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The Power of the 5 Representations for Students Who Are Learning English

National Council of Teachers of Mathematics

2023 National Conference, Washington DC

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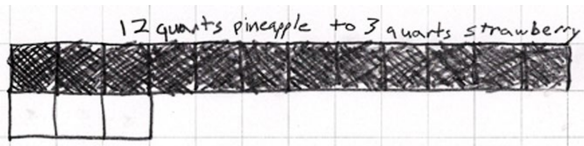
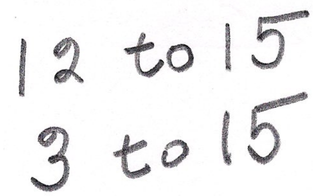
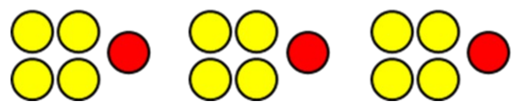
Which One Doesn't Belong

A 	B 
C 1 : 4	D 

Small Group Discussion

How does each representation in “Which One Doesn’t Belong” model the following context?

Laurie is making fruit punch. The recipe says, in order to make 1 batch of the fruit punch, mix 4 quarts of pineapple juice with 1 quart of strawberry juice. In a large container, Laurie makes 3 batches.

Individual Think Time into a Turn-and-Talk

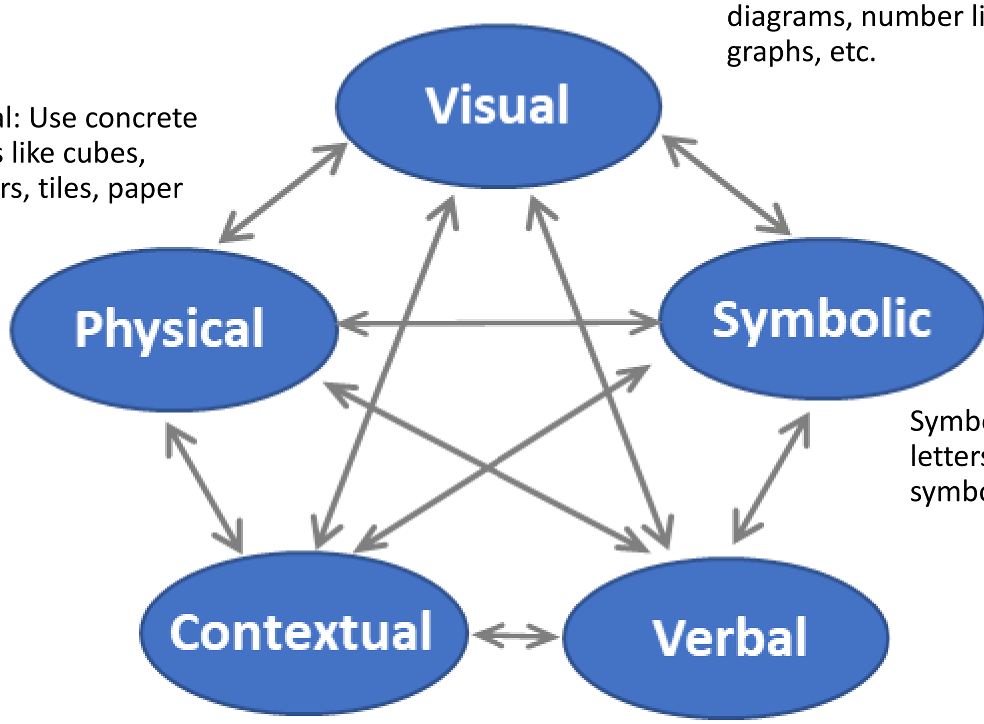
- How did the visual representations help you make sense of the real-world context?

- How did it provide an opportunity to use language to explain your thinking?

Use and Connect Representations

Visual: Create pictures, diagrams, number lines, graphs, etc.

Physical: Use concrete objects like cubes, counters, tiles, paper strips.



Symbolic: Use numbers, letters, tables, and other symbols

Contextual: Use every day, real-world, imaginary, or mathematical situations and contexts.

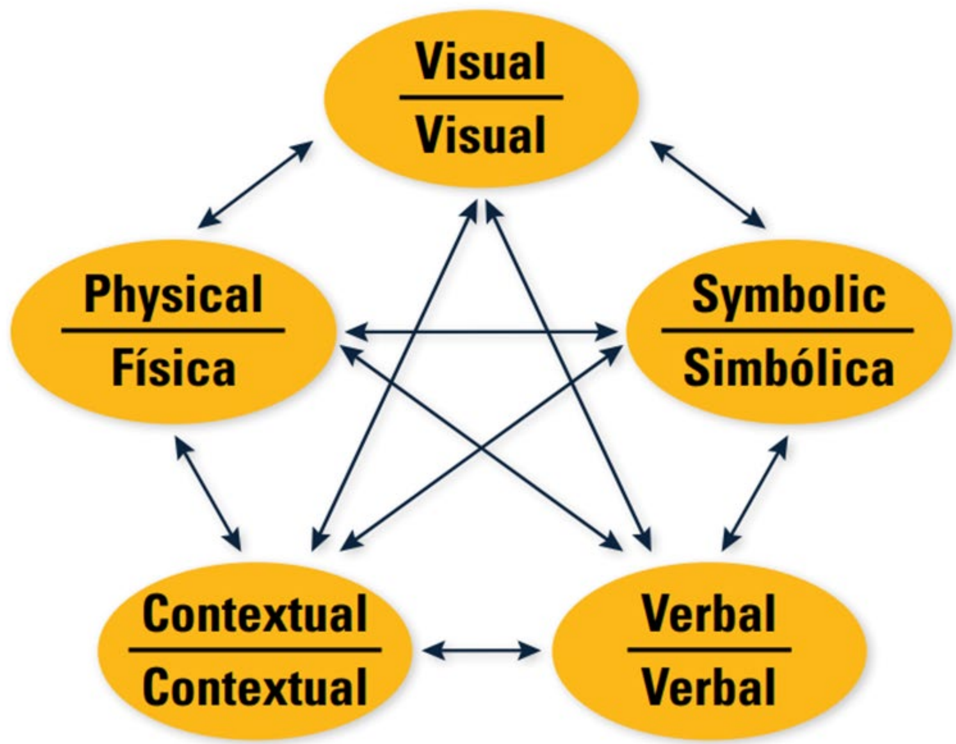
Verbal: Use formal and informal language (words and phrases) during discussion

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

Adapted from [Lesh, Post, Behr (1987) *Representation and translations among representations in mathematics learning and problem solving*] and modified from [Huinker, D. (2015) *Teaching matters: Actions for attaining high-leverage teaching in every mathematics classroom*]

WHAT ARE THE TYPES OF MATHEMATICAL REPRESENTATIONS?

¿CUÁLES SON LOS TIPOS DE REPRESENTACIONES MATEMÁTICAS?



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From *Teaching Math to Multilingual Students: Positioning English Learners for Success*

The authors cull what research says about academic language:

"Since 1991, the National Council of Teachers of Mathematics has advocated for the value and importance of communication for mathematics learning. Moreover, research shows that every student benefits from explicit inclusion of academic language in instruction and curriculum (Genesee, 2006; Gibbons, 2015). Academic language is not limited to vocabulary; it also comprises a set of intellectual and social practices shaped by cultural norms and values, such as understanding different meanings and using expressions in context, comprehending how language is used in academic discourse, and understanding complex sentence structures and syntax along with the cognitive demands of their meanings (Gottlieb & Ernst-Slavit, 2019; Valdes, 2004; Zwiers & Soto, 2017). In the classroom, teachers must attend to the ways they facilitate multilingual learners' acquisition of academic language to ensure they can be mathematically successful (MacDonald, Lord, & Miller, 2019).

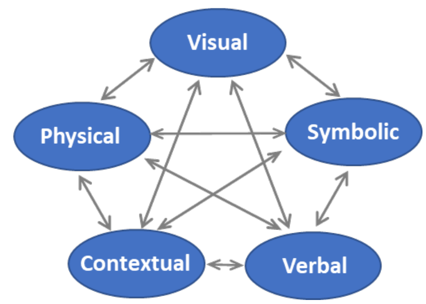
Unfortunately, we do not learn language through "osmosis" or by solely interacting with fluent or native-like speakers; teachers must explicitly teach it (de Jong & Harper, 2005; de Jong, Harper, & Coady, 2013). However, this does not mean that teachers should focus solely on teaching particular words or phrases in isolation, as this is unproductive for student learning and an inefficient use of instructional time (Gibbons, 2015). Said another way, mastering words or phrases alone does not ensure that students can effectively engage in mathematical discourse (Moschkovich, 2002). As a result, teachers must attend to the ways they enhance multilingual learners' acquisition of specialized mathematical language and provide opportunities for language to be used in and across contexts. This instruction must also include opportunities for students to negotiate meaning while drawing on their prior experiences and knowledge (de Jong & Harper, 2005; Zwiers & Hamerla, 2018)."

Chval et al., 2021, p. 103-104

Using and Connecting Mathematical Representations

A High-Leverage Practice for Mathematics Instruction

The National Council of Teachers of Mathematics has named "use and connect mathematical representations" as one of the eight effective teaching practices that should be evident in every math classroom. To ensure that this practice is both high-leverage and equitable, it must be planned for and implemented with intention and without bias. Teachers must be "asking students to make math drawings and use other visual supports to explain and justify their reasoning" (NCTM, 2014, p. 29).



- **Physical:** *Use concrete objects like cubes, counters, tiles, paper strips.*
- **Visual:** *Create pictures, diagrams, number lines, graphs, etc.*
- **Symbolic:** *Use numbers, letters, tables, and other symbols.*
- **Verbal:** *Use formal and informal language (words and phrases) during discussion*
- **Contextual:** *Use every day, real-world, imaginary, or mathematical situations and contexts.*

The fact that one of the eight effective teaching practices is "use and connect mathematical representations" is not a surprise since a seminal piece of research by Lesh, Post, and Behr (1987) showed years ago that five ways to represent a mathematical idea exist in student problem solving and that emphasizing the interconnectedness among the five representations could strengthen student problem-solving skills. Since this time, others have gone on to research and discuss the importance of making connections between representations. According to Anhalt and Cortez (2015), mathematical modeling, in which students use mathematics to explain or interpret physical, social, or scientific phenomena, is an essential component. The Common Core State Standards for Mathematics describes mathematical modeling as "The process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (CCSS, 2010, p. 72).

"Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to oneself and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling" (p. 67). An important question for mathematics teachers is this: "How can we help students learning mathematics to solve everyday problems, rather than teach them only to memorize rules and practice mathematical procedures?" Teaching students by using activities in which they are required to employ representations can help them learn mathematics in real-world problem-solving situations that are useful outside school (Lesh and Doerr, 2003).

Use of representations can also help students gain transferable skills, such as habits of mind that are pervasive across subject matter (Gaimme, 2016, p. 8). Although teachers recognize the value of engaging their students in mathematical modeling, few have had opportunities to experience modeling, and many teachers feel unsure of how to teach it. Modeling is not for science only—it transcends disciplines and affords tools for students to engage with real problems in their community and in society. If you have not yet tried modeling with your students, we hope . . . you "Start big. Start small. Just start" (Gaimme, 2016, p. 92).

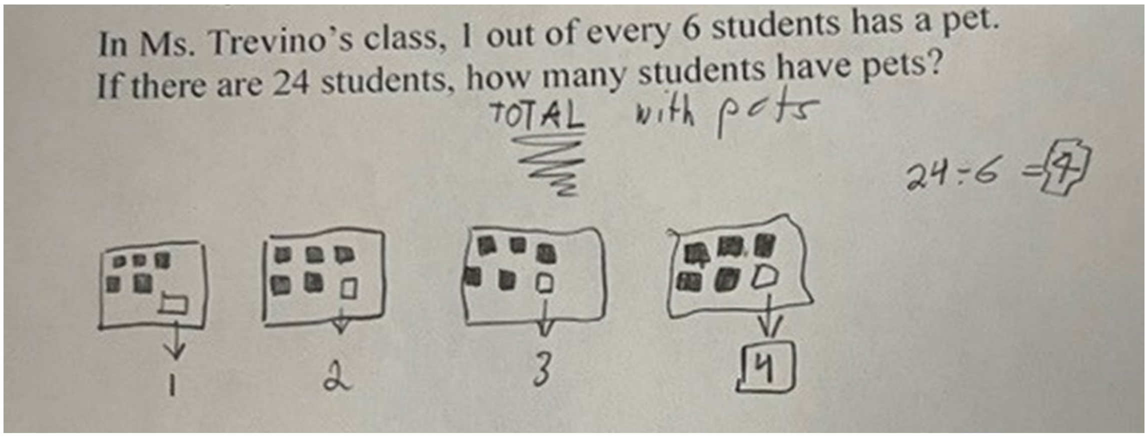
Students who have engaged in the process model appreciate the opportunity to use their own ideas in creating a mathematical solution to a real-world problem and have experiences that help them regardless of what college or career path they follow. When asked to reflect on her experience during a math-modeling task, one precalculus student wrote,

> *It [modeling] helps me remember the math, because then I have some kind of example that can help me think through a problem logically and relate it to something that I know about outside of the classroom. I feel like I can apply this method to a lot of things outside of math, like sciences and literature and history.*

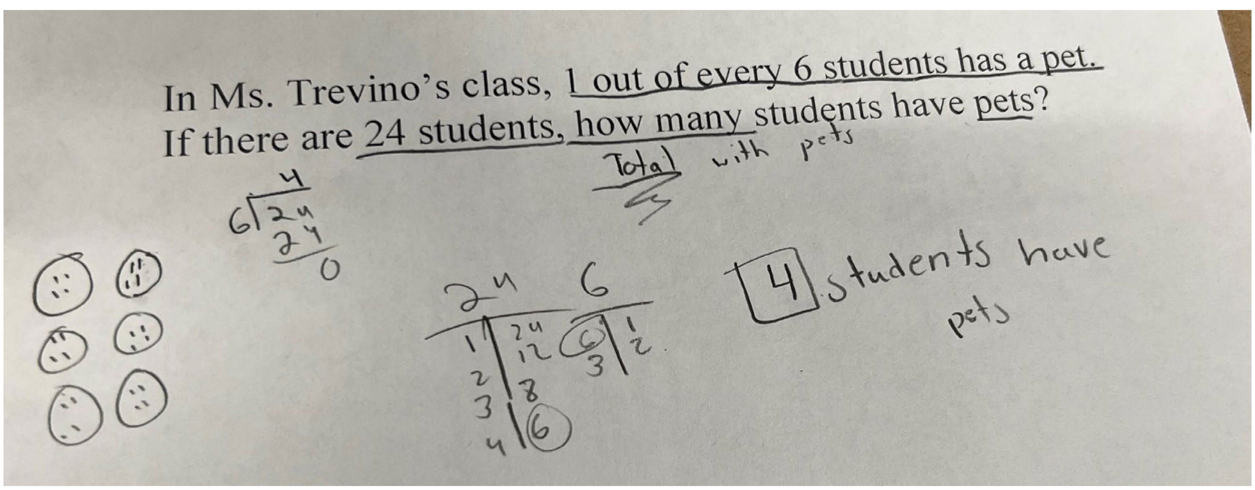
The Task and Student Work

In Miss Trevino's class, 1 out of every 6 students has a pet. If there are 24 students, how many students have pets?

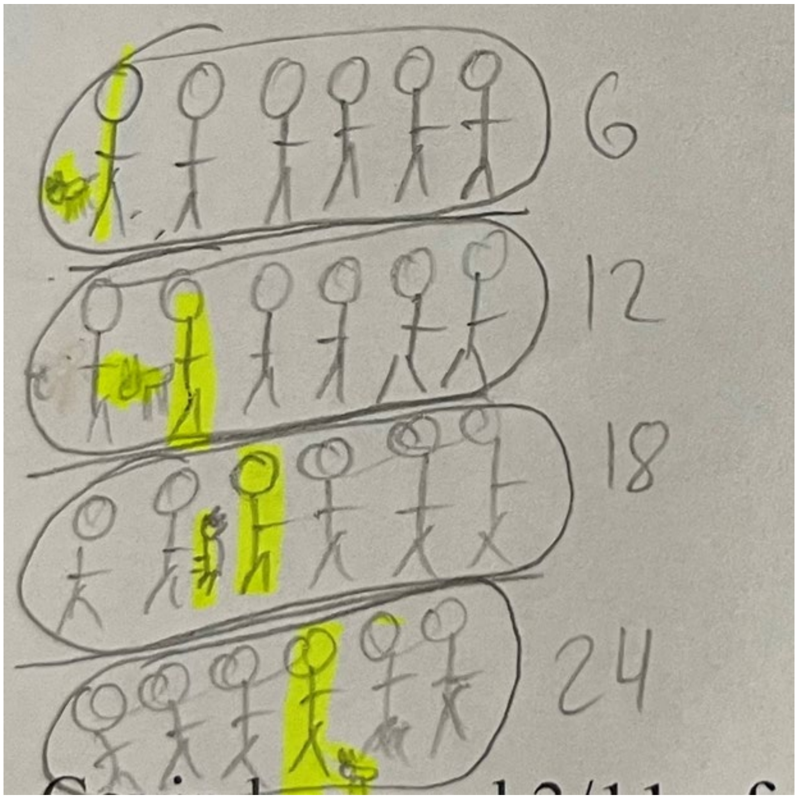
Student Work #1



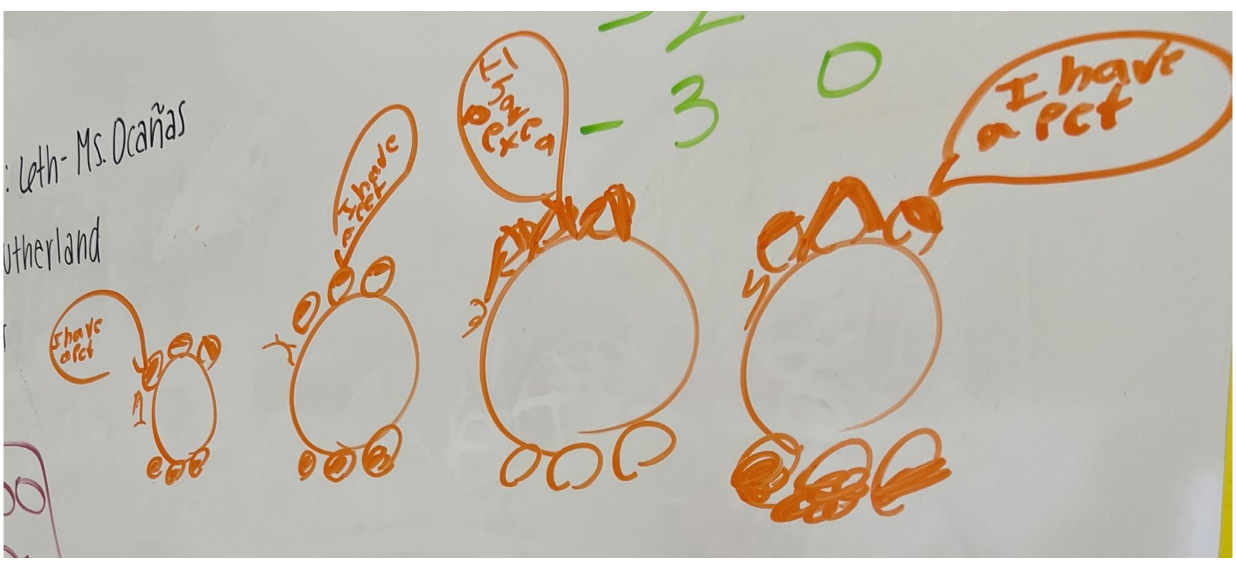
Student Work #2



Student Work #3



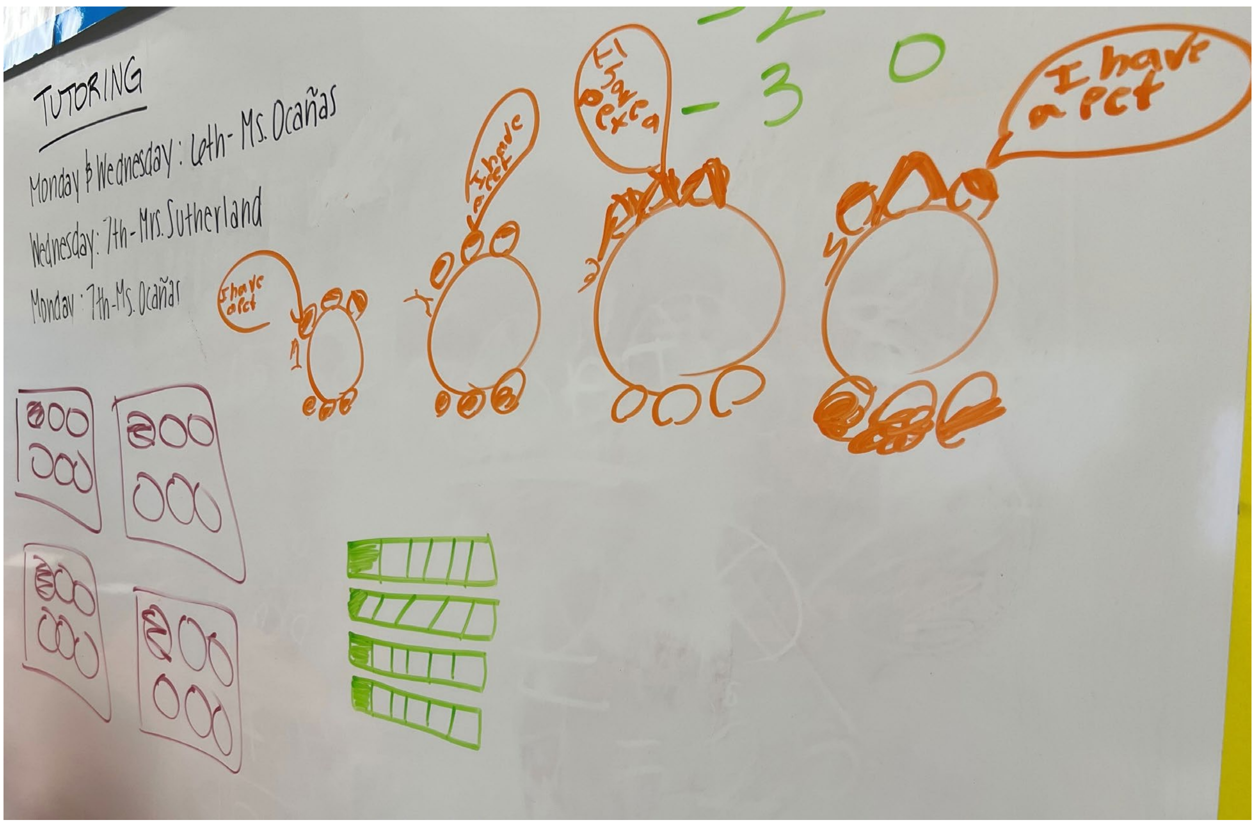
Student Asked to Share First in the Discussion



The Teacher Asked

- The circles represent what?
- I saw drawings and not all of you had the same. Raise your hand if you got something like that.
- If you did not, it means you had something different. Listen as ______ explains their picture.
- Who can say that again?

The Teacher Then Called Two More Students to the Board



- The circles represent what?
- What do the squares represent?
- How does the diagram show 1 out of 6?
- Looking at the diagrams, do both get us the same answer? Why or why not?
- Who can repeat that?

The Three Question Types

Elicit Thinking Questions prompt the student to share information related to their current work, which may include their use and explanation of representations.

Connections Representations Questions prompt students to interpret and make sense of different ways of representing the same mathematical idea.

Explain Mathematical Reasoning Questions prompt students to express in words essential understandings of the math being studied.

Contact Us

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