



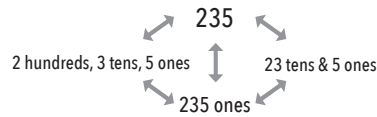
Base Ten Number Progression

QUANTITY: How Many?

RELATIVE MAGNITUDE: How Big? How Close?

Application of Base Ten

- Flexible use of Base Ten understanding to solve problems
- Multiplicative understanding of place value
 $235 = (2 \times 100) + (3 \times 10) + (5 \times 1)$
- Use of Base Ten for flexible and efficient computational strategies
- Extension of Base Ten understanding to decimal and negative numbers



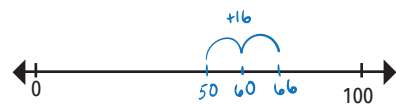
Number Composition by Place Value Parts

Uses place value and number relationships to compare and consider relative magnitude

$$\begin{aligned} 200 + 30 + 5 &= 235 \\ 200 &= 20 \text{ tens} \\ 30 &= 3 \text{ tens} \\ 23 \text{ tens and } 5 \text{ ones} \end{aligned}$$

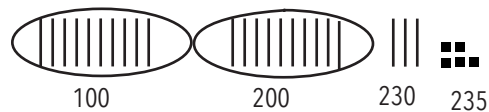
"63 > 59 because 6 tens is more than 5 tens"

Where does 66 go on the number line?



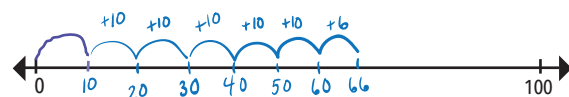
Unitizing with Composite Units

Unitizes to locate and compare numbers



"63 is 4 more than 59"

Where does 66 go on the number line?

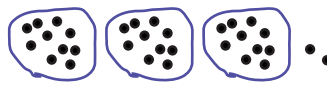


10, 20, 21, 22, 23

Early Unitizing (Sees quantities in groups when ones are visible)

Composes and decomposes quantities in groups

"5 and 3 is 8"
"2 less than 10 is 8"
"3 and 3 is 6 and 2 more is 8"



"10, 20, 30, 31, 32"

"5 is 2 more than 3"

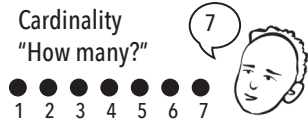
Where does 19 go on the number line?



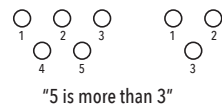
Early Counting (Treats quantities as collections of ones)

Counts by ones to locate and compare numbers

Cardinality "How many?"
"1, 2, 3, 4, 5, 6, 7, 8"



One more/less than a number



"5 is more than 3"

Counts with one-to-one correspondence

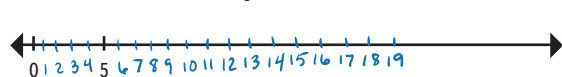
"one" "two" "three" "four" "five" "six" "seven"

Disregards the unit



1, 2, 3, 4, 5

Where does 19 go on the number line?



Precounting

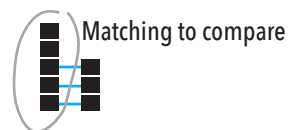
Counts with some correspondence

"one" "two" "three" "four" "five..."

Rote counting

Perceptual comparing

"The second row is bigger"



Matching to compare

Not Counting or Comparing

- Guesses
- Not enough info
- Misinterprets problem situation or task

Underlying Issues/Errors

- Doesn't consider reasonableness of solution
- Sequence error
- Errors in units
- Numeral reversal
- Digit reversal
- Counting Error
- Model error
- Equation error



OGAP Additive Framework

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ogapmathllc.com

Depending upon the strength of additive reasoning, students may move up and down between additive, transitional, counting, and non-additive reasoning and strategies as they interact with new topics or new concepts.

Problem Contexts

Counting

Quantities
Patterns

Additive Situations

Add to
Take from
Put together/Take apart
Compare

Concepts/Properties

Properties and Relationships
Magnitude

Base 10/Place Value

Concepts
Strategies

Problem Structures

Types of Items

Contextual
Non-contextual

Complexity of Addends

Single digit
Multiple digit
Multiples of 10, 100, 1000

Representations

Number Paths
Number Lines
Base 10
Ten Frames
Dot images

Properties and Relationships

Relationship between addition and subtraction
Commutative property
Associative property
Identity
Relationships between models, equations and contexts
Compensation
Constant difference

Addends

Two addends
Three or more addends
Compositions of 10
Relationship between addends

Meanings for Subtraction

Difference
Removal
Distance
Missing addend

Number of steps

Single step
Multi-step

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Both Addends Unknown	Addend Unknown
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ $5 = 5 + 0$ $5 = 1 + 4$ $5 = 4 + 1$ $5 = 2 + 3$ $5 = 3 + 2$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ $5 - 3 = ?$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? $2 + ? = 5$ $5 - 2 = ?$ ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ $3 + 2 = ?$ (Version with "fewer"): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have?	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $? + 3 = 5$ (Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?

K 1 2



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Additive Reasoning Progression – Addition

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Additive

Additive Strategies

Traditional US algorithm

$$\begin{array}{r} 18 \\ +27 \\ \hline 45 \end{array}$$

Transparent algorithms

Decomposes by place value and adds

$$18 + 27 = ?$$

$$18 = 10 + 8$$

$$18 + 20 = 38$$

$$27 = 20 + 7$$

$$38 + 7 = 45$$

$$30 + 15 = 45$$

Partial Sums

$$\begin{array}{r} 18 \\ +27 \\ \hline 30 \\ +15 \\ \hline 45 \end{array}$$

Uses properties

Associative Property
 $7 + 2 + 8 = 7 + (2 + 8)$

Commutative Property
 $6 + 7 = 7 + 6$

Flexible compensation

$$\begin{array}{r} 18 + 27 \\ 20 + 25 \\ \hline 45 \end{array}$$

Fact Fluency

Fact Recall
 $6 + 7 = 13$

Derived Facts
 $6 + 6 = 12$
 $6 + 7 = 13$

Using 10s

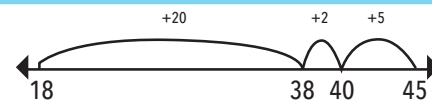
$$\begin{array}{r} 6 + 7 \\ 6 + 4 = 10 \text{ and } 10 + 3 = 13 \end{array}$$

Transitional (Tens)

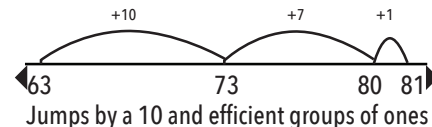
Transitional Strategies

Efficient Use of a Model

$$\begin{array}{r} 18 + 27 \\ 10 + 8 + 20 + 7 \\ \hline 30 + 15 \\ \hline 45 \end{array}$$



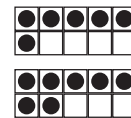
Jumps by multiples of 10 on a number line



Jumps by a 10 and efficient groups of ones

Inefficient decomposition to derive facts

$6 + 7$
"I know $5 + 5$ is 10 and 3 more is 13"



Unitizes on a model

Early Transitional Strategies

Adding inefficiently with or without a model

Adding on

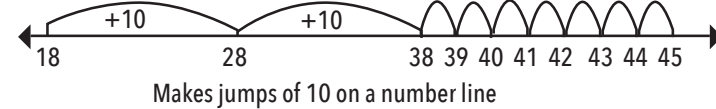
by tens

$$18 + 10 = 28$$

$$28 + 10 = 38$$

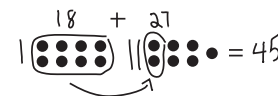
$$38 + 2 = 40$$

$$40 + 5 = 45$$



Makes jumps of 10 on a number line

Combines or counts by 10s using base 10 representations



Counting (Ones)

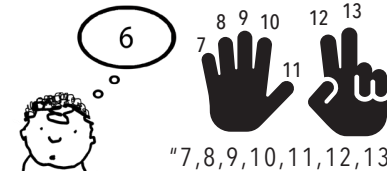
Counting Strategies

Mental counting strategies:

- Count on from first
- Count on from larger



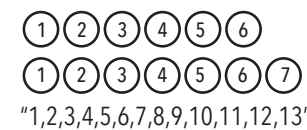
Counting on with physical or visual model: $6 + 7$



"7, 8, 9, 10, 11, 12, 13"

Early Counting Strategies

Direct modeling and counting from 1 with model (count 3 times)



"1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13"

Non-Additive Strategies

- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

Underlying Issues/Errors

- Does not consider reasonableness of solution
- Error in counting, calculation, place value, property, equation, or model
- Units inconsistent or missing



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Additive Reasoning Progression – Subtraction

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Additive

Additive Strategies

Traditional US algorithm

$$\begin{array}{r} 73 \\ -38 \\ \hline 35 \end{array}$$

Transparent algorithms

Decomposes by place value and subtracts

Adding Up Efficiently

$$38 + 2 = 40$$

$$40 + 33 = 73$$

$$2 + 33 = 35$$

Constant Difference:

$$73 - 38$$

$$+2 \quad +2$$

$$75 - 40 = 35$$

Partial Difference:

$$73$$

$$-38$$

$$40 - 5 = 35$$

Fact Fluency

Fact Recall

Derived Facts

Inverse relationship between addition and subtraction
"7 - 2 is 5 because 5 + 2 is 7"

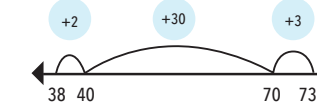
Using 10s: 13 - 8

"13 - 3 = 10 and 10 - 5 = 15"

Transitional (Place Value Parts)

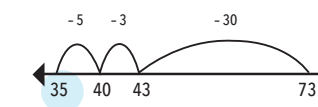
Transitional Strategies

Making efficient jumps by multiples of 10, 100, ...



$$73 - 38 = 35$$

Distance on number line



$$73 - 38 = 35$$

Jump back on number line

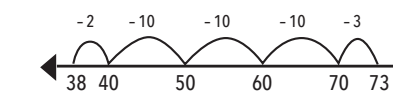
Early Transitional Strategies

Subtracting inefficiently with or without a model

Using Base 10 representations

Jumps by tens on number line

$$73 - 38 = 35$$



$$73 - 38 = 35$$

Adding up by tens

$$73 - 38 = ?$$

$$38 + 2 = 40$$

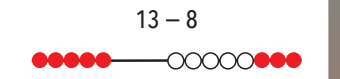
$$40 + 10 = 50$$

$$50 + 10 = 60$$

$$60 + 10 = 70$$

$$70 + 3 = 73$$

$$2 + 10 + 10 + 10 + 3 = 35$$



"13 - 3 is 10 and then 5 less is 5"

Unitizes on a model

Counting (Ones)

Counting Strategies

Counting back (7 - 5)

"7...6, 5, 4, 3, 2"

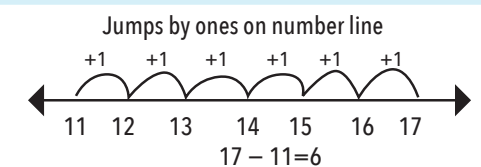


Counting up

"5...6, 7"

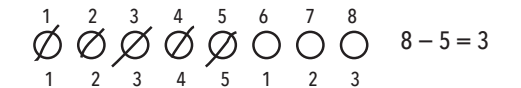
Counting back (7 - 5)

"7...6, 5...that's 2"



Early Counting Strategies

Direct modeling—separating from, separating to, or matching with model (counts 3 times)



Non-Additive Strategies

- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

Underlying Issues/Errors

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Subitizing, Unitizing, Commutativity, Number Composition, Base 10 understanding