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# OGAP Additive Framework

Depending upon the strength of additive reasoning, students may move up and down between additive, transitional, counting, and non-additive reasoning and strategies as they interact with new topics or new concepts.

## Problem Contexts

**Counting**
Quantities
Patterns

**Additive Situations**
Add to
Take from
Put together/Take apart
Compare

**Concepts/Properties**
Properties and Relationships
Magnitude

**Base 10/Place Value**
Concepts
Strategies

## Problem Structures

**Types of Items**
Contextual
Non-contextual

**Complexity of Addends**
Single digit
Multiple digit
Multiples of 10, 100, 1000

**Representations**
Number Paths
Number Lines
Base 10
Ten Frames
Dot images

**Properties and Relationships**
Relationship between addition and subtraction
Commutative property
Associative property
Identity
Relationships between models, equations and contexts
Compensation
Constant difference

**Addends**
Two addends
Three or more addends
Compositions of 10
Relationship between addends

**Meanings for Subtraction**
Difference
Removal
Distance
Missing addend

**Number of steps**
Single step
Multi-step

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add To** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br><br>2 + 3 = ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br><br>2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br><br>? + 3 = 5 |
| **Take From** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br><br>5 – 2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br><br>5 – ? = 3 | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br><br>? – 2 = 3 |
| | **Total Unknown** | **Both Addends Unknown** | **Addend Unknown** |
| **Put Together/ Take Apart** | Three red apples and two green apples are on the table. How many apples are on the table?<br><br>3 + 2 = ? | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br><br>5 = 0 + 5  5 = 5 + 0<br>5 = 1 + 4  5 = 4 + 1<br>5 = 2 + 3  5 = 3 + 2 | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br><br>3 + ? = 5  5 – 3 = ? |
| | **Difference Unknown** | **Bigger Unknown** | **Smaller Unknown** |
| **Compare** | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br><br>("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?<br><br>2 + ? = 5  5 – 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br><br>(Version with "fewer"):<br>Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br><br>2 + 3 = ?  3 + 2 = ? | (Version with "fewer"):<br>Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br><br>(Version with "more"):<br>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?<br><br>5 – 3 = ?  ? + 3 = 5 |

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# OGAP Additive Reasoning Progression – Addition

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Subitizing, Unitizing, Commutativity, Number Composition, Place value understanding

## Additive

### Additive Strategies

**Traditional US algorithm**

$$\begin{array}{r} {\scriptstyle 1\phantom{0}} \\ 18 \\ +27 \\ \hline 45 \end{array}$$

**Transparent algorithms**
Decomposes by place value and adds

18 + 27 = ?

18 = 10 + 8
27 = 20 + 7
30 + 15 = 45

18 + 20 = 38
38 + 7 = 45

**Partial Sums**

$$\begin{array}{r} 18 \\ +27 \\ \hline 30 \\ +15 \\ \hline 45 \end{array}$$

**Uses properties**

Associative Property
7+2+8=7+ (2 + 8)

Commutative Property
6+7=7+6

Flexible compensation
18 + 27
20 + 25
45

**Fact Fluency**

Fact Recall
6+7=13

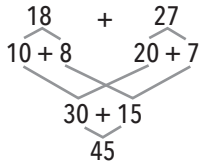
Derived Facts
6+6=12
6+7=13

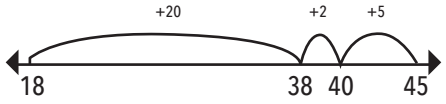
Using 10s
6+7
6+4=10 and 10+3=13

## Transitional (Tens)

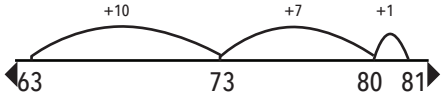
### Transitional Strategies

**Efficient Use of a Model**



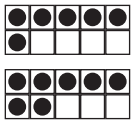

Jumps by multiples of 10 on a number line




Jumps by a 10 and efficient groups of ones

**Inefficient decomposition to derive facts**

6+7
"I know 5+5 is 10 and 3 more is 13"

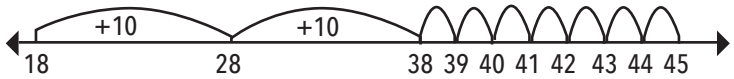


Unitizes on a model

### Early Transitional Strategies

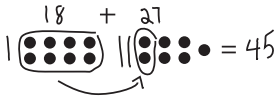
Adding inefficiently with or without a model

Adding on by tens
18+10=28
28+10=38
38+2=40
40+5=45




Makes jumps of 10 on a number line

Combines or counts by 10s using base 10 representations

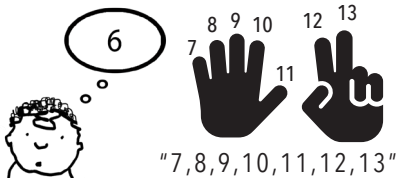



## Counting (Ones)

### Counting Strategies

Mental counting strategies:
- Count on from first
- Count on from larger

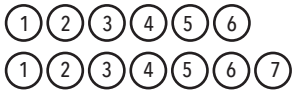

Counting on with physical or visual model: 6+7




"7,8,9,10,11,12,13"

### Early Counting Strategies

Direct modeling and counting from 1 with model (count 3 times)




"1,2,3,4,5,6,7,8,9,10,11,12,13"

### Non-Additive Strategies

- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

### Underlying Issues/Errors

- Does not consider reasonableness of solution
- Error in counting, calculation, place value, property, equation, or model
- Units inconsistent or missing

This product was funded by the National Science Foundation (DRL-1620888). 

# OGAP Additive Reasoning Progression – Subtraction

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Subitizing, Unitizing, Commutativity, Number Composition, Base 10 understanding

## Additive

### Additive Strategies

Traditional US algorithm

$$\begin{array}{r} {\scriptstyle 6\ 1} \\ 7\not{3} \\ -\ 38 \\ \hline 35 \end{array}$$

Transparent algorithms
Decomposes by place value and subtracts

73 – 38 = ?

| | |
|---|---|
| 73 = 70 + 3 | 73 – 30 = 43 |
| 38 = 30 + 8 | 43 – 3 = 40 |
| 40 – 5=35 | 40 – 5 = 35 |

Adding Up Efficiently
38 + 2 = 40
40 + 33 = 73
2 + 33 = 35

Constant Difference:
73 – 38
+2 +2
75 – 40=35

Partial Difference:
73
– 38
40 – 5=35

Fact Fluency
Fact Recall
Derived Facts

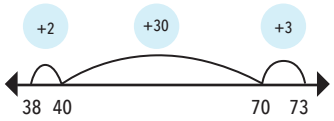
Inverse relationship between addition and subtraction
"7 – 2 is 5 because 5 + 2 is 7"

Using 10s: 13–8
"13 – 3=10 and 10 – 5 = 15"

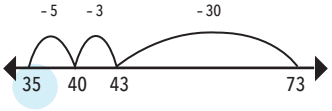
## Transitional (Place Value Parts)

### Transitional Strategies

Making efficient jumps by multiples of 10, 100, ...




73 – 38=35
Distance on number line




73 – 38=35
Jump back on number line

### Early Transitional Strategies

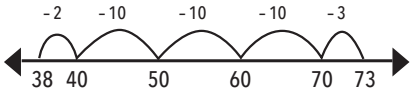
Subtracting inefficiently with or without a model

Using Base 10 representations

73 – 38=35

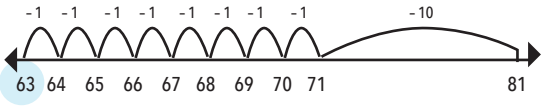
Jumps by tens on number line

73 – 38 = 35

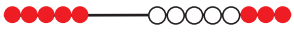



Adding up by tens
73–38=?
38+2=40
40+10=50
50+10=60
60+10=70
70+3=73
2+10+10+10+3=35

81 – 18 = 63




13 – 8



"13 – 3 is 10 and then 5 less is 5"

Unitizes on a model

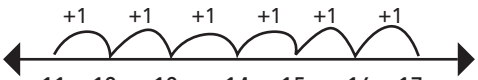
## Counting (Ones)

### Counting Strategies

Counting back (7 – 5)
"7...6,5,4,3,2"
Counting up
"5...6, 7"


Counting back (7 – 5)
"7...6,5...that's 2"

Jumps by ones on number line




17 – 11=6

### Early Counting Strategies

Direct modeling–separating from, separating to, or matching with model (counts 3 times)


8 – 5 = 3

### Non-Additive Strategies

- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

### Underlying Issues/Errors

- Does not consider reasonableness of solution
- Error in counting, calculation, place value, property, equation, or model
- Units inconsistent or missing

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# Base Ten Number Progression

QUANTITY: How Many? — RELATIVE MAGNITUDE: How Big? How Close?

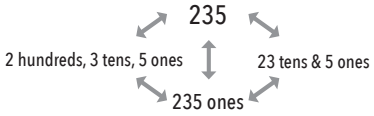
The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Subitizing, Mental Representations, Conservation, Base Ten Understanding

## Base Ten

### Application of Base Ten

- Flexible use of Base Ten understanding to solve problems




- Multiplicative understanding of place value
  235 = (2 x 100) + (3 x 10) + (5 x 1)
- Use of Base Ten for flexible and efficient computational strategies
- Extension of Base Ten understanding to decimal and negative numbers

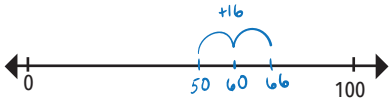
### Number Composition by Place Value Parts

200 + 30 + 5 = 235
200 = 20 tens
30 =3 tens
23 tens and 5 ones

Uses place value and number relationships to compare and consider relative magnitude

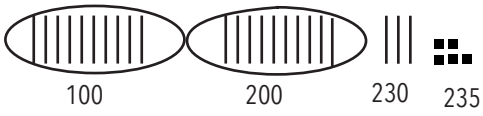
"63>59 because 6 tens is more than 5 tens"

Where does 66 go on the number line?




## Transitional

### Unitizing with Composite Units

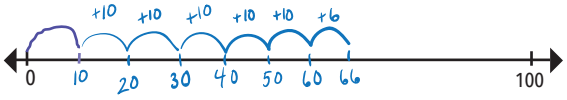



10, 20, 21, 22, 23

Unitizes to locate and compare numbers

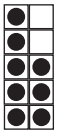
"63 is 4 more than 59"

Where does 66 go on the number line?

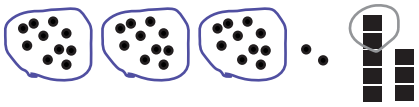



### Early Unitizing (Sees quantities in groups when ones are visible)

Composes and decomposes quantities in groups



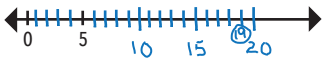
"5 and 3 is 8"
"2 less than 10 is 8"
"3 and 3 is 6 and 2 more is 8 "



"10, 20, 30, 31, 32"

"5 is 2 more than 3"

Where does 19 go on the number line?

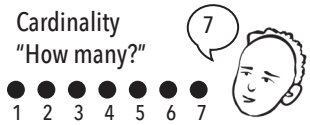



## Counting

### Early Counting (Treats quantities as collections of ones)

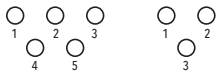
"1,2,3,4,5,6,7,8"

Cardinality
"How many?"




Counts by ones to locate and compare numbers

One more/less than a number



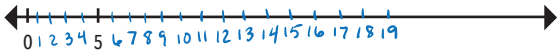

"5 is more than 3"

Counts with one-to-one correspondence


Disregards the unit

1, 2, 3, 4, 5

Where does 19 go on the number line?




### Precounting

Counts with some correspondence


Rote counting

Perceptual comparing

"The second row is bigger"

Matching to compare

**Not Counting or Comparing**

- Guesses
- Not enough info
- Misinterprets problem situation or task

**Underlying Issues/Errors**

- Doesn't consider reasonableness of solution
- Sequence error
- Errors in units
- Numeral reversal
- Digit reversal
- Counting Error
- Model error
- Equation error

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