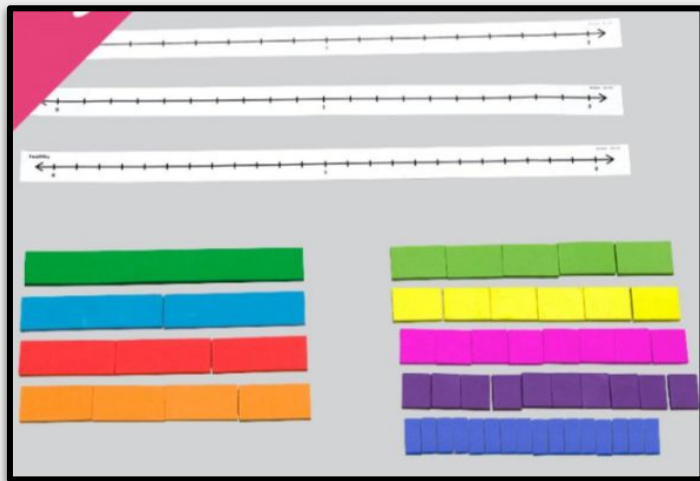
# Manipulatives

$$1 \frac{1}{5} - \frac{4}{5} =$$

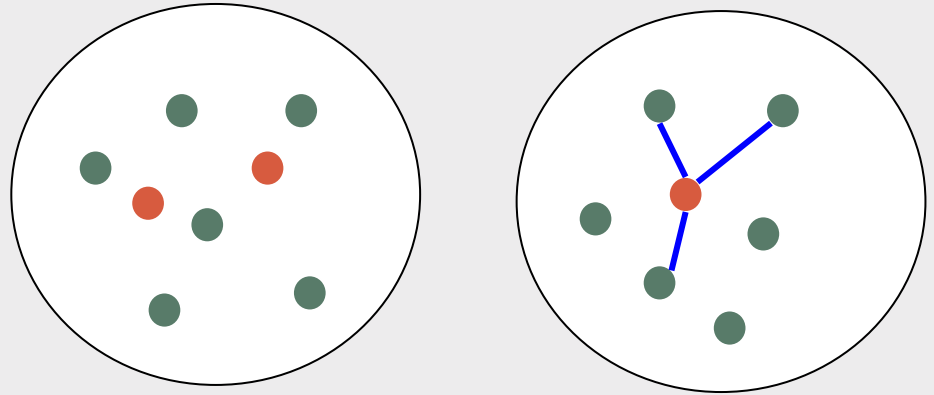
$$1 \frac{3}{4} - 1 \frac{5}{8} =$$

Discuss with your table.

How do the pictorial representations and the manipulatives help to better assess and understand student thinking?



# Building Connections



Dr. Karen Karp

Graphic recreated from Kate Fanelli who is the math accessibility specialist for Michigan's Integrated Mathematics Initiative



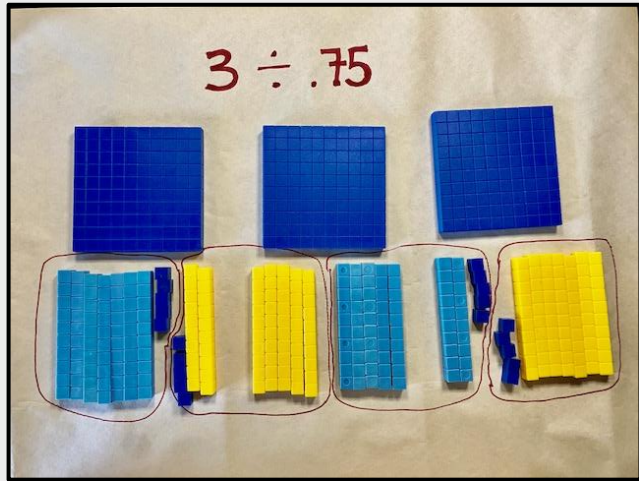
# Most Important Job of the teacher is:

- Provide a problem that allows students to connect what they know to representations that further learning.
- Time for student thinking (for solving and sharing with a partner)
- Scribe Student thinking (visually)
- Allow for student talk
- Make Connections Explicit

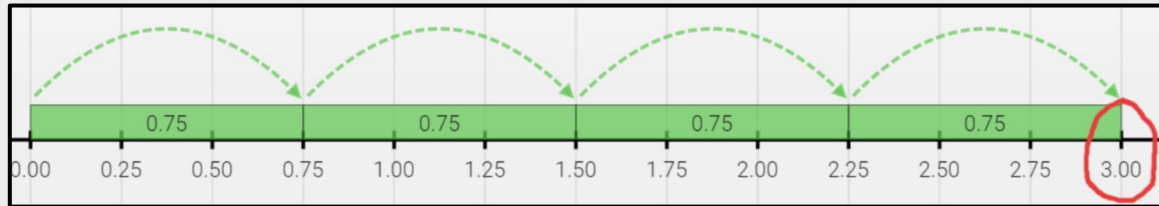


# Concurrent Representations Make Connections

Physical actions, drawings or diagrams, symbolic representations.



$3 \times 100 = 300$  Multiply the dividend by 100  
 $.75 \times 100 = 75$  Multiply the divisor by 100  
 $300 \div 75 = 4$  Divide





*“Using multiple  
representations creates a  
mental residue” –*



Karp 2022

$$4\frac{1}{3} - 2\frac{2}{3}$$

$$\begin{array}{r} 4\frac{1}{3} \quad 2\frac{2}{3} \\ \wedge \quad \wedge \\ 2 \quad 2\frac{1}{3} \quad 2\frac{1}{3} \quad \frac{1}{3} \end{array}$$

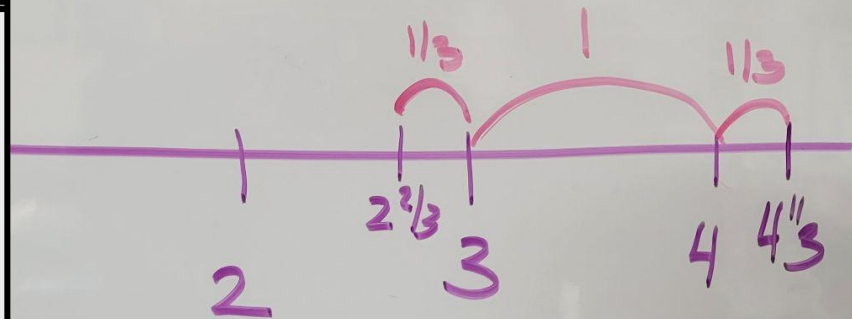
$$2\frac{1}{3} - 2\frac{1}{3} = 0$$

$$2 - \frac{1}{3} = 1\frac{2}{3}$$

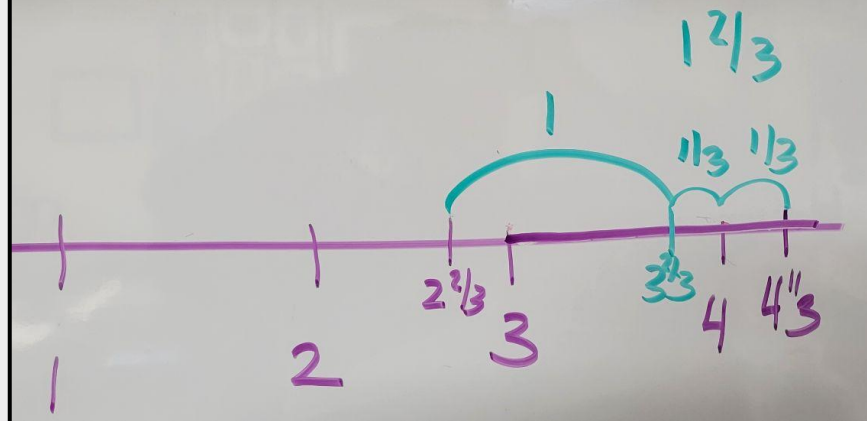
$$4\frac{1}{3} - 2\frac{1}{3} = 2$$

$$2 - \frac{1}{3} = 1\frac{2}{3}$$

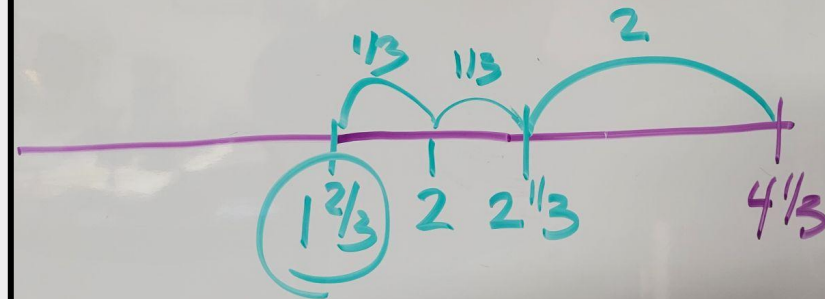
$$4\frac{1}{3} - 2\frac{2}{3}$$



$$4\frac{1}{3} - 2\frac{2}{3}$$



$$4\frac{1}{3} - 2\frac{2}{3}$$



*“Diagram literacy is the ability to “read” a diagram. Students who can create diagrams to express their thinking, and interpret diagrams to receive new information, are working at a conceptual, representational level to make connections, look for and make use of structures (Common Core Standard of Mathematical Practice #7), and model mathematically (Standard of Mathematical Practice #4).”*

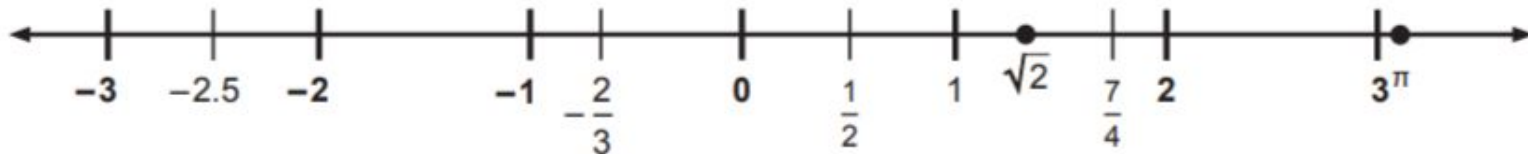


***Kate Fanelli***

# Number Lines

*Help students build understanding that fractions are numbers with magnitude*  
(Fuchs et al., 2013; Siegler et al., 2010)

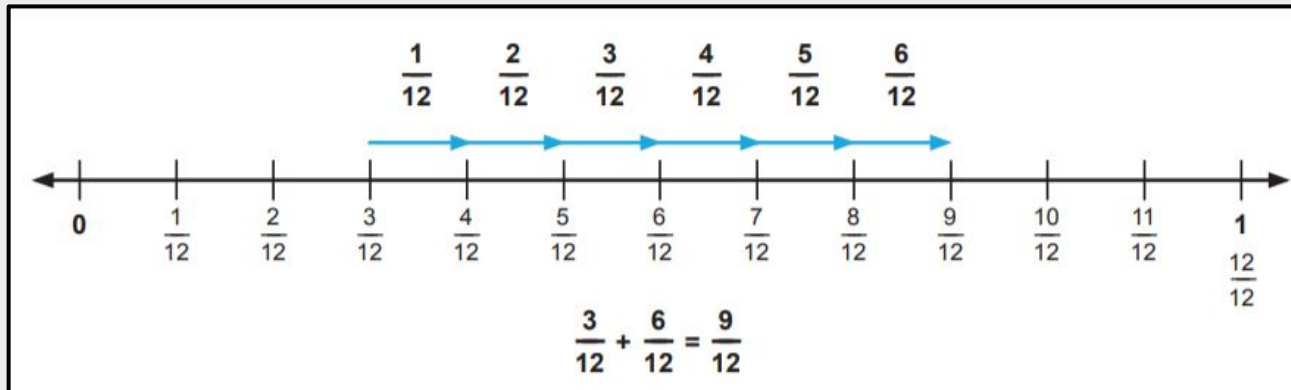
Example 4.1. Number line representing magnitudes of whole, positive, negative, rational, and irrational numbers.





# Number Lines

*An intervention program that strongly emphasized the number line representation for fractions found larger gains for at-risk learners than a program that focused primarily on the part-whole approach.* (Fuchs et al., 2013; Siegler et al., 2010)

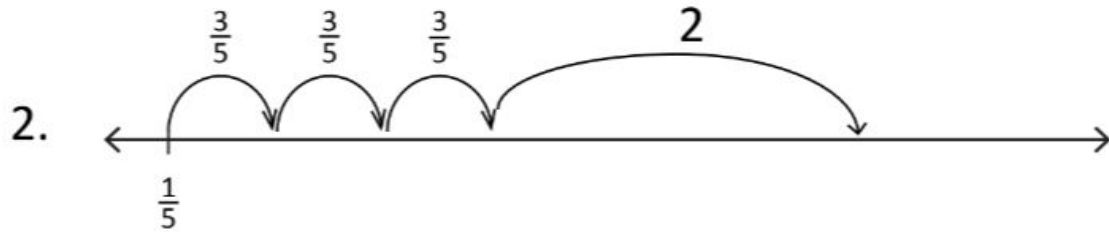
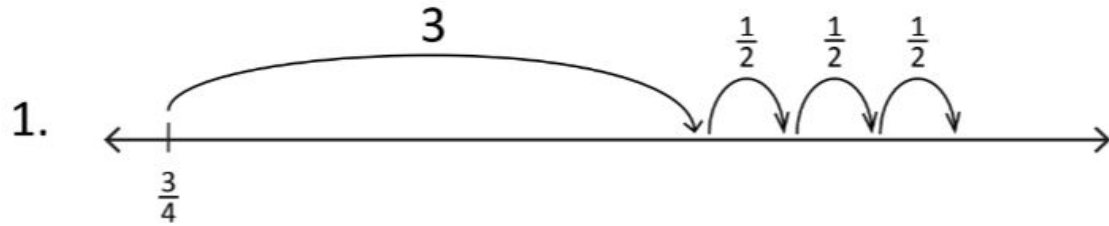


# Protocol For Math Activators

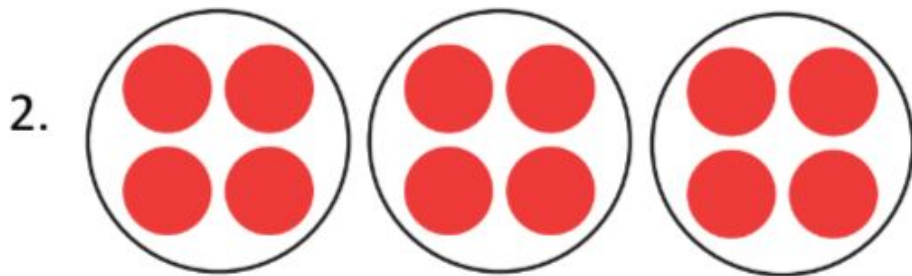
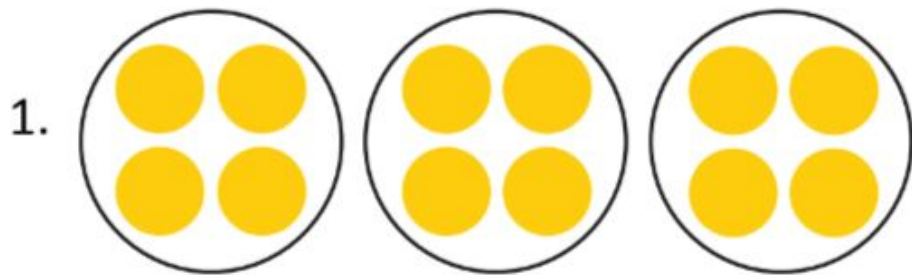
1. The teacher shows the slide.
2. The students solve the problem.
3. The teacher asks for answers and scribes all the answers that students provide.
4. The students turn and talk.
5. The teacher asks for students to share their answers.
6. Teacher conducts a conversation connecting strategies and representations.



What is the end point?



What equations show the value of the integer chips?



# Practice

1

**Choose an activator**

From your table

2

**Choose a person to be the facilitator**

Take turns

3

**Students solve then scribe student thoughts**

Discuss how to scribe thinking for maximum student understanding





What is the missing number?



+ \_\_\_\_\_ =



A hexagon represents one whole.



# Synthesis/Closing up

1

Focus on student thinking

THIS TAKES  
PRACTICE

2

Use visuals to represent thinking

3

Make connections explicit

