## Activity 1: Murals (*20 minutes*)

| **Instructional Routine:** Co-Craft Questions (MLR5) | |
|---|---|
| **Addressing:** NC.M2.A-CED.2; NC.M2.F-BF.1 | **Building Towards:** NC.M2.A-REI.1 |

In the previous lesson, students learned that an inverse variation relationship is one where the product of the two variables is constant. Students wrote equations to represent these relationships primarily in the form $y = \frac{k}{x}$. In this activity, students will explore the different ways of expressing the same inverse variation relationship. They will conclude that $xy = k$, $y = \frac{k}{x}$, and $x = \frac{k}{y}$ are all ways of representing the relationship. The goal here is for students to recognize—by repeatedly calculating the value of one quantity and then the value of the other quantity—that a particular form of equation might be handy for finding one quantity but not so handy for finding the other.

When answering the first couple of parts of the first question (finding the area each artist will paint given the number of artists), students can use various strategies to efficiently reason about the answers. They might draw diagrams, use proportional reasoning, or think in terms of multiplication (asking, for example, "3 times what number equals 460?"). They might also write an equation such as $n \cdot A = 460$, substitute the number of artists for $n$, and then solve the equation.

As students progress through the parts, they will likely notice that some strategies become less practical for finding the value of interest. One strategy, however, will remain efficient: dividing 460 by the number of artists, or evaluating $460 \div n$ or $\frac{460}{n}$ at the given values of $n$.

Likewise, when answering the third question (finding the number of artists given the area to paint), students could start out with a variety of strategies and find the number of artists without much effort. Later, however, the number of artists becomes a bit more cumbersome to find except when using division (that is, dividing 460 by the given area, or evaluating $460 \div A$ or $\frac{460}{A}$ at the given values of $A$).

### Step 1 (Launch)

- Introduce the art form of murals and how they can be used to empower and celebrate a city and its residents. For example, share this video with students: https://bit.ly/TrailblazersMural.
- Use the *Co-Craft Questions* routine by reading the description of the situation aloud and asking students, "What mathematical questions could be asked about this situation?" Give students 1–2 minutes to jot down questions, then call on two or three students to share their questions aloud. Consider assigning a question from a student to the whole class.
- Give students a few minutes of quiet work time on questions 1–4 and then time to share their responses with their partner.

> **Monitoring Tip:** Identify students who use the approaches mentioned in the narrative and select them to share their strategies during discussion. Include at least one student who does not typically volunteer.

## Student Task Statement

DO THE MATH

To the right is a mural titled "Bloom"[1] by artist Osiris Rain. It can be found at the corner of N. Davidson and 35th Street in Historic NoDa, Charlotte, North Carolina. This piece of street art covers 460 square feet of the building (46 feet x 10 feet).


Imagine a scenario where Osiris needed to have the 460-square-foot mural completed in one day. To complete the project in time, he needs a team of artists to help him, each artist taking an equal area of mural.

1. Determine the area of the building each artist would paint each day to have it complete in time if there were:
    a. Two artists
    b. Three artists
    c. Five artists
    d. Eight artists

2. Write an equation that would make it easy to find $A$, the area of the building in square feet, for each artist if there are $n$ artists.

3. Determine the number of artists needed if each artist were to paint the following area of the building. Be prepared to explain or show your reasoning.
    a. 65.714 square feet
    b. 115 square feet
    c. 46 square feet
    d. 33 square feet

4. Write an equation that would make it easy to find the number of artists, $n$, if each artist paints a section that is $A$ square feet.

[1] Rain, O. (2017). *Bloom* [Acrylic paint.] https://artwalksclt.com/noda-east

### Step 2 (Discuss)

- Select students to present their strategies for solving either set (or both sets) of questions. Start with students using the least straightforward approach and end with those who wrote $A = \frac{460}{n}$ for the first set of questions (or $n = \frac{460}{A}$ for the second set of questions).
- Emphasize that isolating the variable that we're interested in—before we substitute any known values—can be an efficient way for solving problems. Once we pin down the variable of interest first and see what expression is equal to it, we can simply evaluate that expression and bypass some tedious steps.
- Highlight that isolating a variable is called "solving for a variable." In the context of painting the mural, if we want to know the area of the mural each artist would be responsible for, we can solve for $A$. If we want to know how many artists would be needed, we can solve for $n$.

**RESPONSIVE STRATEGIES**

Use color coding and annotations to highlight connections between representations. As students share their reasoning, capture their thinking on a visible display. Invite students to describe connections they notice between approaches that use diagrams, proportional reasoning, or multiplication, with those who used an equation. For example, ask "Where does *A* appear in this (diagram)?"

*Supports accessibility for: Visual-spatial processing*

**PLANNING & REFLECTION NOTES**

# Lesson 2: Solving Inverse Variation Equations

## PREPARATION

| Lesson Goals | Learning Target |
|---|---|
| • Create equations in one variable that represent an inverse variation relationship.<br>• Solve problems involving inverse variation relationships. | • I can create and solve equations given an inverse variation relationship. |

### Lesson Narrative

In the previous lesson, students learned to recognize and represent inverse variation relationships. In this lesson students will explore the different ways to represent the relationship. They will also create and solve one variable equations involving inverse variation.

First, students recall the constant of proportionality for an inverse variation relationship is defined as the product of the two related values. They use this definition to calculate and compare the constant of proportionality for three different inverse variation functions. In Activity 1, students reason with a contextual situation and conclude that there are different ways to write the equation given what values they know and which values they want to know. In Activity 2, students justify the steps in solving an inverse variation equation. Finally, in the last activity, students create and solve inverse variation equations.

**What comments or feedback from the End-of-Unit 3 Student Surveys will you focus on during this lesson?**

## Focus and Coherence

| Building On | Addressing |
|---|---|
| **NC.M1.A-CED.4:** Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.<br><br>**NC.M1.A-REI.3:** Solve linear equations and inequalities in one variable. | **NC.M2.A-CED.1:** Create equations and inequalities in one variable that represent quadratic, square root, inverse variation, and right triangle trigonometric relationships and use them to solve problems.<br><br>**NC.M2.A-CED.2:** Create and graph equations in two variables to represent quadratic, square root and inverse variation relationships between quantities.<br><br>**NC.M2.A-REI.1:** Justify a chosen solution method and each step of the solving process for quadratic, square root and inverse variation equations using mathematical reasoning.<br><br>**NC.M2.A-REI.2:** Solve and interpret one variable inverse variation and square root equations arising from a context, and explain how extraneous solutions may be produced.<br><br>**NC.M2.F-IF.9:** Compare key features of two functions (linear, quadratic, square root, or inverse variation functions) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).<br><br>**NC.M2.F-BF.1:** Write a function that describes a relationship between two quantities by building quadratic functions with real solution(s) and inverse variation functions given a graph, a description of a relationship, or ordered pairs (include reading these from a table). |

## Agenda, Materials, and Preparation

- **Bridge** (*Optional, 5 minutes*)
- **Warm-up** (*10 minutes*)
- **Activity 1** (*20 minutes*)
  - "Trailblazers Mural with Mural Artist Kiara Sanders" video: https://bit.ly/TrailblazersMural
- **Activity 2** (*10 minutes*)
- **Activity 3** (*15 minutes*)
- **Activity 4** (*15 minutes*)
- **Lesson Debrief** (*10 minutes*)
- **Cool-down** (*5 minutes*)
  - M2.U4.L2 Cool-down (print 1 copy per student)

Technology is required for Practice Problem #6. Using the link, https://bit.ly/M2U4L2PP6, add the Desmos activity to the class account or create a unique code. Find more information on creating a class or code in Unit 4, Lesson 5, Station E.

# LESSON

### Bridge (*Optional, 5 minutes*)

**Building On:** NC.M1.A-REI.3

In this lesson, students will solve equations of the form $y = \frac{k}{x}$. The reasoning is similar to solving equations such as $12 = \frac{x}{5}$, where students may have multiplied each side by 5. The purpose of the bridge is to remind students of this strategy. This task aligns to Check Your Readiness problem 2.

### Student Task Statement

DO THE MATH

Solve each of the following equations. What strategies could be used to find the value of the variable in each of these equations? Think about strategies that might work for some but not all and some that would work for all.

1. $\frac{t}{20} = 10$
2. $\frac{p}{1.4} = 0.5$
3. $12 = \frac{x}{5}$
4. $-\frac{2}{3} = \frac{r}{4}$

## Warm-up: The Constant of Proportionality (*10 minutes*)

| | |
|---|---|
| **Instructional Routine:** Discussion Supports (MLR8) - Responsive Strategy | |
| **Addressing:** NC.M2.F-IF.9 | |

In the previous lesson, students learned to identify the constant of proportionality by multiplying the $x$ and $y$ values from a coordinate point on the graph or from a table of values. They also learned to recognize it as the value of $k$ in the equation $y = \frac{k}{x}$. The purpose of this warm-up is to work towards fluency in identifying and comparing the constant of proportionality given different representations.

**RESPONSIVE STRATEGY**

Use *Discussion Supports* by displaying sentence frames to support students when they explain their strategy. For example, "First, I ____ because ..." or "I noticed ____ so I ..."

*Discussion Supports* (MLR8)

### Step 1 (Launch)

- Ask students to arrange themselves in pairs or use visibly random grouping. **Students will stay in these groups for the remainder of the lesson.**
- Provide students a minute of individual think time and then time to share their thinking with their partner.

**Monitoring Tip:** As students compare with partners, monitor for different ways students reason with the representations. They may:

- Identify a point and multiply the $x$ and $y$ values
- Compare a single point from each representation such as $(4, 12.5)$, $(4, 6)$, and $(4, 18)$
- Create the same representation (for example, students may write equations for "a" and "b" or create tables for "b" and "c")

Ask a few students if they would be willing to share their findings. Look to elevate the voices of students who have not volunteered contributions recently.

## Student Task Statement

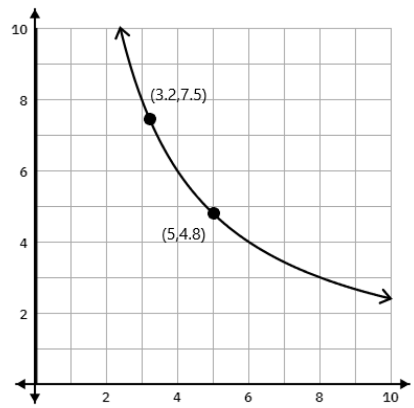
DO THE MATH

Each of the following represents an inverse variation relationship. List the relationships in order from least to greatest based on the constant of proportionality.

a.

| $x$ | $y$ |
|---|---|
| 2 | 25 |
| 4 | 12.5 |
| 10 | 5 |
| 20 | 2.5 |

b.




c. $y = \frac{72}{x}$

## Step 2 (Discuss)

- Ask previously identified students to share how they reasoned with the representations. Record their thinking for all to see.
  - If most students used the same reasoning, suggest one of the other strategies mentioned in the Monitoring Tip. Provide students time to think about the approach and discuss with a partner before discussing as a whole class.
- Facilitate a whole-class discussion with the purpose of highlighting ways to identify the constant of proportionality from different representations. Here are some questions for discussion:
  - "What might you say to another student to explain how to find the constant of proportionality?" (The constant of proportionality is the product of $x$ and $y$.)
  - "Could another point on the table or graph have been used to find the constant of proportionality?" (Yes. The constant of proportionality is the same for all $(x, y)$ pairs.)
  - "Which representation do you prefer to have if asked to identify the constant of proportionality?" (Responses may vary.)

## PLANNING & REFLECTION NOTES

## Activity 1: Murals (*20 minutes*)

| **Instructional Routine:** Co-Craft Questions (MLR5) | |
|---|---|
| **Addressing:** NC.M2.A-CED.2; NC.M2.F-BF.1 | **Building Towards:** NC.M2.A-REI.1 |

In the previous lesson, students learned that an inverse variation relationship is one where the product of the two variables is constant. Students wrote equations to represent these relationships primarily in the form $y=\frac{k}{x}$. In this activity, students will explore the different ways of expressing the same inverse variation relationship. They will conclude that $xy=k$, $y=\frac{k}{x}$, and $x=\frac{k}{y}$ are all ways of representing the relationship. The goal here is for students to recognize—by repeatedly calculating the value of one quantity and then the value of the other quantity—that a particular form of equation might be handy for finding one quantity but not so handy for finding the other.

When answering the first couple of parts of the first question (finding the area each artist will paint given the number of artists), students can use various strategies to efficiently reason about the answers. They might draw diagrams, use proportional reasoning, or think in terms of multiplication (asking, for example, "3 times what number equals 460?"). They might also write an equation such as $n \cdot A = 460$, substitute the number of artists for $n$, and then solve the equation.

As students progress through the parts, they will likely notice that some strategies become less practical for finding the value of interest. One strategy, however, will remain efficient: dividing 460 by the number of artists, or evaluating $460 \div n$ or $\frac{460}{n}$ at the given values of $n$.

Likewise, when answering the third question (finding the number of artists given the area to paint), students could start out with a variety of strategies and find the number of artists without much effort. Later, however, the number of artists becomes a bit more cumbersome to find except when using division (that is, dividing 460 by the given area, or evaluating $460 \div A$ or $\frac{460}{A}$ at the given values of $A$).

### Step 1 (Launch)

- Introduce the art form of murals and how they can be used to empower and celebrate a city and its residents. For example, share this video with students: https://bit.ly/TrailblazersMural.
- Use the *Co-Craft Questions* routine by reading the description of the situation aloud and asking students, “What mathematical questions could be asked about this situation?” Give students 1–2 minutes to jot down questions, then call on two or three students to share their questions aloud. Consider assigning a question from a student to the whole class.
- Give students a few minutes of quiet work time on questions 1–4 and then time to share their responses with their partner.

> **Monitoring Tip:** Identify students who use the approaches mentioned in the narrative and select them to share their strategies during discussion. Include at least one student who does not typically volunteer.

## Student Task Statement

DO THE MATH

To the right is a mural titled “Bloom”[1] by artist Osiris Rain. It can be found at the corner of N. Davidson and 35th Street in Historic NoDa, Charlotte, North Carolina. This piece of street art covers 460 square feet of the building (46 feet x 10 feet).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1. Determine the area of the building each artist would paint each day to have it complete in time if there were:
    a. Two artists
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2. Write an equation that would make it easy to find $A$, the area of the building in square feet, for each artist if there are $n$ artists.

3. Determine the number of artists needed if each artist were to paint the following area of the building. Be prepared to explain or show your reasoning.
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4. Write an equation that would make it easy to find the number of artists, $n$, if each artist paints a section that is $A$ square feet.

[1] Rain, O. (2017). *Bloom* [Acrylic paint.] https://artwalksclt.com/noda-east

### Step 2 (Discuss)

- Select students to present their strategies for solving either set (or both sets) of questions. Start with students using the least straightforward approach and end with those who wrote $A = \frac{460}{n}$ for the first set of questions (or $n = \frac{460}{A}$ for the second set of questions).
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- Highlight that isolating a variable is called "solving for a variable." In the context of painting the mural, if we want to know the area of the mural each artist would be responsible for, we can solve for $A$. If we want to know how many artists would be needed, we can solve for $n$.

**RESPONSIVE STRATEGIES**

Use color coding and annotations to highlight connections between representations. As students share their reasoning, capture their thinking on a visible display. Invite students to describe connections they notice between approaches that use diagrams, proportional reasoning, or multiplication, with those who used an equation. For example, ask "Where does A appear in this (diagram)?"

*Supports accessibility for: Visual-spatial processing*

**PLANNING & REFLECTION NOTES**

## Activity 2: Reasoning Algebraically with Inverse Variation (*10 minutes*)

| | | |
|---|---|---|
| **Instructional Routines:** Notice and Wonder; Discussion Supports (MLR8) - Responsive Strategy; Compare and Connect (MLR7) | | |
| **Building On:** NC.M1.A-REI.3; NC.M1.A-CED.4 | **Addressing:** NC.M2.A-REI.1 | **Building Towards:** NC.M2.A-REI.2 |

In this activity, students start by revisiting how to solve linear equations involving fractions. The reasoning used to solve these equations is similar to the reasoning used to solve inverse variation equations. Lastly, students demonstrate the equivalency of the equations $xy = k$, $y = \frac{k}{x}$, and $x = \frac{k}{y}$ given that $x$, $y$, and $k$ are not 0.

### Step 1 (Launch)

- Display the equations $8 = \frac{k}{3}$ and $8 = \frac{3}{x}$. Engage students in the *Notice and Wonder* routine by asking students what they notice and wonder about the two equations. Encourage students to record what they notice and wonder in their Student Workbooks.
- Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice and wonder with their partner.

**RESPONSIVE STRATEGY**

Display the following sentence frames to support students with the *Notice and Wonder* routine: "I noticed ____ because...," "I wonder why ____ , "The equations are the same/different because ____."

*Supports accessibility for: Language; Organization*

*Discussion Supports* (MLR8)

- Ask students to share the things they noticed and wondered. As you are listening to student responses, look for opportunities to highlight the structure of the equations. For example, both equations show 8 is equivalent to a fraction. In the first equation, some number $k$ is divided by 3, while in the second equation, 3 is divided by some number $x$.

### Step 2 (Launch)

- Tell students that in this activity they will look at how solving these two equations is similar and how it is different.
- Ask students to work individually on question 1 and provide a couple of minutes of quiet work time.
- Ask students to compare solutions with their partner. If the solutions are different, work together to come to an agreement and then complete questions 2–4.
- Remind partner sets to make a visual display of their work as they will have the opportunity to view their classmates' thinking as part of the *Compare and Connect* routine in Step 3.

### Student Task Statement

DO THE MATH

1. Solve each of the following equations. Be prepared to share your reasoning with your partner.

   a. $8 = \frac{k}{3}$

   b. $16.2 = \frac{k}{5}$

2. Noah was solving the equation $8 = \frac{3}{x}$. Analyze Noah's work and provide justification for each step.

| | |
|---|---|
| $8 = \frac{3}{x}$ | Original equation |
| $8x = 3$ | |
| $x = \frac{3}{8}$ | |

3. How is the way Noah solved $8 = \frac{3}{x}$ similar to how you solved the equations in question 1? How is it different?

4. Use Noah's reasoning to solve $y = \frac{k}{x}$ for $x$.

| | |
|---|---|
| $y = \frac{k}{x}$ | Original equation |
| | |
| | |

   a. How are the three equations related?

   b. Are there any restrictions on the value of $x$, $y$, or $k$? Explain.

### Step 3 (Discuss)

- Use the *Compare and Connect* routine to facilitate a whole-class discussion about questions 2 and 3. The purpose of this discussion is to use a variety of words and phrases to make sense of the steps in solving equations when there is a variable in the denominator.
  - Invite students to share their responses to question 2. Listen for and amplify language students use to justify the steps.
  - Ask students to share the similarities and differences they recognized in the way the equations in question 1 were solved and the way Noah solved his equation in question 2. Invite students to refer to their responses to question 3 during this discussion. (In question 2, the variable was in the denominator, so you have to first multiply each side by the variable, and then divide by a number to isolate the variable.)
  - Invite students to share the steps to solve $y=\frac{k}{x}$ for $k$. Emphasize the reasoning used and how the equations created are equivalent.
  - Ask students what restrictions they considered for the value of $x$, $y$, or $k$ and why they are necessary for saying the equations are equivalent. (Neither $x$, $y$, nor $k$ can be 0. $x$ and $y$ can't be 0 because they both show up in the denominators in question 4. $k$ can't be 0 because if it were, then $y$ would have to be 0 in the original equation.)
- Tell students that the equation $y=\frac{k}{x}$ is the most common form of an equation representing inverse variation; however, it is not the only form. The equation $xy=k$ is beneficial when you have the two values and are looking for the constant of proportionality. The equation $x=\frac{k}{y}$ is beneficial when looking for the value of the input variable.

**PLANNING & REFLECTION NOTES**

## Activity 3: Boyle's Law (*15 minutes*)

| | |
|---|---|
| **Instructional Routines:** Three Reads (MLR6); Co-Craft Questions (MLR5) | |
| **Building On:** NC.M2.A.CED.2; NC.M2.F-BF.1 | **Addressing:** NC.M2.A-CED.1; NC.M2.A-REI.2 |

In this activity, students are given two variables that are inversely related and a set of related values. Students use their understanding of the definition of inverse variation and the values given to identify the constant of proportionality. Next, they write a two-variable equation to represent the relationship.

### Step 1 (Launch)

- Use the *Three Reads* routine together with the *Co-Craft Questions* routine. Encourage students to write their thoughts and misconceptions in their Student Workbook throughout the routine.
  - First Read: Without displaying the problem, read the problem aloud to the class:

    > Boyle's Law states that the pressure of an ideal gas increases as its container volume decreases. The volume of an ideal gas, $V$, varies inversely with the pressure of the gas, $P$, when the temperature remains the same.
    >
    > A balloon with a volume of 2.0 L is filled with a gas at 44 pounds per square inch.

    - Ask students: "What is this situation about? What is going on here?" Let students know the focus is just on the situation, not on the numbers. (For example, students might say, "it's about balloons filled with gas" or "it's about volume and pressure.")
    - Spend less than a minute scribing their understanding of the situation in a place where all can see. Do not correct students, but do clarify any unfamiliar words; visuals often help.
  - Second Read: Display the problem and ask a student volunteer to read it aloud to the class again.
    - Ask: "What are the quantities in this situation? A quantity is something that can be counted or measured."
    - Again, spend less than a minute scribing student responses. Encourage students to identify quantities that are named in the problem explicitly, and any quantities that may be implicit. For each quantity (for example, "volume"), ask students to add details (for example, "volume of 2 liters").
  - Finally, after a third and final time reading the description, invite students to think of their own mathematical questions about the situation. Provide students 1–2 minutes of individual think time.
  - Ask for volunteers to share their questions with the whole class. Listen for and amplify any questions involving the relationship between the pressure of gas and its volume.
- Have students work with their partner to complete questions 1–3, or assign a student-generated question in place of question 2.

**Monitoring Tip:** As pairs work, monitor for how students approach question 2. Students may:
- Guess and check by substituting values for pressure
- Create a table of values for volume and pressure
- Create and solve an equation

Let students know that they may be asked to share later. Include at least one student who does not typically volunteer.

***Advancing Student Thinking***: Students may question what the phrase "varies inversely" means. Tell students it is another way to describe an inverse variation relationship. Ask students what that information tells them about the equation.

## Student Task Statement

DO THE MATH

Boyle's Law states that the pressure of an ideal gas increases as its container volume decreases. The volume of an ideal gas, $V$, varies inversely with the pressure of the gas, $P$, when the temperature remains the same.

A balloon with a volume of 2.0 L is filled with a gas at 44 pounds per square inch.

1. Write an equation to represent the volume of the balloon, $V$, given the pressure, $P$.

2. If the pressure is reduced to 7.3 pounds per inch, what would be the volume of the balloon?

3. If the volume of the balloon is 5 L, what would be the pressure?

### Step 2 (Discuss)

- Facilitate a whole-class discussion. Begin by inviting students to share their equations from question 1 and explain their reasoning. Discuss questions such as:
  - "How did you know the form of the equation?" (The information given was that the volume and the pressure are inversely related. This means that the relationship can be represented by an equation such as $y = \frac{k}{x}$.)
  - "How did you find the constant of proportionality?" (The constant of proportionality is the product of the two related values 2 and 44.)
  - "What other equations can represent the relationship between volume and pressure?" ($PV = 88$, $P = \frac{88}{V}$)
- Ask previously identified students to share their responses and reasoning to question 2. Here are some key observations to highlight:
  - For the strategy of guess and check using the equation from question 1, ask students to share how they decided what numbers to try. Based on the result, how did they adjust for their next number?
  - If students write an equation such as $2 \cdot 44 = 7.3 \cdot V$, ask students to justify why the two products can be set equal to one another. Next, focus on how they solved the equation.
  - For the strategy of creating and solving an equation such as $V = \frac{88}{7.3}$, ask students to share how they knew where to substitute the given information into the equation and focus on the reasoning used in the solving process.

**PLANNING & REFLECTION NOTES**

## Activity 4: When the Variables Vary Inversely (*15 minutes*)

| **Building On:** NC.M2.A-CED.2; NC.M2.F-BF.1 | **Addressing:** NC.M2.A-CED.1; NC.M2.A-REI.2 |
|---|---|

In this activity, students use what they learned in the previous activity to create and solve equations.

### Step 1 (Launch)

- Ask students to decide who will be Partner A and who will be Partner B.
- Give students 6 minutes to independently complete the questions in their column.
- Ask students to compare their equations and missing values for each question. Equations should be the same for each question, derived from different data points. If they are not the same, ask students to take turns sharing their reasoning and finding a common solution.

**Student Task Statement**

Decide with your partner who will complete the column labeled "Partner A" and who will complete the column labeled "Partner B." For each situation, write an equation and find the missing value. When each partner has completed their column, compare answers. Equations for each numbered question should be the same for Partner A and Partner B. If the equation or missing value are not the same, take turns explaining your reasoning and find a common solution.

| Partner A | Partner B |
|---|---|
| 1. $y$ varies inversely with $x$, and $y = 28$ when $x = 8$. Find $y$ when $x = 4$.<br>Equation: ________________ | 1. $y$ varies inversely with $x$, and $y = 32$ when $x = 7$. Find $y$ when $x = 4$.<br>Equation: ________________ |
| 2. $y$ varies inversely with $x$, and $y = 10.54$ when $x = 2$. Find $x$ when $y = 4$.<br>Equation: ________________<br>(*continued*) | 2. $y$ varies inversely with $x$ and $y = 6.2$ when $x = 3.4$. Find $x$ when $y = 4$.<br>Equation: ________________ |

DO THE MATH

| | |
|---|---|
| 3. In an electric circuit, the current, $c$, varies inversely with the resistance, $r$. The current is 40 amps when the resistance is 12 ohms. Find the current when the resistance is 20 ohms.<br><br>Equation: ________________ | 3. In an electric circuit, the current, $c$, varies inversely with the resistance, $r$. The current is 48 amps when the resistance is 10 ohms. Find the current when the resistance is 20 ohms.<br><br>Equation: ________________ |
| 4. The amount of time, $t$, it takes to download a typical audiobook varies inversely with the rate, $r$, at which data is transferred. If it takes 28 seconds to download an audiobook at a rate of 10 megabytes per second (Mbps), how long would it take to download if the rate is 17 Mbps?<br><br>Equation: ________________ | 4. The amount of time, $t$, it takes to download a typical audiobook varies inversely with the rate, $r$, at which data is transferred. If it takes 20 seconds to download an audiobook at a rate of 14 megabytes per second (Mbps), how long would it take to download if the rate is 17 Mbps?<br><br>Equation: ________________ |

## Are You Ready For More?

DO THE MATH

The frequency (number of swings per second), $F$, of a pendulum varies inversely with the square root of the length in meters, $L$, of the pendulum.

1. Which of the following represents the relationship between frequency and length? Select **all** that apply.

   a. $F = 30\sqrt{L}$
   b. $F\sqrt{L} = 30$
   c. $F = \frac{30}{L^2}$
   d. $F = \frac{30}{\sqrt{L}}$
   e. $L = \frac{30}{\sqrt{F}}$

2. What is the frequency if the length of the pendulum is 0.5 meters?

3. What is the length of the pendulum if the frequency is 10 swings per second?

### Step 2 (Discuss)

- Select pairs to share their responses and reasoning. Prioritize any partner groups who individually used different strategies and ask them to compare their approaches. Give pairs discussion time to compare and contrast different approaches to their method to deepen understanding and connection. Highlight explanations that:
  - Calculated the product of the two values to find the constant of proportionality
  - Substituted the given value into the equation and solved to find the other value

**PLANNING & REFLECTION NOTES**

## Lesson Debrief (*10 minutes*)

The purpose of this lesson is for students to understand what it means when two values vary inversely and how to use the information to solve problems. Students connect this relationship to there being a constant of proportionality, calculate the constant of proportionality, and write an equation to describe the relationship. There are three equivalent forms of the inverse variation equation. Depending on which form the student is using, they may need to solve the equation using algebraic reasoning.

Choose what questions to focus the discussion on, whether students should first have an opportunity to reflect in their workbooks or talk through these with a partner, and what questions will be prioritized in the full class discussion.

To highlight the key ideas from this lesson and the connections to earlier lessons, discuss questions such as:

- "Suppose you are given that $y$ varies inversely with $x$. What additional information do you need to know to write an equation?" (There is a constant of proportionality that is the product of $x$ and $y$.)
- "If $y = 7$ and $x = 10$, what is the constant of proportionality?" ($7 \cdot 10 = 70$)
- "What is an equation that relates $x$ and $y$?" (There are three: $y = \frac{70}{x}$, $x = \frac{70}{y}$, and $xy = 70$.)
- "How could you show that $y = \frac{70}{x}$ is equivalent to $x = \frac{70}{y}$?" (Solve $y = \frac{70}{x}$ for $x$ by multiplying each side by $x$ and then dividing each side by $y$.)
- "If $y = 5$, what is the value of $x$?" ($x = \frac{70}{5} = 14$ or $5 = \frac{70}{x} \implies 5x = 70 \implies x = 14$)

## Student Lesson Summary and Glossary

The number of hours it takes to build an outdoor storage building varies inversely with the number of people building it. If it takes 18 hours with 3 people, how long will it take if there are 5 people?

We know the number of hours, $h$, varies inversely with the number of people, $p$; therefore, there is a constant of proportionality. We can calculate the constant of proportionality by multiplying the two related values. In this case, the constant of proportionality is $18 \cdot 3 = 54$.

The equation $h = \frac{54}{p}$ shows the number of hours, $h$, needed to build the outdoor storage building given the number of people, $p$.

To find the number of hours if there are 5 people, substitute the value 5 into the equation and evaluate.

$$h = \frac{54}{5} = 10.8 \text{ hours.}$$

If the building needed to be built in 9 hours, how many people would you need? To solve this, substitute 9 in for the number of hours, $h$, and solve.

| | |
|---|---|
| $9 = \frac{54}{p}$ | original equation |
| $9p = 54$ | multiply both sides by $p$ |
| $p = \frac{54}{9}$ | divide both sides by 9 |

You would need 6 people to build the outdoor storage building in 9 hours.

The relationship between the number of hours, $h$, and the number of people, $p$, can also be expressed with the equations $p = \frac{54}{h}$ and $hp = 54$. If we wanted to repeatedly calculate the number of people needed to build the building in a certain amount of time, we might prefer to use the equation $p = \frac{54}{h}$. For example, to find the number of people needed to build the building in 9 hours, we could write $p = \frac{54}{9}$. This gives the answer $p = 6$ without the need for algebra.

## Cool-down: Board Breaking (*5 minutes*)

**Addressing:** NC.M2.A-CED.1; NC.M2.A-REI.2

**Cool-down Guidance:** More Chances
Students will have more opportunities to understand the mathematical ideas in this cool-down, so there is no need to slow down or add additional work to the next lessons. Instead, use the results of this cool-down to provide guidance for what to look for and emphasize over the next several lessons to support students in advancing their current understanding.

### Cool-down

In martial arts, it is found that the force, $f$, needed to break a board varies inversely with the length, $l$, of the board. Write and solve an equation to answer the following question:

If it takes 5 pounds of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long?

**Student Reflection:**

Thinking about something that matters deeply to you (e.g., pollution, poverty). Give an example of a relationship that may vary inversely.

**INDIVIDUAL STUDENT DATA**

**SUMMARY DATA**

## NEXT STEPS

## TEACHER REFLECTION

**What went well in the lesson? What would you do differently next time? What happened today that will influence the planning of future lessons?**

**What do you notice about the topics students wrote about in their reflection today? How might you use those in order to build more urgency around the link between mathematics and real life?**

## Practice Problems

1. A company has found that the number of boxes of chocolates sold, $s$, varies inversely with the price, $p$. When the price is $5, then 800 boxes of chocolates are sold.

    a. Write an equation for the number of boxes sold, $s$, given the price, $p$.

    b. The company can make a maximum of 2500 boxes of chocolates. What price should they set if they are to sell all 2500 boxes?

2. If $y$ varies inversely with $x$, and $y = 10$ when $x = 6$:

    a. What is the value of $y$ when $x = 15$?

    b. What is the value of $x$ when $y = 3$?

3. If $y$ varies inversely with $x$, and $y = 16$ when $x = 6.25$:

    a. What is the value of $y$ when $x = 30$?

    b. What is the value of $x$ when $y = 500$?

4. In some martial arts, the strength and precision of punches is determined by an athlete's ability to break a board. The force, $f$, measured in pounds of pressure, required to break a board varies inversely with the length, $l$, of the board. If 10 pounds of pressure is required to break a board 18 inches long, how many pounds of pressure is required to break a board 1 foot long?

5. For cylinders with equal volume, the height of the cylinder varies inversely with the area of the circular base.

    a. Write an equation for the height of the cylinder, $h$, given the area of the base, $A$.

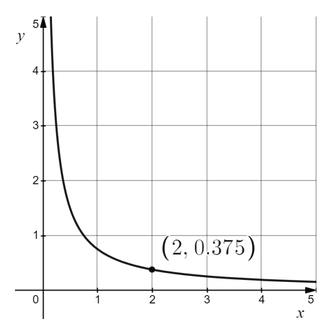
    b. What does $k$ represent in this situation?

    c. One jumbo can of tomatoes has a height of 6 inches and a base area of $4\pi$ square inches. If a new can is designed to hold the same amount of tomatoes with a height of 3 inches (to fit on a shorter shelf), what is the area of the base?

6. Investigate what an inverse variation is to develop an understanding of inverse variation functions and their graphs by using the idea of serving pizza at a party through Desmos activity M2U4L2PP6. Your teacher will provide you with a code or assign this activity to your class account.

7. Given the graph, write the inverse variation equation.

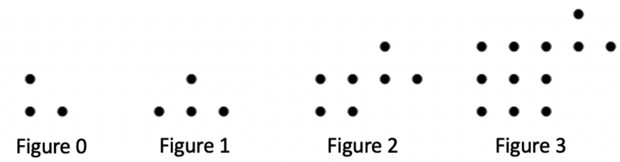
    (From Unit 4, Lesson 1)




8. Simplify the expression $(x-2)(y+3)(y-5)$.

    (From Unit 3)

9. Here is a pattern of dots.




a. Complete the table.

| Figure | Total number of dots |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |

b. How many dots will there be in figure 10?

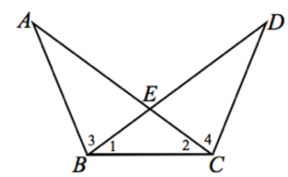
c. How many dots will there be in figure $n$?

(From Unit 3)

10. Given: $\angle 3 \cong \angle 4$ and $\overline{AE} \cong \overline{DE}$

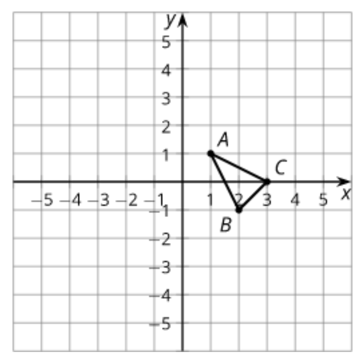
Prove: $\angle 1 \cong \angle 2$

(From Unit 2)




11. Reflect triangle $ABC$ over the $y$-axis. Call this new triangle $A'B'C'$. Then translate triangle $A'B'C'$ 3 units up and 2 units right. Call the resulting triangle $A''B''C''$. What are the coordinates of the vertices of triangle $A''B''C''$?

(From Unit 1)




12. Solve each of the following equations.

a. $20 = \frac{m}{4}$

b. $32 = \frac{n}{2}$

c. $-\frac{3}{4} = \frac{x}{3}$

(Addressing NC.M1.A-REI.3)