2023 NCTM ANNUAL MEETING & EXPOSITION
Oct. 25-28, Washington, DC

# Building Community Through Math Modeling and Tech Tools in High School


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*NC School of Science and Math & COMAP*

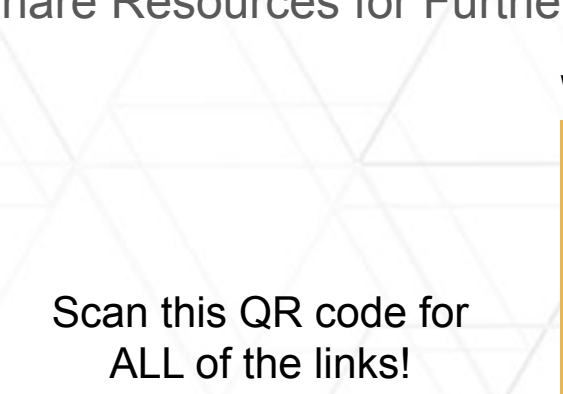
@mathmodeling

# Plan for Our Session

- Share Your ideas about Building Community (As we go!)
- Engage in Mathematical Modeling!
- Use a Variety of Tech Tools As we Go!
- Share Resources for Further Investigation

**Workshop Landing Page**

Scan this QR code for ALL of the links!



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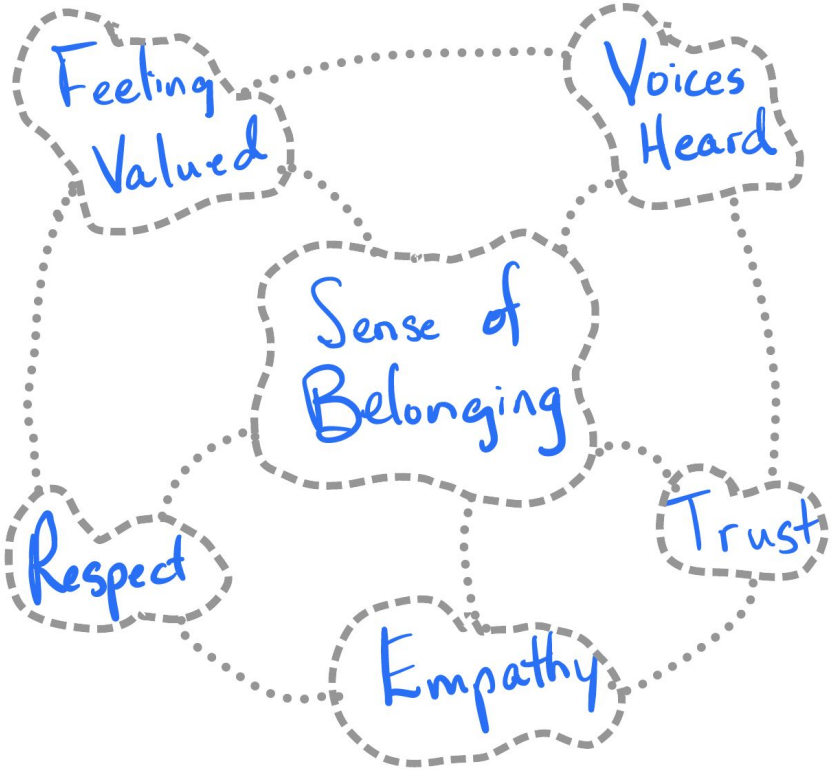
# *Catalyzing Change in High School Mathematics*

A mathematical model is a mathematical representation of a particular real-world process or phenomenon that is under examination, in an attempt to describe, explore, or understand it.

# Building Community



# The Problem

*Suppose that you have a large population that you wish to test for a certain characteristic in their blood or urine. Each test will be either positive or negative. How can we reduce the number of tests needed to screen everyone and thereby reduce the costs?*

Two Minute Table Group Brainstorm Go!

# Share your best ideas via Poll Everywhere!

Scan this QR code for ALL of the links!

Workshop Landing Page

**To use your phone:**

Text #: 37607

Message: lauriecavey722

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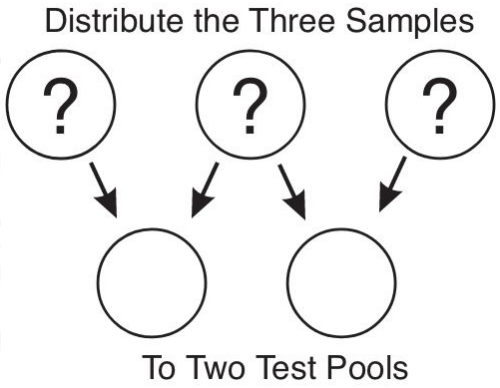
## Enter an idea about how to approach The Problem.

Nobody has responded yet.

Hang tight! Responses are coming in.

# The Three Samples Version

*Suppose that you have samples from **three** individuals where you know in advance that **only one is infected** (but you don't know which one). Let's take TWO samples and POOL them for testing purposes. Your job is to figure out which of the three samples is infected!*



# Learn about the invention and first use of Pooled Testing.

***Solving a Problem:***
***The Invention of Pooled Testing***

From
AMS :: Feature Column :: Pooling strategies for COVID-19 testing
By David Austin

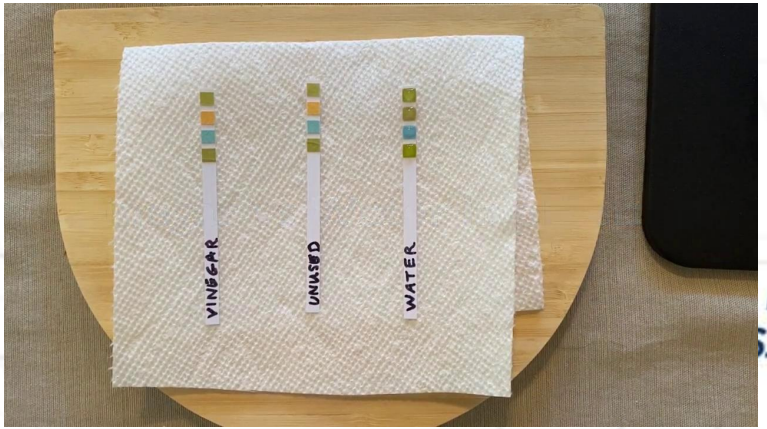
***In the 1940s Syphilis was a problem in the general population.***


A 1941 poster encourages syphilis testing (Library of Congress)

***Economist Robert Dorfman hired by the US Government to screen recruits for the US Army***

Dorfman's solution:
Organize the Samples into Groups

***Problem: 25 people need to be tested and there are only enough resources: money, test strips, time, chemicals to run 10 tests.***

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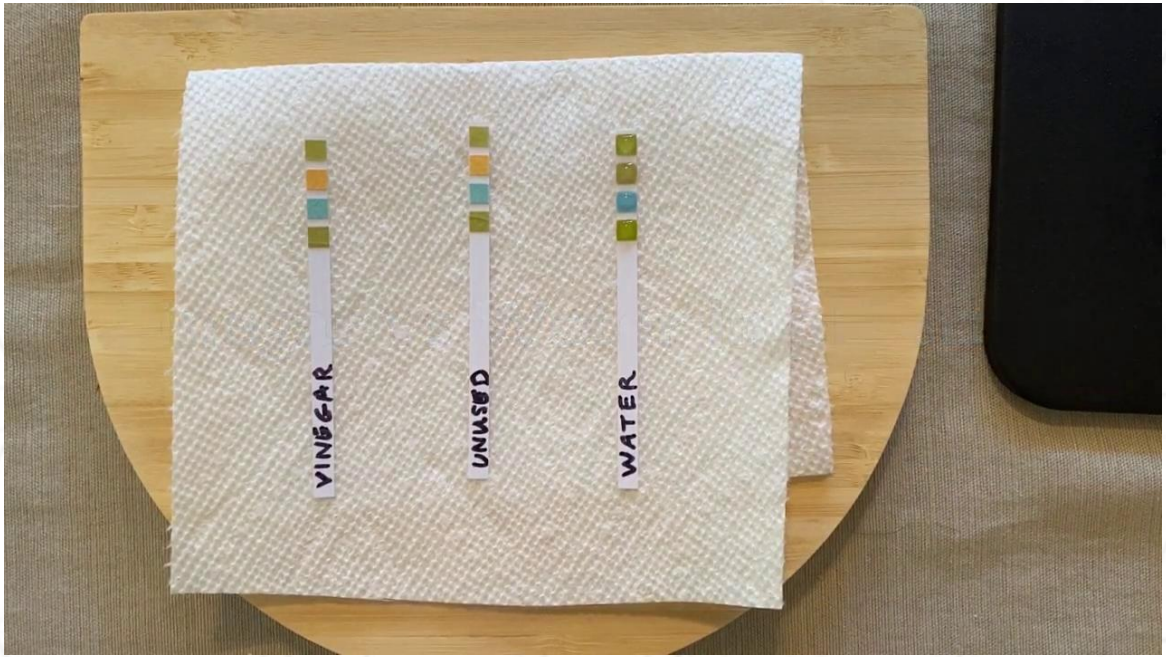
What was Dorfman's Idea?
Pool the Samples in order to decrease the number of tests needed.



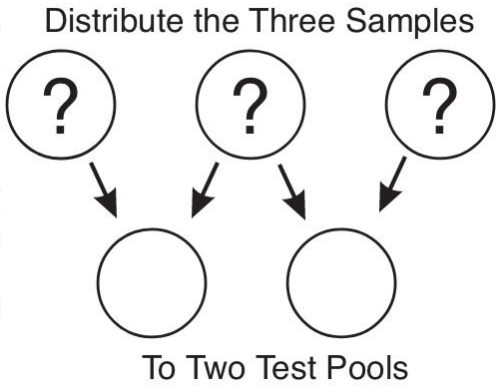

1 min 15 sec

https://youtu.be/AKs2iEiNkCs - youtube link

# The Set-Up

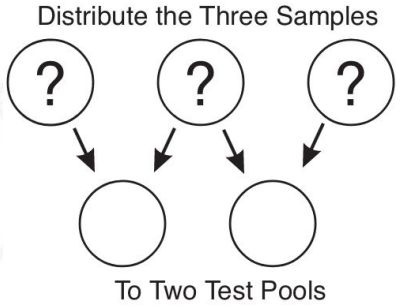
Table groups have a testing kit that will enable you to:

- Represent the original samples with two vials of water and one vial of vinegar
- Combine (Pool) two sample at a time in TWO empty vials for testing.
- Use pH strips to test your pooled samples



# Open your Kits!

- Practice dripping water with a pipette.
- Fill two vials up to the ‘1’ mark with water and secure tops on the vials.
- Wait for us to give you a vinegar vial.
- Mix the vials up in your hand so you don’t know which is which.
- Place the samples in one row of the tray
- Combine the contents of 2 vials by putting 6 drops from each sample in one of the “pool” vials as demonstrated in the diagram
- Test the Pools using pH strips

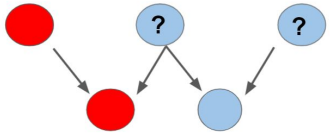


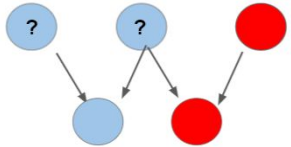
# Prepare for Whole-Group Discussion

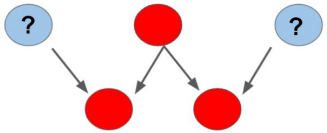
- Which of the original vials has the vinegar?
- How do you know?
- Use your Table Group's designated **Jamboard Slide** to sketch a diagram to help explain your thinking

# Discussion:

Which of the original vials has the vinegar?
How do you know?



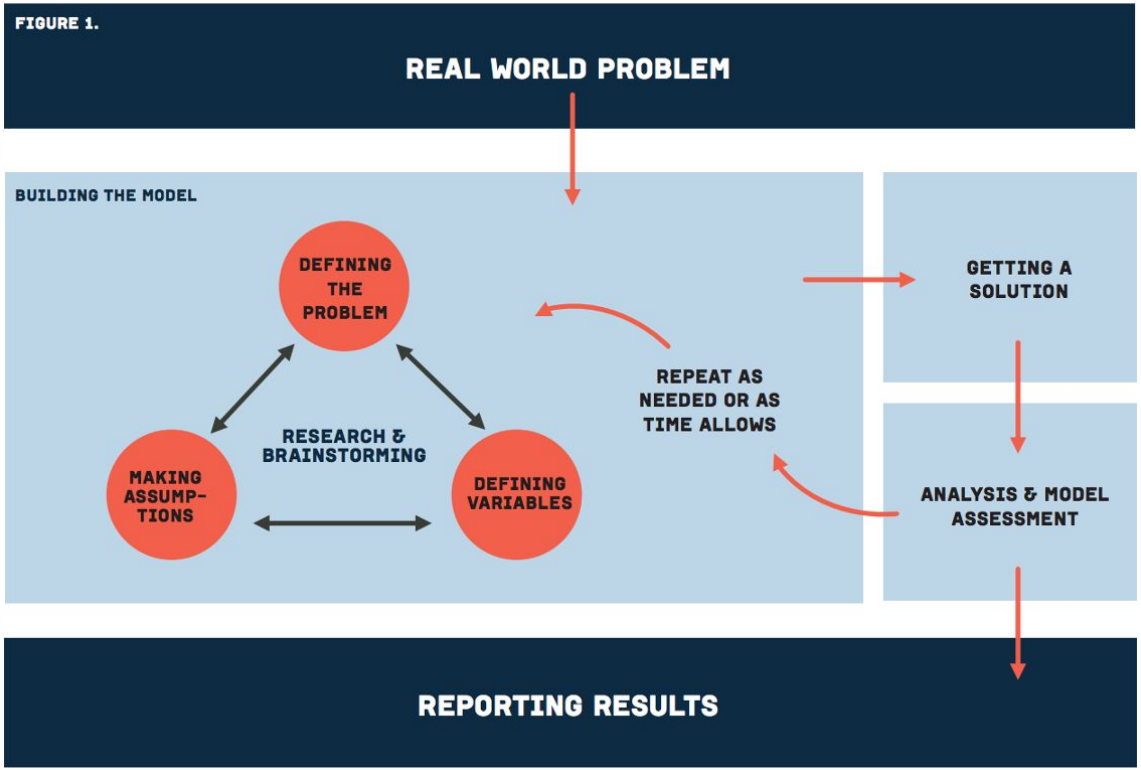



# Accessing the Testing Kits

There is a link to a Google Form on the Workshop Landing Page that you can use to request a kit for your own classroom!

# Modeling Cycle from *Math Modeling Handbook* (SIAM/COMAP)

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# Catalyzing Change in High School Mathematics, p. 3

“When students engage in mathematical modeling, they often have the opportunity to leverage mathematics to understand and critique the world. Mathematical modeling is the creative, often collaborative, process of developing these representations.

Modeling always requires decision making that involves determining which aspects of the phenomenon to include in the model and which to suppress or ignore and what kind of mathematical representation to use.”

# The Problem

*Suppose that you have a large population that you wish to test for a certain characteristic in their blood or urine. Each test will be either positive or negative. How can we reduce the number of tests needed to screen everyone and thereby reduce the costs?*

# What is the simplest version of the simplest problem?

Suppose there are 2 people in the population to be tested.

Let’s consider the possible cases (we don’t know who or how many are infected this time!)

For each possible case, determine the number of tests needed if we combine the two samples into one.

Talk to 2-3 of your neighbors

# Share using Jamboard

Pick any one of the slides titled, “The Simplest Version” to share the ideas generated in your “elbow neighborhood”

# What if we only have 2 samples, how many tests will we need?

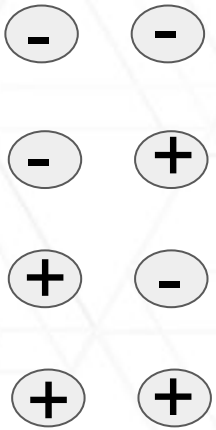
Both Negative:  Need ? Tests

One Negative & One Positive:  Need ? Tests

One Positive & One Negative:  Need ? Tests

Two Positives:  Need ? Tests

# What if we only have 2 samples, how many tests will we need?




Both Negative: Need 1 Tests

One Negative & One Positive: Need 3 Tests

One Positive & One Negative: Need 3 Tests

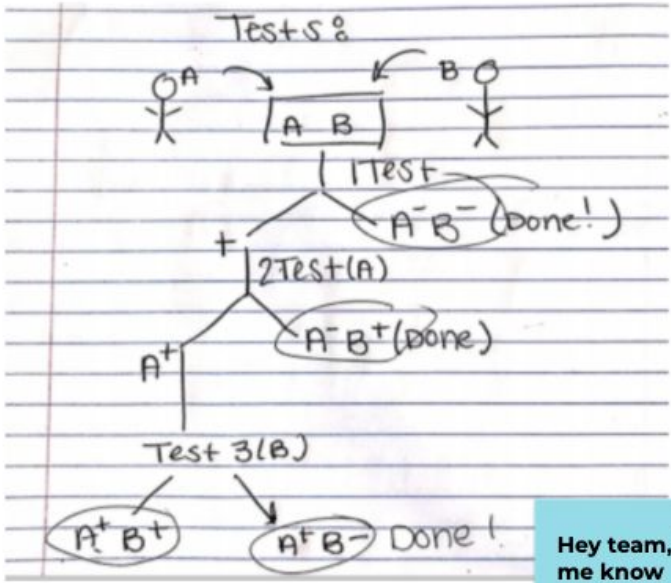
Two Positives: Need 3 Tests

**Group Alpha's Board**

Possibilities:

2 more tests needed | 1 more test needed | 0 more tests needed

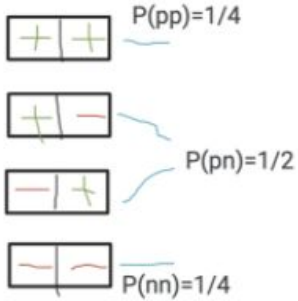
I like your picture. I am just wondering if there are really four possibilities?? Not sure!




Hey team, let me know what you think about this!

| | P | N |
|---|---|---|
| P | PP | PN |
| N | PN | NN |

I'm not sure how to write an equation for this, but the double negative is the only case where no more testing is needed.




This is the best I can think of for equations for the probability.

Let's write a function (model), *T(p)*, for the simplest version (2 people)

Assume *T(p)* is the total number of tests needed when the probability of a positive test is *p*

**1 Test—Group Test is Negative** (*only 1 way for this to occur*)

**3 Tests—Group Test is Positive + Test Individuals** (3 ways this could occur)

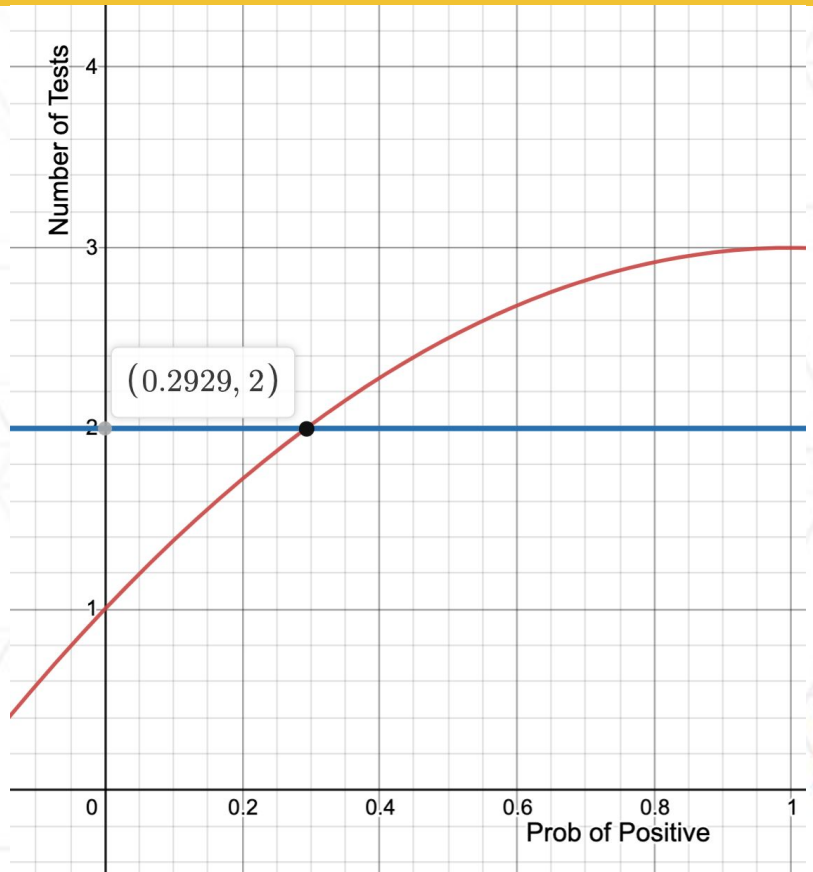
Expected Number of Tests = Sum of (Event * Probability of Event)

$$T(p) = 1(1-p)^2 + 3(1-p)p + 3p(1-p) + 3p^2$$

Graph this model in Desmos - Consider your domain in context.

When is it a good idea to pool the samples?

# Graph of Model T(p)



# Moving Forward - Consider larger populations

We explore a specific scenario where we have more than two people in the population and we pool a specific number of tests. This helps us understand the connection between the total number of tests required and the pool size.

Or goal is to create a mathematical model (function) for the total number of tests required as a function of the pool size.

## Example with Larger Number of Samples

Suppose we have a total of 30 samples; we want to identify the 2 positive samples within the 30 in the population.

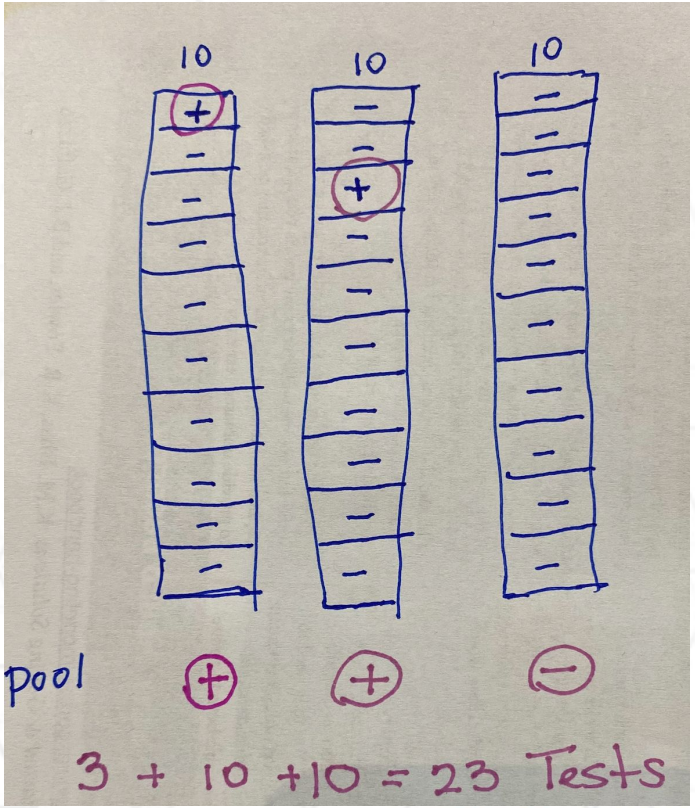
How many tests are needed if we have a pool or **group size of 10** pooled samples?

Sketch a diagram to help explain your reasoning.

Suppose we have a total of 30 samples;
we want to identify the 2 positive samples
within the 30 in the population.
**How many tests are needed if we have a group size of 10 pooled samples?**

## Sample Student Response

We need 3 pooled tests.
If positive samples are in separate groups, we need 20 individual tests.
For a total of **23** tests.




Note: This is the worst case.

# Exploring Various Group Sizes

As a class, we continue with this specific example of population size of 30 in which there are 2 positive samples.

We will explore **various group sizes** to see which group size is optimal; that is, which group size results in the least number of tests.

# As a class, Collect Data w/Population = 30 w/2 +'s

Population size = 30

Probability of positive Tests = 2/30

| N = Population Size | p = Prob of +ive Test | G = Group Size | T = #Tests |
|---|---|---|---|
| 30 | 0.067 | 10 | **23** |
| 30 | 0.067 | 6 | |
| 30 | 0.067 | 5 | |
| 30 | 0.067 | 3 | |
| 30 | 0.067 | 2 | |

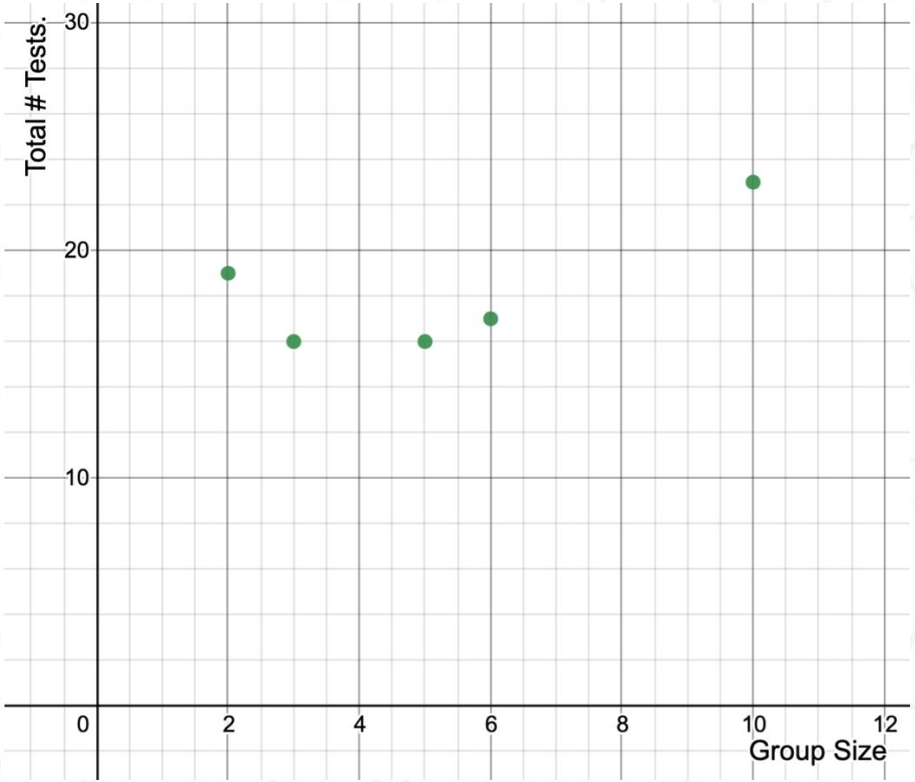
# As a class, Collect Data w/Population = 30 w/2 +'s

Population size = 30

Probability of positive Tests = 2/30

| N = Population Size | p = Prob of +ive Test | G = Group Size | T = #Tests |
|---|---|---|---|
| 30 | 0.067 | 10 | 23 |
| 30 | 0.067 | 6 | 17 |
| 30 | 0.067 | 5 | 16 |
| 30 | 0.067 | 3 | 16 |
| 30 | 0.067 | 2 | 19 |

# Graph Data



# We move on to

Create a function model for the total number of tests as a function of the group size.

This helps us find the optimal group size; that is the group size that minimizes the total number of tests required.

# Revisit Padlet w/Your Ideas for Community Building

# Link to COMAP Materials and Kit Request Form

- Padlet with COMAP Materials for Pooled Testing
  - **Password**: CovidMath

- Interested in your own Pooled Testing kit?
  - Complete this form to request a kit from

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# More Resources

- NCCTM Centroid Article *Online Tools to Support Math Modeling and Community Building*
- COMAP Consortium Article *Everybody's Problem The Blood Testing Problem*
- GAIMME Report and Modeling Handbooks
- MT Article *Mathematical Modeling in the High School Curriculum*
- NCTM/SIAM/COMAP Committee Web Page
- NCSSM Teaching Contemporary Mathematics Conference, Durham NC
- Anja Greer Conference on Math and Technology, Exeter, NH
- Other Modeling Sessions here in DC:

**Session 541** - *Math Modeling as a Vehicle to Catalyze Change* - **9:30 - 10:30 am**
**Saturday 10/28** Jenn Suh, Ben Galluzzo & Maria Hernández

# Looking beyond HS

Should we include info on how colleges and universities are including modeling in their curricula or pathways?

If so - Here is a link to a JustEquations report.

# Thank you!

Contact us:

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@mathmodeling