Math with "Bad" Words (& Phrases)

NCTM 2023

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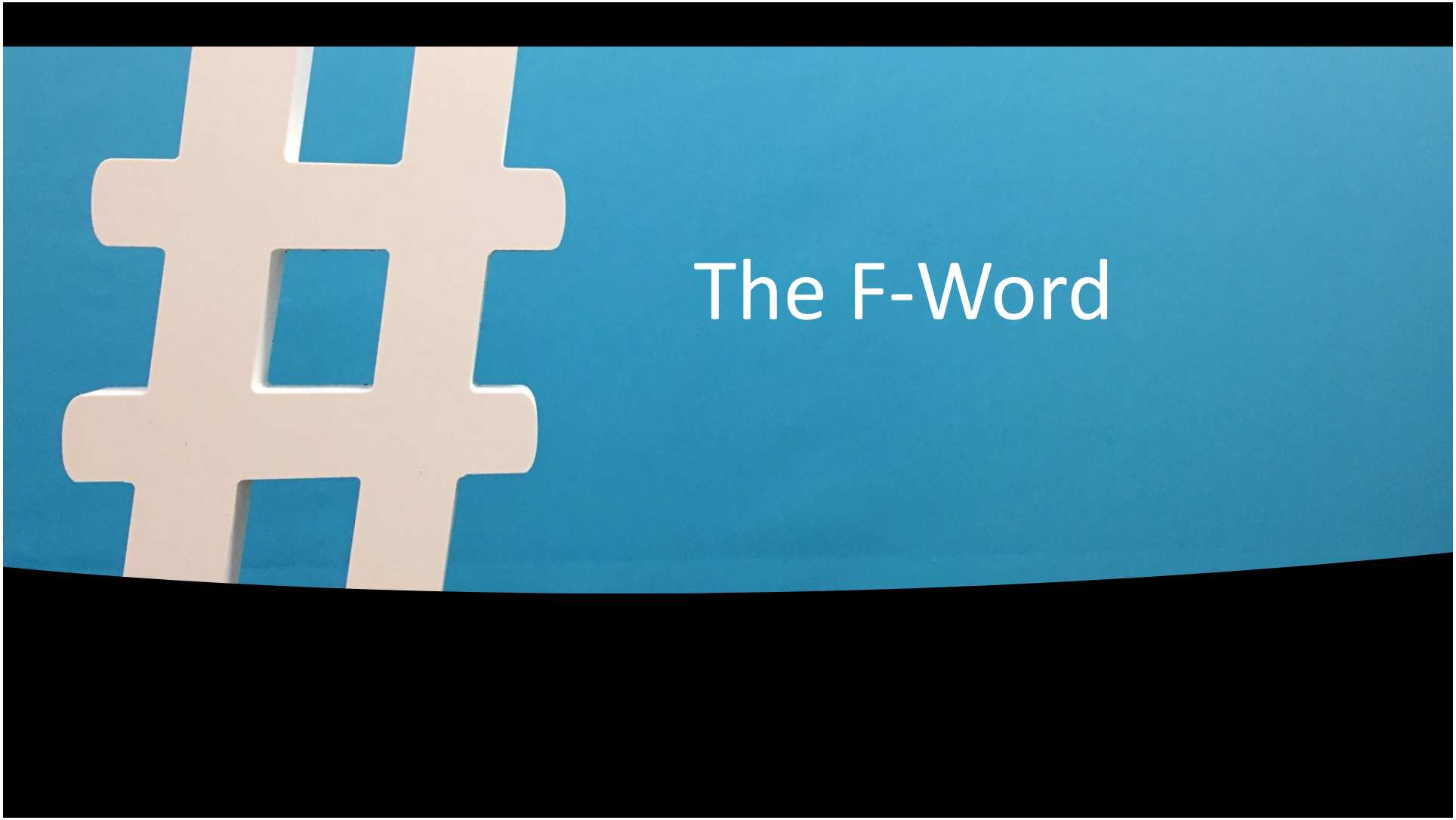
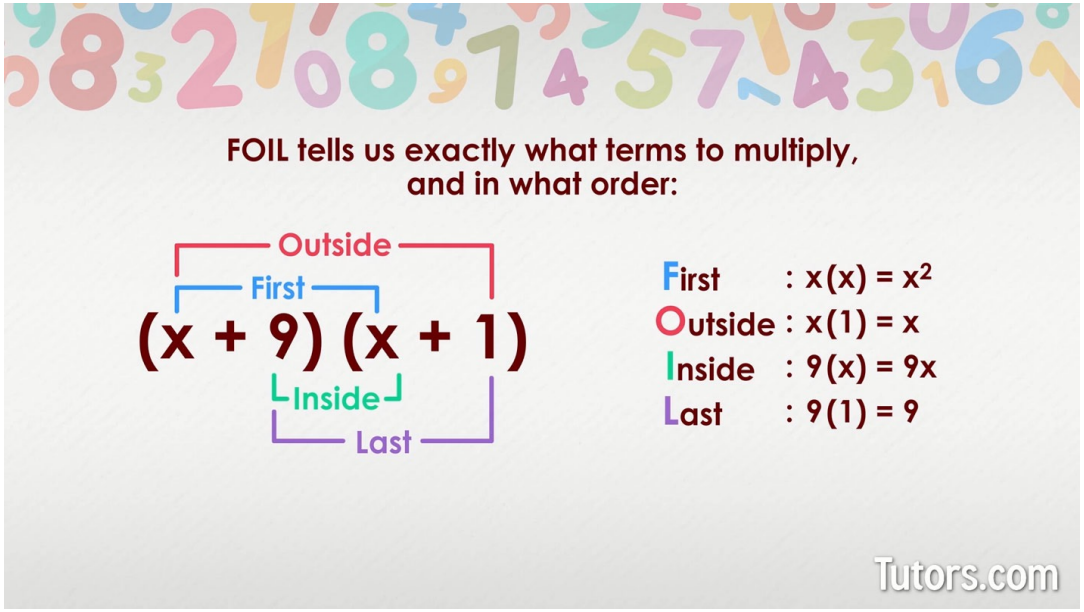
UNIVERSITY


The F-Word

FOIL



$(2x + 3y)(x - 2y)$	Original Problem
$(2x+3y)(x -2y)$	Multiply the First terms: $(2x)(x) = 2x^2$
$2x^2$	
$(2x +3y)(x -2y)$	Multiply the Outside terms: $(2x)(-2y) = -4xy$
$2x^2 -4xy$	
$(2x +3y)(x -2y)$	Multiply the Inside terms: $(3y)(x) = 3xy$
$2x^2 -4xy +3xy$	
$(2x +3y)(x -2y)$	Multiply the Last terms: $(3y)(-2y) = -6y^2$
$2x^2 -4xy +3xy -6y^2$	
$2x^2 -xy -6y^2$	Combine like terms: $-4xy +3xy = -xy$
$2x^2 -xy -6y^2$	Solution.

Pinterest.com

What the F-OIL????

To multiply two binomials, find the sum of the products of

F the *First* terms,
O the *Outer* terms,
I the *Inner* terms, and
L the *Last* terms.

EXAMPLE

4 **Find $(2x - 3)(x + 1)$.**

F O I L

$(2x - 3)(x + 1) = (2x)(x) + (2x)(1) + (-3)(x) + (-3)(1)$ FOIL method

$= 2x^2 + 2x - 3x - 3$ Multiply.

$= 2x^2 - x - 3$ Combine like terms.

The Distributive Property can be used to multiply any two polynomials.

EXAMPLE

5 **Find $(3x - 2)(2x^2 + 7x - 4)$.**

$(3x - 2)(2x^2 + 7x - 4) = 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4)$ Distributive Property

$= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8$ Distributive Property

$= 6x^3 + 17x^2 - 26x + 8$ Combine like terms.

Three special products are $(a + b)^2 = a^2 + 2ab + b^2$,
$(a - b)^2 = a^2 - 2ab + b^2$, and
$(a + b)(a - b) = a^2 - b^2$.

DISTRIBUTIVE PROPERTY!!!!

- $(3)(x+1)$
- $(x+9)(x+1)$
- $(x^2+9x+4)(x+1)$
- $(x^2+9x+4)(x^2-4x+1)$

All these expressions can be rewritten using the distributive property.

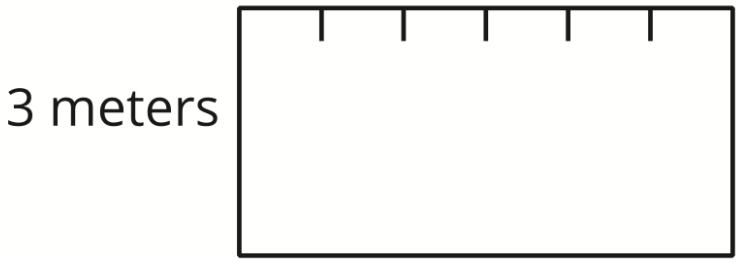
Why do we teach something different for multiplying two binomials?

What Can We Do Instead?

Let's Build from Elementary School

3rd grade

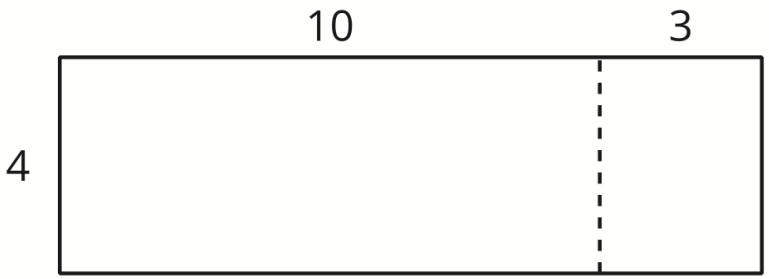
- Find the area

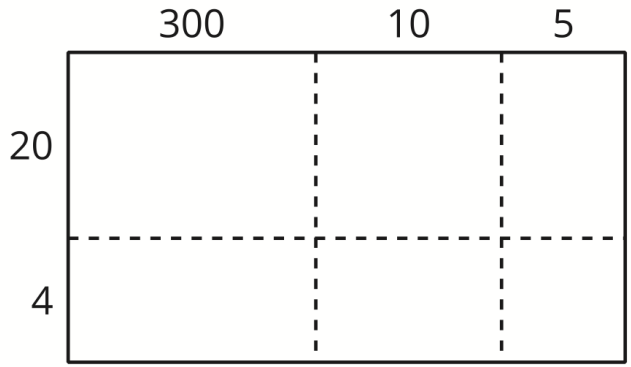


What is the area of this rectangle in square meters?

Source: Illustrative Mathematics Curriculum

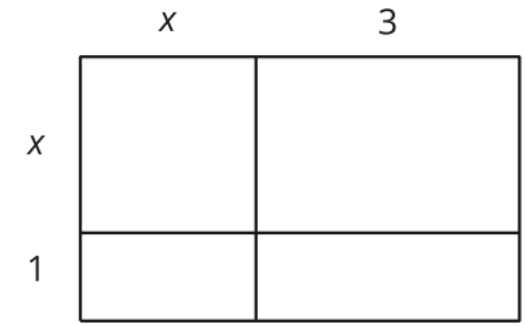
Moving to 4th and 5th Grade

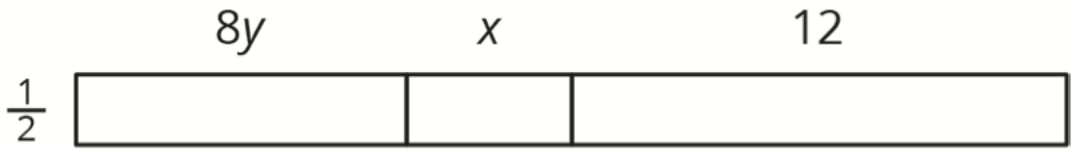




Source: Illustrative Mathematics Curriculum

How do the area diagrams support student learning?





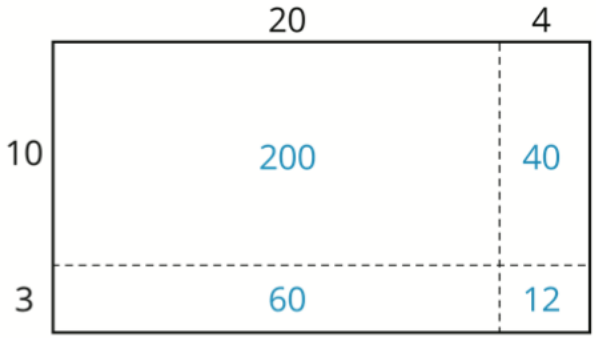


Diagram 1

$$
\begin{array}{r}
3x - 5 \\
2x + 1 \overline{)\,6x^2 - 7x - 5} \\
\underline{-6x^2 - 3x} \\
-10x - 5 \\
\underline{10x + 5} \\
0
\end{array}
$$

$2x$	$6x^2$	
1		

Source: Illustrative Mathematics Curriculum

Debrief

- How do the diagrams support student learning?
- How does “FOIL” impact student learning?

Diagrams support the understanding of mathematical concepts and procedures while helping students solve problems

If you want to FOIL…

PITMASTER'S CHOICE FOIL

- Extra Thick for Legendary BBQ
- 18" Widge for Large cuts of meat
- Made in USA

EVERYDAY FOIL

- Tough and durable for everyday meals
- Easy Open and stay closed box
- Made in USA

FOIL SHEETS

- Pre cut, single sheets
- Line Pans for Easy Cleanup
- Made in USA

RECYCLED FOIL

- Made of 100% Recycled Aluminum
- Same great quality as Everyday foil
- Easy Open and Stay Closed Box

GRILL FOIL

- Heavy Duty for Ultimate Grilling
- Food wont stick to non stick coating
- 18" Width Covers Grill Grates

HEAVY DUTY FOIL

- Thick and durable to prevent rips and tears
- Easy Open and Stay Closed Box
- Made in USA

NON-STICK FOIL

- Food wont stick to non stick coating
- Easy Open and Stay Closed Box
- Made in USA

https://www.reynoldsbrands.com/products/aluminum-foil


The C-Word

Cancel

$$3x = 9 \qquad \frac{x}{3} = 6$$

$$8x + 3 = 21 \qquad 2x - 3 = 19$$

“Cancel the three”

It gets worse…

Cancel…

Common Factors

Like terms $x^2 + 4x - 4x + 24$

The x's

$$-5x + 7y = 11$$
$$5x - 3y = -19$$

$$\frac{12}{20} = \frac{3 \cdot 4}{5 \cdot 4}$$

Multiplication $(-2)(-3) = 6$ The negatives "cancel"

Cancellation Law

In an algebraic structure A with a binary operation $\cdot$, the left and right cancellation laws respectively hold if for all x, y, z

$$x \cdot y = x \cdot z \Rightarrow y = z,$$

$$x \cdot y = z \cdot y \Rightarrow x = z.$$

Cancellation law. *Encyclopedia of Mathematics.* URL: http://encyclopediaofmath.org/index.php?title=Cancellation_law&oldid=37349

If you want to say "cancel" you must solve equations like this….

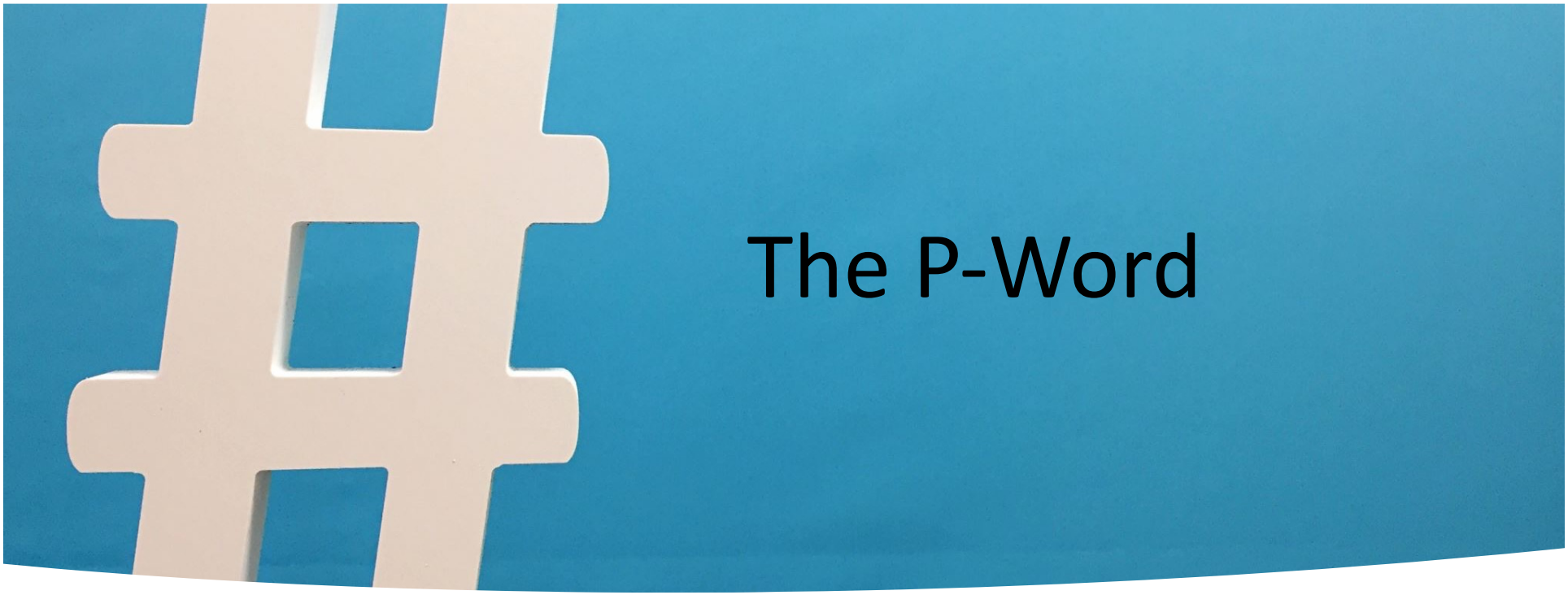
$$8x + 3 = 21$$
$$8x + \cancel{3} = 18 + \cancel{3}$$

$$3x = 9$$
$$\cancel{3} \cdot x = 3 \cdot \cancel{3}$$

What Can We Do Instead?

Name the mathematics students are doing

- Let's engage in the Mathematical Practice – Attend to Precision
- Try the Bad Word "Cancel" Activity by just practicing naming the mathematical operations to break the habit of using "cancel".


The P-Word

Plug In

» Find the value of the expression $8x + 3$, when $x = 6$.

» Let's "plug 6 in for x"

Other “Plug Ins”

- When solving an equation, how do you check your answer?
- When solving a system of equations for x, how do you find y?
- How do you evaluate a formula for particular values, such as A=(l)(w)?
- How do I evaluate a derivative at a point?
- When writing the table form of a function, how do you determine x and y values?
- When solving a system of equations, using substitution how you do write the equations with one variable?

What Can We Do Instead?

Substitution Property of Equality

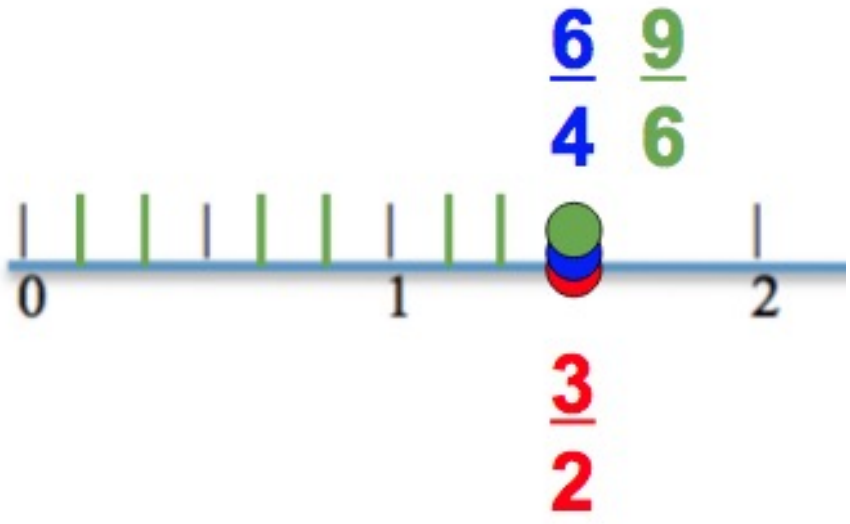
» If *a*=*b*, then *b* can be **substituted** in for *a*


The R-Word

Reduce

$$\frac{9}{6} \qquad \frac{6}{4} \qquad \frac{3}{2}$$

Did the value of the fraction really get smaller?



What Can We Do Instead?

Rewrite your fraction in an equivalent form

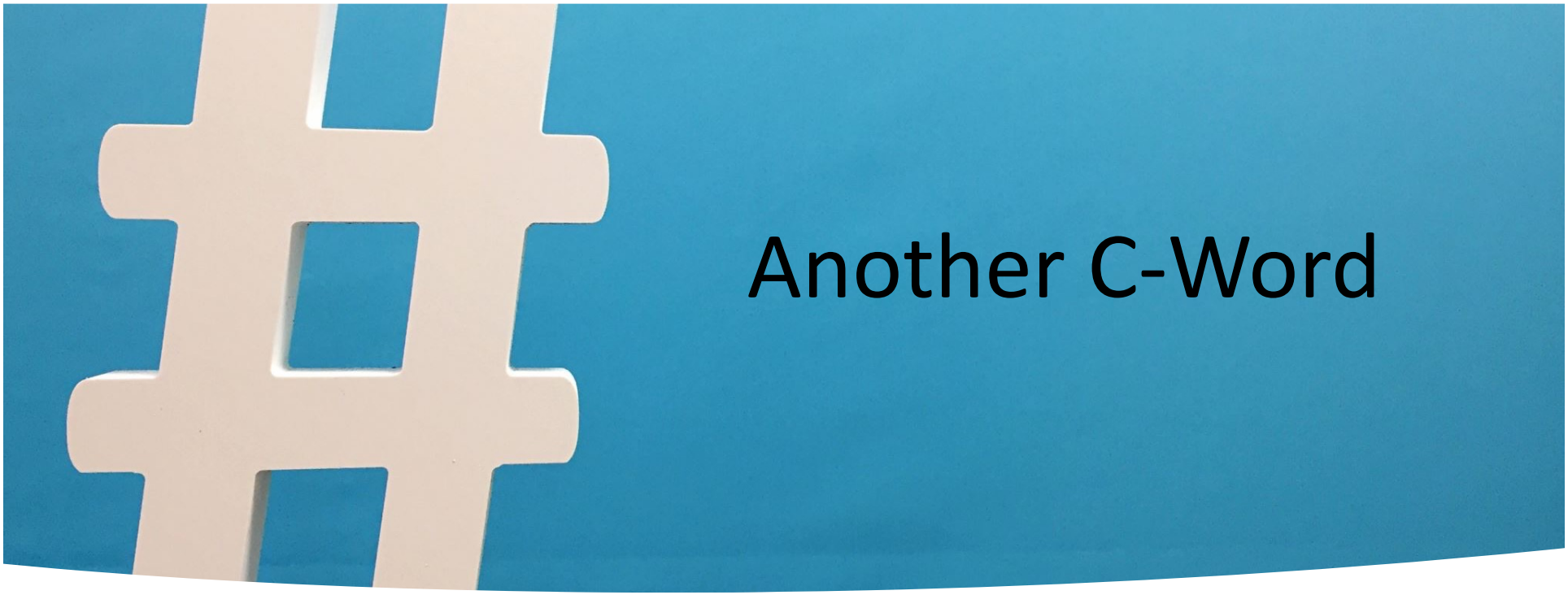
$$\frac{36}{48} = \frac{18}{24} = \frac{9}{12} = \frac{3}{4}$$

'Rewrite in an equivalent form' is really helpful…

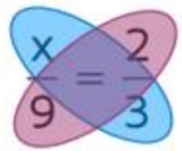
$3(x + 1)$

$x^2 + 4x - 4x + 24$

$2x - 3 = 19$


Another C-Word

Cross-Multiply



Cross-Multiply

$$(x)(3) = (2)(9)$$ Set the cross-products equal to each other.

$$\frac{3x}{3} = \frac{18}{3}$$ Simplify.

Divide both sides by 3 to get x by itself.

$$\boxed{x = 6}$$

Students will now cross-multiply this too.

$$\frac{4}{9} + \frac{2}{3}$$

Cross-Multiply….what???

$$\frac{2}{3} \div \frac{4}{5} = \frac{10}{12}$$

Process Addition or Subtraction using Cross-Multiply method.

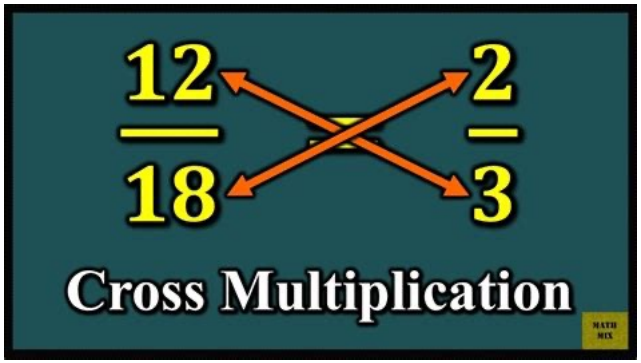
$$= \frac{1}{4} + \frac{2}{3}$$

$$= \frac{(1)(3)}{(4)(3)} + \frac{(2)(4)}{(4)(3)}$$

$$= \frac{3}{12} + \frac{8}{12}$$

$$= \frac{(3+8)}{12}$$

$$= \frac{11}{12}$$



Cross Multiplication confusion!!!

- » Create common errors for students.
- » "Short-cut" for a longer solution method.
- » Historically, developed from the 'rule of three' to teach students by rote.

  (Cocker's Artimetick, 1678)

- » Non-sensical with labels.

$$\frac{x}{7\text{ hours}} = \frac{90\text{ miles}}{3\text{ hours}}$$

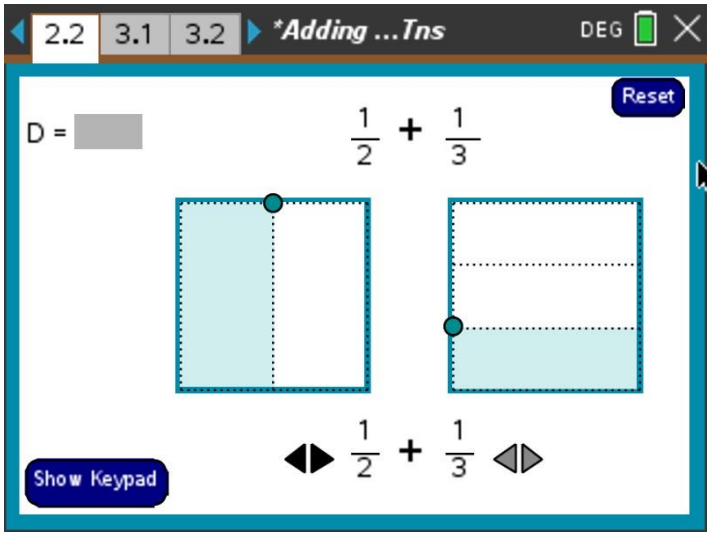
Cross-multiplying yields

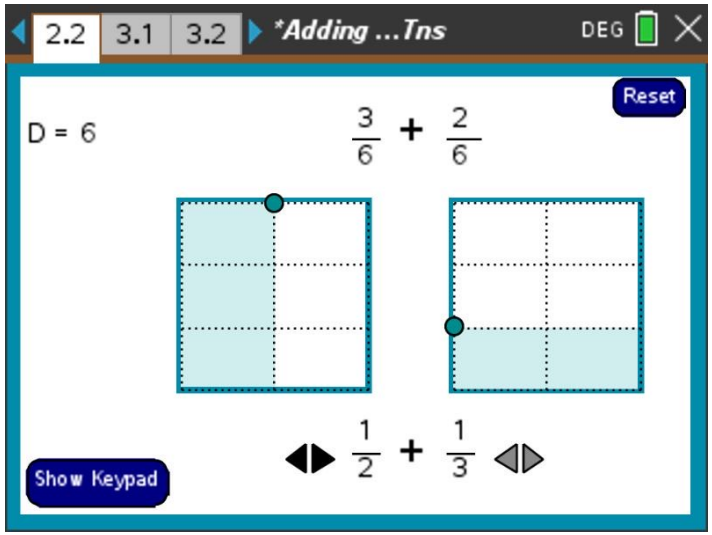
$$3 \cdot x\text{ hours} = 90 \cdot 7\text{ hours miles}$$

What Can We Do Instead?

Fixing Cross Multiply with Fractions

Start with visuals and let students build algorithms from a position of sense making.





Source: Building Concepts from education.ti.com

Fixing Cross Multiply with Solving Proportions

Proportions as equivalent Fractions **Proportions as solving equations**

$$\frac{x}{7} = \frac{15}{21}$$

$$\frac{7}{x} = \frac{21}{15}$$


An M-Phrase

Move the Decimal

$12.3 \cdot 10 = 123.0$

12.3 and 123.0

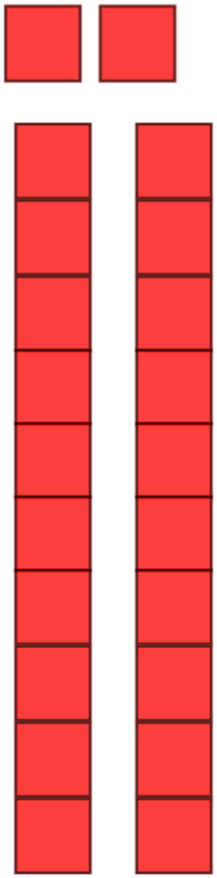
ones	tenths
ones	tenths

The decimal is **ALWAYS** between the ones and the tenths place!

The digits move because….

- 12.3 – the two is a ones digits
- 123.0 – the two is a tens digit

We've changed the place value each digit represents!!!



What Can We Do Instead?

Scientific Notation

- How can we rewrite 12.3×10^4 without saying "move the decimal"?
  - Which way do the digits move?

  - If I'm multiplying by a power of ten, then the digits move to the left four places…

    because their place value has grown by a factor of 10^4

$$12.3 \times 10^4 = 123{,}000$$

12.3×10^4

3	Tenths
2 ●	Ones
1	Tens
	Hundreds
	Thousands
	Ten thousands
	Hundred thousands
	Millions

12.3×10^4

0	Tenths
3 ●	Ones
2	Tens
1	Hundreds
	Thousands
	Ten thousands
	Hundred thousands
	Millions

12.3×10^4

0	Tenths
0 ●	Ones
3	Tens
2	Hundreds
1	Thousands
	Ten thousands
	Hundred thousands
	Millions

$$12.3 \times 10^4$$

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Tenths
		1	**2**	**3**	**0**	**0** ●	**0**

12.3×10^4

0	Tenths
0 ●	Ones
0	Tens
0	Hundreds
3	Thousands
2	Ten thousands
1	Hundred thousands
	Millions

Scientific Notation

- How can we rewrite 3.45×10^{-4} without saying "move the decimal"?
  - Which way do the digits move?

  - If I'm _____ by a power of ten, then the digits move to the _____ _____ places…

    because their place value has _____ by a factor of 10^{-4}

$$3.45 \times 10^{-4} = 0.000345$$

Scientific Notation

- How can I write 1,230,000 using scientific notation without saying "move the decimal"?

  Goal is to rewrite as $a \times 10^n$ where $1 < a < 10$.

  1.23×10^6

  - So, we need 1,230,000 to become $1.23 \times 10^?$

  - Question 1 – How many places did the place value change by?
    - Millions to ones is 6 places
  - Question 2 – Does the *a* value need to be bigger or smaller in standard form?
    - Bigger so my exponent must make the number bigger, 6

1,230,000

0	Hundredths
0	Tenths
0	Ones
0	Tens
0	Hundreds
0	Thousands
3	Ten thousands
2	Hundred thousands
1	Millions

1,230,000

0	Hundredths
0	Tenths
0 .	Ones
0	Tens
0	Hundreds
3	Thousands
2	Ten thousands
1	Hundred thousands
	Millions

1,230,000

0	Hundredths
0	Tenths
0 ●	Ones
0	Tens
3	Hundreds
2	Thousands
1	Ten thousands
	Hundred thousands
	Millions

1,230,000

0	Hundredths
0	Tenths
0 ●	Ones
3	Tens
2	Hundreds
1	Thousands
	Ten thousands
	Hundred thousands
	Millions

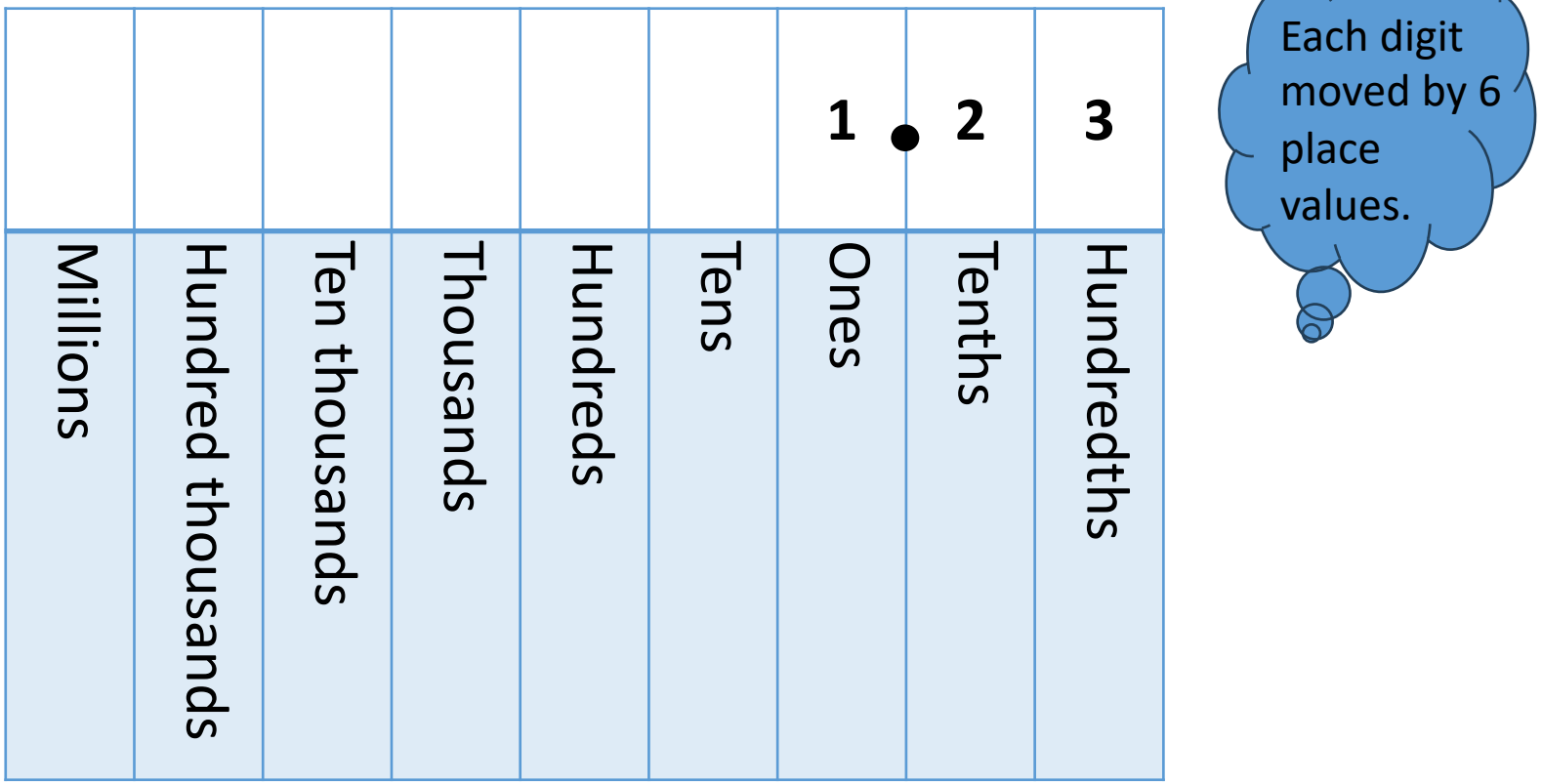
1,230,000

0	Hundredths
0	Tenths
3 ●	Ones
2	Tens
1	Hundreds
	Thousands
	Ten thousands
	Hundred thousands
	Millions

1,230,000

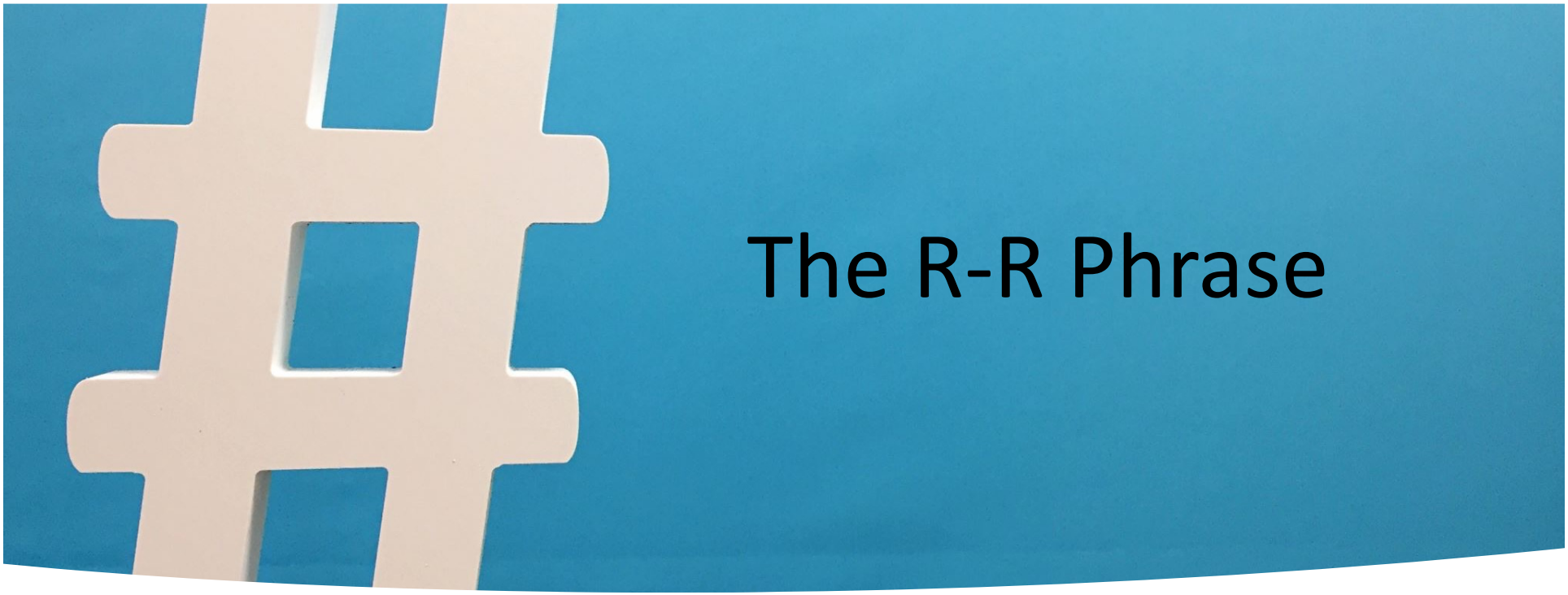
0	Hundredths
3	Tenths
2 ●	Ones
1	Tens
	Hundreds
	Thousands
	Ten thousands
	Hundred thousands
	Millions

1,230,000


Millions
Hundred thousands
Ten thousands
Thousands
Hundreds
Tens
Ones
Tenths
Hundredths
1
2
3
Each digit moved by 6 place values.

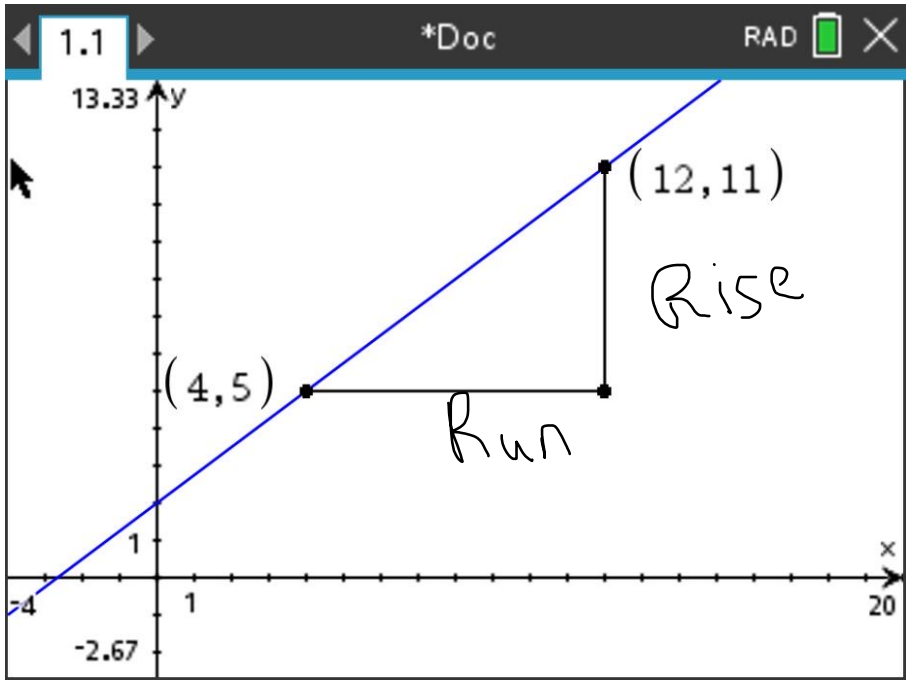
Move the Digits

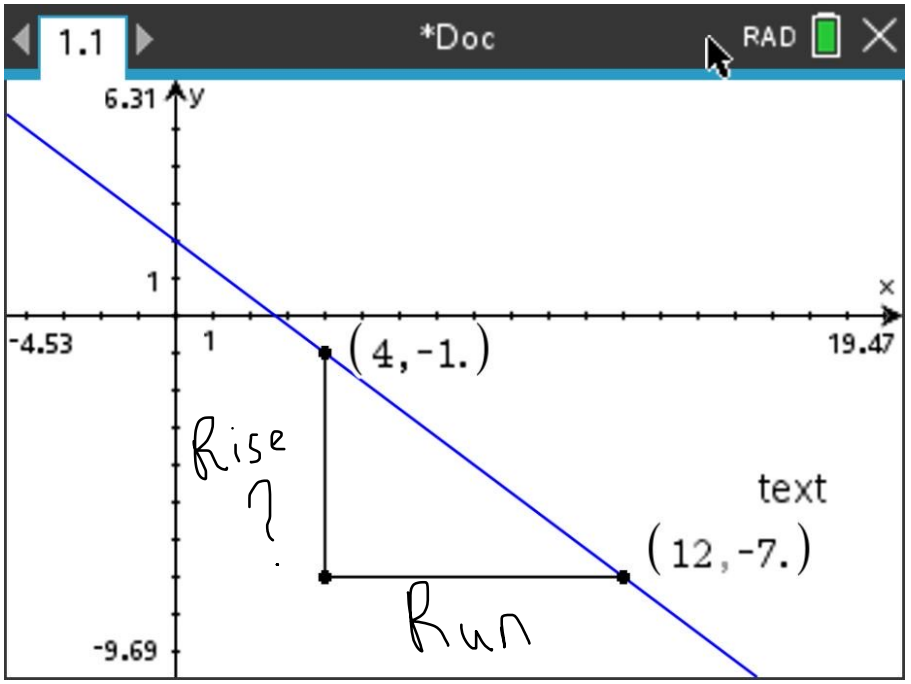
- Try the Bad Word “Move the Decimal” Activity to make sense of moving digits to their proper place value.



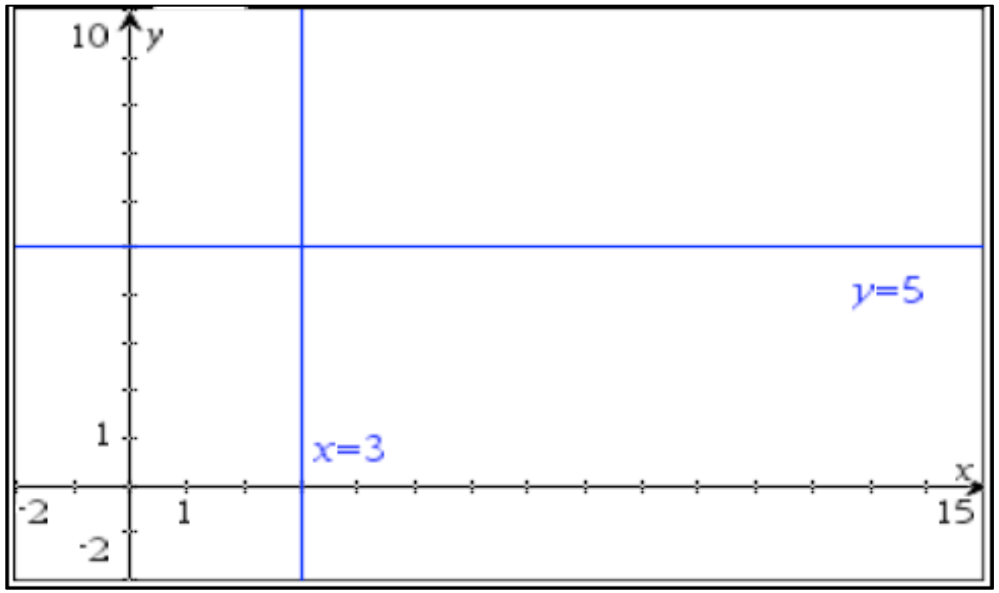
The R-R Phrase

Rise Over Run





Rise over Run



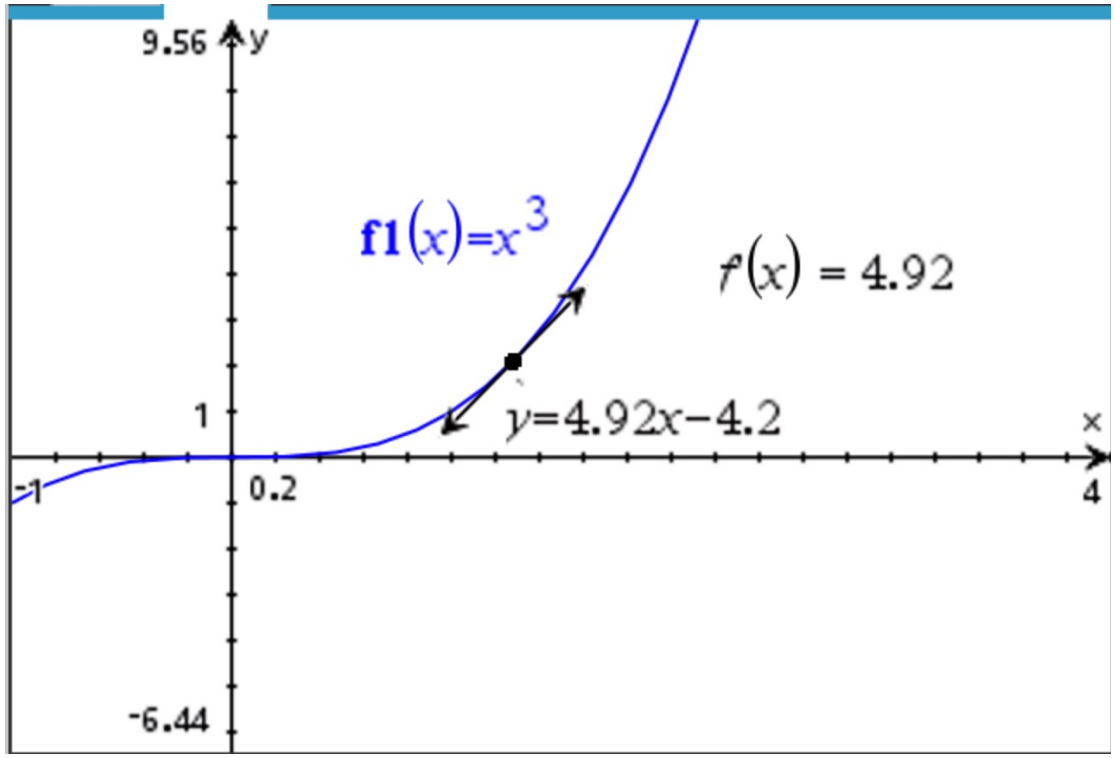
- Running with no rising means a zero slope?
- Rising with no running means an undefined slope?

Rise over Run?

x	y
1	4
2	6
3	8
4	10
5	12
6	14

Eventually….

There is no
Rise or Run



What Can We Do Instead?

Slope is a Rate

When there are *A* units of one quantity for every *B* units of another quantity, a *rate* associated with the ratio *A : B* is $\frac{A}{B}$ units of the first quantity per 1 unit of the second quantity. (Note that the two quantities may have different units.) The associated *unit rate* is $\frac{A}{B}$. The term *unit rate* is the numerical part of the rate; the "unit" is used to highlight the 1 in "per 1 unit of the second quantity." Unit rates should not be confused with unit fractions (which have a 1 in the numerator).

Progressions for Common Core State Standards – Ratio and Proportional Relationships, 2011

Making Sense of Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

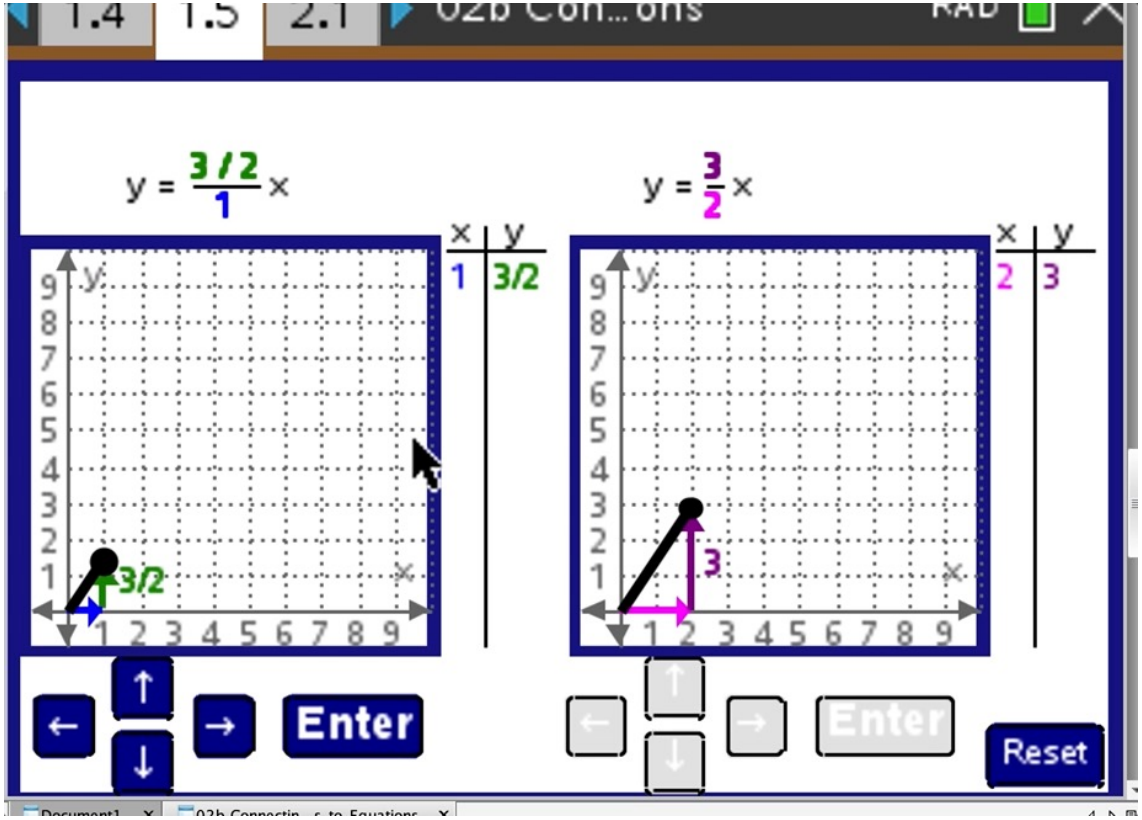
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y - y_1 = m(x - x_1)$$

Making Sense of Slope

7th grade

- A collection of equivalent ratios can be graphed in the coordinate plane.
- The graph represents a proportional relationship.
- The unit rate appears in the equation and graph as the slope of the line, and in the coordinate pair with first coordinate 1.

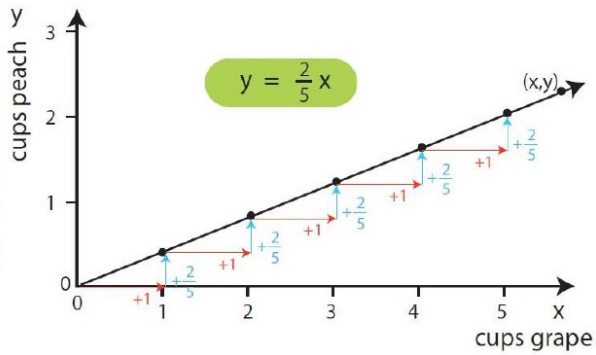


Making Sense of Slope

Correspondence among a table, graph, and equation of a proportional relationship

For every 5 cups grape juice, mix in 2 cups peach juice.

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$



On the graph: For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

When you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

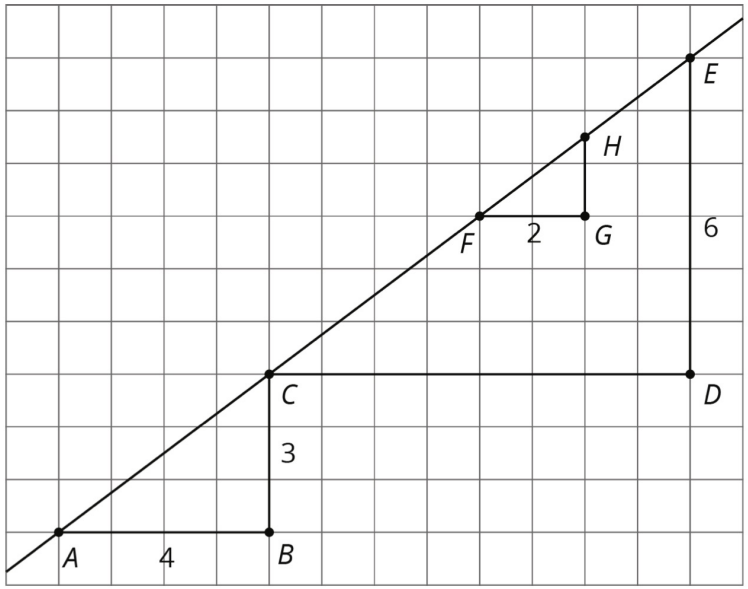
Starting from $(0, 0)$, to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$ $y = \frac{2}{5}x$

Progressions for Common Core State Standards – Ratio and Proportional Relationships, 2011

Making Sense of Slope

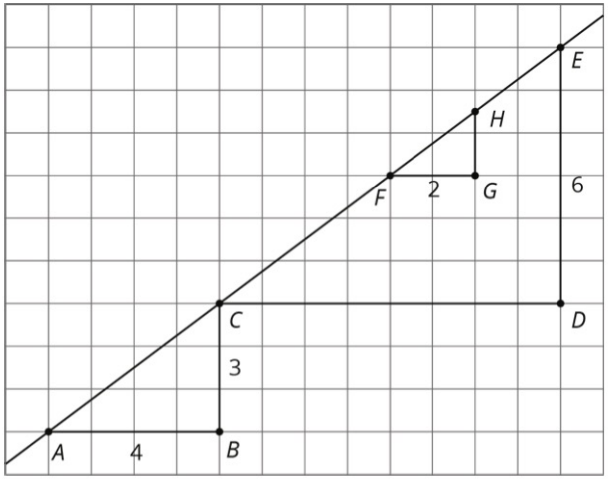
- What Mathematical Questions can you ask about this image?



Source: Illustrative Mathematics Curriculum

Meeting Slope

1. The figure shows three right triangles, each with its longest side on the same line. Your teacher will assign you two triangles. Explain why the two triangles are similar.



2. Complete the table.

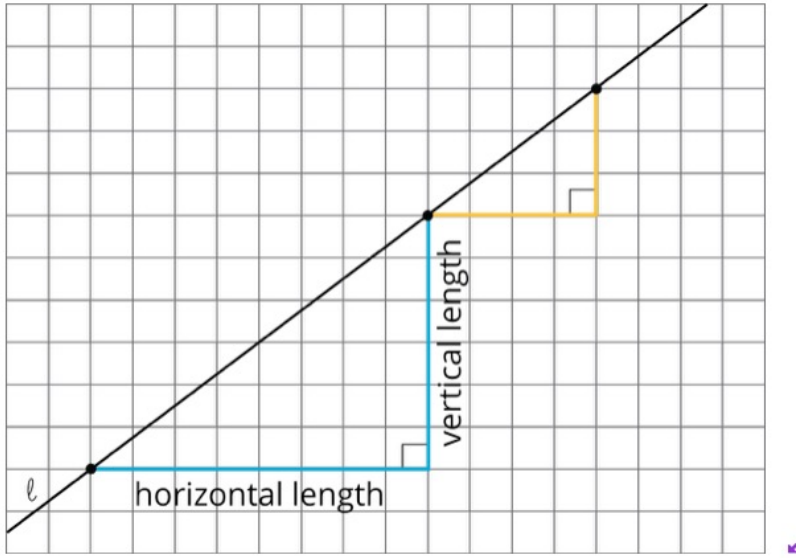
triangle	length of vertical side	length of horizontal side	(vertical side) ÷ (horizontal side)
ABC			
CDE			
FGH			

Source: Illustrative Mathematics Curriculum

Synthesizing Slope

- The quotients from the table are the same as finding the internal ratios of corresponding sides of similar triangles.
- Whenever we have a (non-vertical or non-horizontal) line, we can construct triangles where the quotient of length of the vertical side and the horizontal side will always be the same.
- This number is called **slope**

Slope is the vertical length ÷ horizontal length

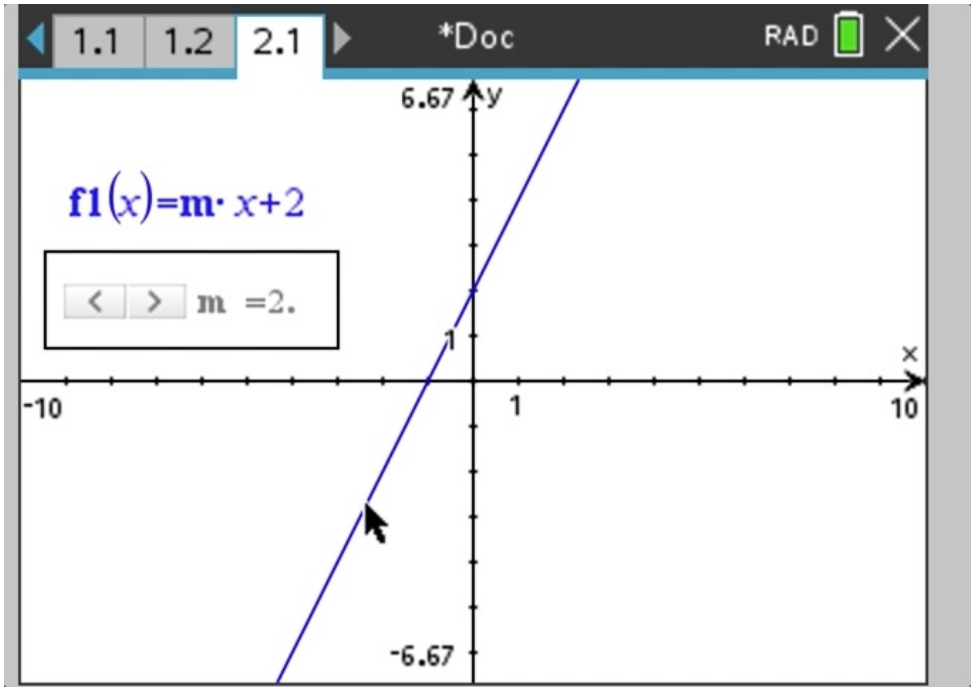


Note: This is a mathematical convention to define slope in this way

Source: Illustrative Mathematics Curriculum

Explore Negative Slope Dynamically

What Do You Notice/Wonder?



Address Negative Slope in Context

Noah put $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after he takes

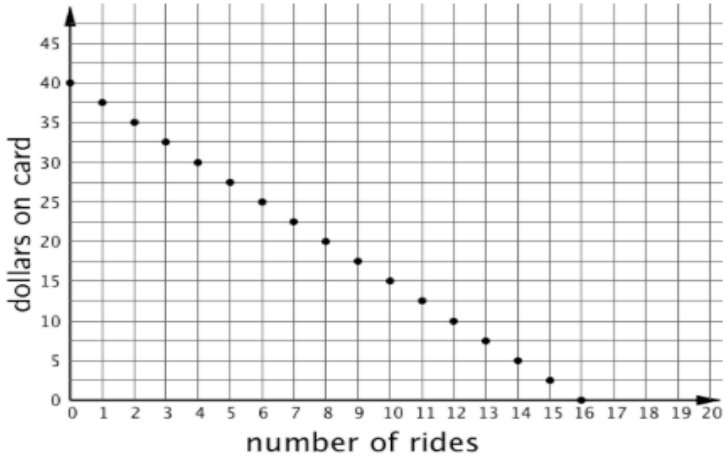
   a. 0 rides?

   b. 1 ride?

   c. 2 rides?

   d. x rides?

2. Graph the relationship between amount of money on the card and number of rides.



Why does it make sense to say the slope of this graph is – 2.5 rather than 2.5?

Source: Illustrative Mathematics Curriculum

If you want to use Rise over Run….

A Final C Phrase

Clear (the fraction)

By what number would you multiply to clear the fraction?

$$\frac{1}{4}x+2=\frac{3}{4}x$$

Clear (the decimal)

$$0.25x + 2 = 0.75x$$

$$100(0.25x + 2) = 100(0.75x)$$

$$25x + 200 = 75x$$

$$25.x + 200. = 75.x$$

What Can We Do Instead?

Clear….

» Rewrite the  in an equivalent form.

BAD WORDS ROUND UP

STOP IMMEDIATELY

- FOIL
- CANCEL
- PLUG IN
- REDUCE
- CROSS MULTIPLY
- MOVE THE DECIMAL
- RISE OVER RUN
- CLEAR…

USE INSTEAD

- DISTRIBUTE
- NAME THE OPERATION
- SUBSTITUTE
- REWRITE
- NOTHING!
- MOVE THE DIGITS
- RATE OF CHANGE
- REWRITE

Key Takeaway

Use Proper Academic Vocabulary

For the benefit of helping **students**

MAKE SENSE

of Mathematics



Thank you

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