### Session Number: 213

## Problem Solving + Problem Posing: Opening Access and Opportunities for All with Deeper Instruction

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Presented at NCTM 2023, Washington, DC Thursday, October 26, 2023, 2:45pm - 4:00pm DC Convention Center, 150B Link to original problems



https://tinyurl.com/3unmnvkw

During today's session, reflect on how you engaged in the problems. How can you pose problems in order to:

- Increase access to the mathematics?
- Invite all student voices into the problem solving?
- · Share authority over the mathematics?

**P1** 



"By recognizing and valuing the strengths and contributions of every student, we empower them mathematically in the classroom."

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$$

This work is supported by a grant from the National Science Foundation, award #1660809, ATMALA.

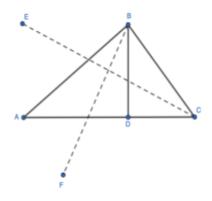
For more information, visit <a href="http://atmala.weebly.com">http://atmala.weebly.com</a>

## Think - Pair - Share

## P3 How can we pose this problem in a different way to:

- Increase access to the mathematics?
- Invite more student voices into the problem solving?

Let ABC be a right triangle with hypotenuse AC. Let BD be the altitude to the hypotenuse. Let CE be the angle bisector of  $\angle DCB$ , and BF be the angle bisector of  $\angle DBA$ . Prove  $EC \perp FB$ .



Link to slides



https://tinyurl.com/msrhhfd

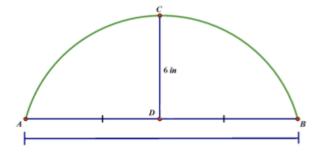
P4	Consider the following array		
	3	4	5
	5	12	13
	7	24	25
	9	40	41
	11	60	61

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NCTM 2023, Martinez-Cruz, Contreras, Turner, Session 213 October 26, Washington, DC

**P5** 



Link to slides



https://tinyurl.com/msrhhfd

#### **ORIGINAL PROBLEMS**

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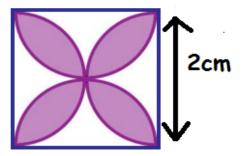
# Problem Solving + Problem Posing: Opening Access and Opportunities for All with Deeper Instruction

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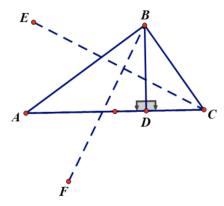
Link to slides: https://tinyurl.com/msrhhfd

1. Quietly and only in your head (<u>no writing</u>), think about how you can find the area of the purple region below. Once you have one strategy, try to think of more ways.



2. (The Mathematics Teacher, June 2011, C28). A point (x, y) is called integral if both x and y are integers. How many points on the graph of  $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$  are integral points?

3. Let ABC be a right triangle with hypotenuse AC. Let BD be the altitude to the hypotenuse. Let CE be the angle bisector of  $\angle DCB$ , and BF be the angle bisector of  $\angle DBA$ . Prove  $EC \perp FB$ .



- 4. Consider the following triples Consider the following triples
  - 3, 4, 5
  - 5, 12, 13
  - 7, 24, 25
  - 9, 40, 41
  - 11, 60, 61

...

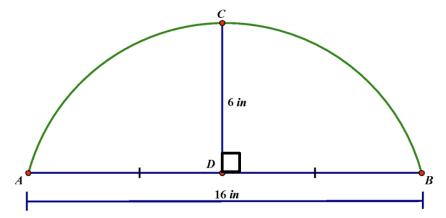
- a) Find several patterns.
- b) Can we find four integers a, b, c and d such that

$$a^2 + b^2 + c^2 = d^2$$
?

c) Five integers, a, b, c, d and e, such that

$$a^2 + b^2 + c^2 + d^2 = e^2$$
?

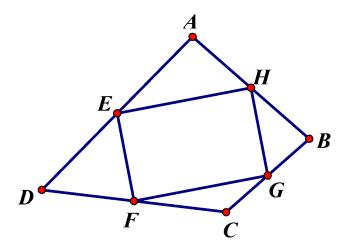
5. A moving company needs to replace a circular glass mirror that broke during a move. The company was able to determine the following information: AB is a chord that is 16in long. D is the midpoint of AB, and CD is a perpendicular segment to AB that is 6in long. Determine the length of the diameter.



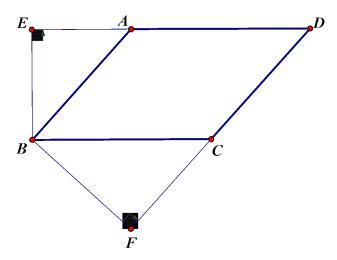
- 6. What is the smallest positive integer x such that 17 is a factor of x + 11 and 11 is a factor of x + 17? The Mathematics Teacher, January 2010, Calendar
- 7. Scott spent half of his monthly allowance in a jacket. Then he used one third of what was left to buy shoes. After these buying these two items he ended with \$60. How much was the jacket? How much money does he receive over two months?

8. Evaluate 
$$\sum_{k=1}^{200} (-1)^k k^2$$

- 9. Construct any quadrilateral A, B, C, D. Next, construct the midpoints of each side. Now, connect the four midpoints (clockwise or counterclockwise) to create a new quadrilateral. What can we say about this second quadrilateral?
- 10. Let ABCD be a quadrilateral. Let E, F, G, and H be the midpoints of its sides (as shown). Assume that quadrilateral EHGF is a rectangle. What can be said about the diagonals of quadrilateral ABCD. Prove it.



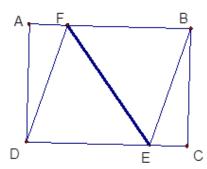
11. Let ABCD be a parallelogram. Assume EB is perpendicular to line AD, and BF is perpendicular to line CD. If AB = 4m, BC = 5m and AE = 3m, find CF.



12. (MT, 4 07, C25). Suppose x and y are positive rational numbers that satisfy

x + y = 5. What is the smallest possible value of the expression  $\frac{1}{x} + \frac{1}{y}$ ?

13. (Posamentier, & Salkind, 1988, 3.5 p. 12) On sides AB and DC of rectangle ABCD, points F and E are chosen so that FBED is a rhombus. Assume AB = 16m and BC = 12m, find EF.



14. In this figure,  $\overline{DF} \perp \overline{CE}$  and  $\angle CAB \cong \angle CDE$ . Determine the perimeter and area of triangle ABC. Units are yards.

